

Nonlinear Robust Controller for Rotating Stall and Surge in Axial Flow Compressors

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Abstract

In this paper, a robust globally asymptotically stabilizing feedback law is proposed for the control of rotating stall and surge in axial flow compressors. This is done in two steps. First, a globally asymptotically stabilizing parameterized feedback is proposed. Then a numerical design procedure is developed to compute the feedback parameters in accordance with some pre-specified robustness criterions. Simulations are proposed to validate the proposed schemes.

Keyword Nonlinear system, Robust stabilization, Axial flow compressors, Rotating Stall and Surge.

1 Introduction

Rotating stall and surge are two flow instabilities occurring in engine compression systems. They may dramatically decrease engine's effectiveness or even cause damage to the engine's components.

Typically, rotating stall is a local two-dimensional compression system oscillation characterized by regions of flow that rotate at a fraction of the compressor rotor speed while surge is an axisymmetric one-dimensional global compression system oscillation that involves axial flow oscillation.

For maximum compressor performance, operating conditions are such that the pressure rise in the compressor corresponds in the (flow, pressure)-plan to the maximum pressure on the stable axisymmetric branch corresponding to a given throttle opening. In practice, however, because of unavoidable uncertainties and external disturbances, this operating point may be perturbed into instable regions causing the instabilities mentioned above.

That is the reason why, in recent years, the development of integrated control system has led to a significant activity in modeling [1, 2, 3, 4] and control [5, 6, 7] and more recently in [8, 9, 10] of flow compression systems. An overview of related works

can be found in [11, 12].

Recently a robust control design approach initially developed in [13] has been applied to robustly control rotating stall and surge in axial flow compressors [14]. The performances of the resulting closed loop system have been compared to many existing stabilizing feedbacks [15, 10, 6]. This comparison clearly shows that robust design is highly beneficial in terms of performance under disturbances and/or system uncertainties.

The justification of the present work lies in the following points :

- The basic idea behind the design procedure used in [14] is to put the system into the following triangular form

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \quad ; \quad x_1 \in \mathbb{R}^2 \\ \dot{x}_2 &= u \quad ; \quad x_2 \in \mathbb{R} \end{aligned}$$

and by using x_2 as controller for sub-state x_1 , a feedback law is designed such that in the ideal case, one has $x_2 = \alpha(x_1)$. Then u is used to regulate the manifold $z := x_2 - \alpha(x_1) = 0$. Robustness is therefore defined by the "amount of decoupling" that is possible to achieve between some external disturbances w and the regulated variable z .

It is clear however, that the decoupling one really needs to perform is the one between the external disturbances w and some regulated output $z = h(x)$ that may reflect concrete specifications concerning the dynamic of the system's state. There is obviously no reason for the choice $z = x_2 - \alpha_1(x)$ to express such considerations.

- The feedback proposed in [14] is quite complicated and a deep comprehension of the technical background of [13] is necessary to understand the design procedure.

This paper is organized as follows. First, system equations are recalled in section 2. In section 3,

a globally asymptotically stabilizing feedback law is proposed. This law is defined by four parameters. Section 4 is then dedicated to a numerical algorithm that enables a robustness-oriented choice of these parameters. Simulations are finally proposed in section 5 in order to illustrate the pertinence of the whole design scheme.

2 The system's model

The post-stall transients in axial flow compressors can be reasonably captured by the one-mode Galerkin approximation model proposed in [3]

$$\dot{A} = -\frac{\sigma}{2}A \left[\frac{1}{4}A^2 + 2\Phi + \Phi^2 \right] \quad (1)$$

$$\dot{\Phi} = -\frac{3}{2}\Phi^2 - \frac{1}{2}\Phi^3 - \frac{3}{4}A^2 \left[1 + \Phi \right] - \Psi \quad (2)$$

$$\dot{\Psi} = -u \quad (3)$$

where A is the normalized stall cell amplitude of angular variation capturing the nonuniformity in the flow, Φ is the circumferentially averaged axial mass flow while Ψ is the total-to-static pressure rise. u is the control variable given by : $u = \frac{1}{\beta^2}(\Phi - \Phi_T)$ where Φ_T is the mass flow through the throttle. The objective is to globally asymptotically stabilize the equilibrium $(A, \Phi, \Psi) = 0$.

3 Design of a parameterized globally asymptotically stabilizing feedback law

Let us define the following reference functions :

$$\Phi_r \quad : \quad \text{Solution of} \quad \Phi^2 + 2\Phi = \lambda_A \pi_\eta(A) \quad (4)$$

$$\Psi_r \quad : \quad \text{such that}$$

$$\frac{d}{dt}(\Phi - \Phi_r) = -\lambda_\phi(\Phi - \Phi_r) \quad (5)$$

whenever $\Psi = \Psi_r$ in (2)

where $\pi_\eta(\cdot)$ is the function depicted on Figure 1 (typically $\pi_\eta(A)$ is to be used with small η in order to accelerate the convergence of A for small values of $A > \eta$). Now, considering the new coordinate system :

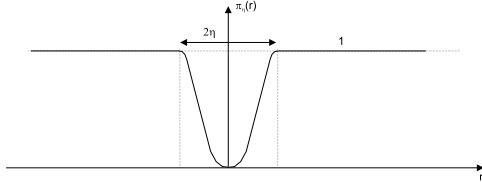


Figure 1: Definition de $\pi_\eta(\cdot)$

$$A; \quad e_\phi = \Phi - \Phi_r; \quad e_\psi = \Psi - \Psi_r$$

together with the state feedback defined by :

$$u = -\dot{\Psi}_r(A, \Phi) + \lambda_\psi \left[\Psi - \Psi_r(A, \Phi) \right] \quad (6)$$

the system equations become :

$$\begin{aligned} \dot{A} &= -\frac{\sigma}{2}A \left[\frac{1}{4}A^2 + \lambda_A \pi_\eta(A) + e_\phi^2 \right] \\ &\quad - \sigma A e_\phi \sqrt{1 + \lambda_A \pi_\eta(A)} \end{aligned} \quad (7)$$

$$\dot{e}_\phi = -\lambda_\phi e_\phi - e_\psi \quad (8)$$

$$\dot{e}_\psi = -\lambda_\psi e_\psi \quad (9)$$

It is then clear by definition of e_ϕ and e_ψ that the asymptotic stability of the closed loop system (7)-(9) is equivalent to that of the original system (1)-(3) under the control law (6).

Proposition 1

The feedback law (6) globally asymptotically stabilizes the origin $(0, 0, 0)$ of the dynamical system (1)-(3). Furthermore, the following is a Lyapunov function for the corresponding closed loop system :

$$V := \frac{1}{2} \left[\mu_A A^2 + \mu_\phi e_\phi^2 + e_\psi^2 \right] \quad (10)$$

whenever μ_ϕ and μ_A satisfy

$$\mu_\phi \leq \frac{\lambda_\phi \lambda_\psi}{4} \quad ; \quad \frac{\mu_A}{\mu_\phi} \leq \frac{\lambda_\phi}{16(1 + \lambda_A)\sigma} \quad (11)$$

4 Robustness-oriented parameter tuning

The feedback proposed in the preceding section is defined by the four positive parameters $\lambda_A, \lambda_\phi, \lambda_\psi$ and η . The aim of the present section is to propose a numerical algorithm enabling these parameters to be chosen in accordance with some pre-specified robustness criterion.

4.1 General theoretical background

Consider a perturbed parameterized closed loop system of the form :

$$\dot{x} = f(x, p) + g(x)\delta \quad (12)$$

where $f(x, p)$ is a **nominal asymptotically stable** closed loop dynamic **depending on some parameter vector** $\mathbf{p} \in \mathbb{R}^{n_p}$ and $\delta \in \mathbb{R}^{n_\delta}$ stands for a vector of disturbances and/or uncertainties.

Let some penalty function $L(x, p, \delta)$ be given such that the corresponding Hamiltonian :

$$H(x, \lambda, p, \delta) := L(x, p, \delta) + \lambda^T \left[f(x, p) + g(x)\delta \right] \quad (13)$$

admits a maximum **as a function of** δ , namely :

$$\hat{\delta}(x, \lambda, p) := \text{Arg} \max_{\delta \in \Delta} H(x, \lambda, p, \delta) \quad (14)$$

where $\Delta \subset \mathbb{R}^{n_s}$ is some admissible set of disturbances.

Denote by $\hat{H}(x, \lambda, p)$ the corresponding maximal value. It is then a known fact [16, 17] that if a positive definite function $V(x)$ can be found such that :

$$\hat{H}\left(x, \frac{\partial V}{\partial x}, p\right) \leq 0 \quad (15)$$

for all x , then the closed loop system (12) satisfies the following property for all $x(0) \in \mathbb{R}^n$, for all $t \geq 0$ and for all admissible disturbances and/or uncertainties $\delta(\cdot) \in \Delta^{[0, t]}$:

$$\begin{aligned} \int_0^t L(x(\tau), p(\tau), \delta(\tau)) d\tau &\leq V(x(0)) - V(x(t)) \\ &\leq V(x(0)) \end{aligned} \quad (16)$$

Consider some regulated output

$$z := h(x, p, \delta) \quad \text{s.t} \quad h(0, p, 0) = 0$$

to be "decoupled" from the disturbances and/or uncertainties. Then L is given by

$$L_\gamma(x, p, \delta) := h^T(x, p, \delta)h(x, p, \delta) - \gamma^2 \delta^T \delta \quad (17)$$

and using (16) shows that starting from the regulated position $x(0) = 0$, the L_2 -gain is bounded by γ^2 , that is, for all $t \geq 0$

$$\boxed{\int_0^t z^T(\tau)z(\tau)d\tau \leq \gamma^2 \int_0^t \delta^T(\tau)\delta(\tau)d\tau} \quad (18)$$

4.1.1 Robustness Oriented tuning of p : Let $V_{nom}(x, p)$ be a Lyapunov function for the nominal closed loop system $\dot{x} = f(x, p)$. The key point is to search a parameter vector p such that (15) holds for $V = V_{nom}$. Therefore, if we define $\Theta_\gamma(x, p)$ by :

$$\Theta_\gamma(x, p) := \hat{H}_\gamma\left(x, \frac{\partial V_{nom}}{\partial x}(x, p), p\right) \quad (19)$$

then the following defines an ideal choice for p , say p_{opt} :

$$p_{opt} := \hat{p}(\gamma_{opt}) \quad (20)$$

where

$$\begin{aligned} \hat{p}(\gamma) &:= \text{Arg} \left\{ \min_{p \in \mathcal{P}} \max_{x \in \mathcal{X}} \Theta_\gamma(x, p) \right\} \\ \gamma_{opt} &:= \min \left\{ \gamma \geq 0 \text{ s.t. } \max_{x \in \mathcal{X}} [\Theta_\gamma(x, \hat{p}(\gamma))] \leq 0 \right\} \end{aligned} \quad (21)$$

where \mathcal{P} is some admissible set for p contained in the one that insures the stability of the nominal closed loop system while \mathcal{X} is some subset of interest of \mathbb{R}^n (typically, a neighbourhood of the origin).

4.2 The design procedure's implementation

The key idea is to consider discrete sets \mathcal{P} and \mathcal{X} and to perform the optimization involved above off line by an exhaustive search over these discrete sets. In the case of the axial flow compressor where \mathcal{P} and \mathcal{X} are of dimension 4 and 3 respectively, this is still quite feasible even for a relatively dense grid.

More precisely, let \mathcal{P} and \mathcal{X} be defined as follows :

$$\begin{aligned} \mathcal{P} &:= \left\{ 0.1, 1.2, 2.3, \dots, 10 \right\}^3 \times \left\{ 0.01, 0.0644, \dots, 0.5 \right\} \\ \mathcal{X} &:= \left\{ -1, -0.5, -0.05, 0, +0.05, +0.5, +1 \right\}^3 \end{aligned}$$

Assuming that the elements of \mathcal{P} and \mathcal{X} are indexed, equations (20)-(21) become $p_{opt} := \hat{p}(\gamma_{opt})$ with $\hat{p}(\gamma) := p^{j_0}$ where

$$\begin{aligned} j_0 &:= \text{Arg} \left\{ \min_{j=1}^{\text{card}(\mathcal{P})} \max_{i=1}^{\text{card}(\mathcal{X})} \Theta_\gamma(x^i, p^j) \right\} \\ \gamma_{opt} &:= \min \left\{ \gamma \geq 0 \text{ s.t. } \max_{i=1}^{\text{card}(\mathcal{X})} [\Theta_\gamma(x^i, \hat{p}(\gamma))] \leq 0 \right\} \end{aligned}$$

The following algorithm use a dichotomic search for γ_{opt} and hence $p_{opt} = \hat{p}(\gamma_{opt})$:

ALGORITHM

1. $\gamma_{max} \leftarrow 10, \gamma_{min} \leftarrow 0.01,$
 2. $\varepsilon = 1.0e - 6$
 3. **While** $(|\gamma_{max} - \gamma_{min}| > 0.01)$ **do**
 - (a) $\gamma \leftarrow (\gamma_{max} + \gamma_{min})/2$
 - (b) **If** $\left(\max_{i=1}^{\text{card}(\mathcal{X})} \Theta_\gamma(x^i, \hat{p}(\gamma)) \leq 0 \right)$ **then**
 $\gamma_{max} \leftarrow \gamma$ **else** $\gamma_{min} \leftarrow \gamma$ **end if**
 4. **End while**
-

4.3 Application to the axial flow compressor

4.3.1 Equations rewriting: The theoretical background above is applied to the control of the axial flow compressor using the following correspondences : $x := (A, \Phi, \Psi)^T, p := (\lambda_A, \lambda_\phi, \lambda_\psi, \eta)^T$ and

$$\begin{aligned} V_{nom}(x, p) &:= \frac{1}{2} \left[\mu_A(p)x_1^2 + \mu_\phi(p) \left(x_2 - \Phi_r(x_1, p_1) \right)^2 + \right. \\ &\quad \left. \left(x_3 - \Psi_r(x_1, x_2, p_1, p_2) \right)^2 \right] \quad [\text{see (10)}] \\ \mu_\phi(p) &:= \frac{p_2 p_3}{4} \quad ; \quad \mu_A(p) := \frac{p_2^2 p_3}{64(1 + p_1)\sigma} \end{aligned}$$

As for the nominal closed loop system $\dot{x} = f(x, p)$, it is readily given by (1)-(3) in which u is given by (6).

4.3.2 Robustness related definitions: The regulated output z with respect to which robustness is to be obtained is taken of the following form : $z := h(x, p, \delta) := [c_1 x_1, c_2 x_2, c_3 x_3, c_4 f_3(x, p)]^T$ s.t. $\sum_{i=1}^4 c_i^2 = 1$. It remains to define the way disturbances and uncertainties affect the system dynamic. For, we consider additive disturbances. Namely, in (12), $g(x)$ and δ are given by :

$$g(x) = \text{diag} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} ; \delta := \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} ; \Delta := \mathbb{R}^3 \quad (22)$$

This is a classical disturbance rejection formulation. In this case, $\hat{\delta}(x, \frac{\partial V_{nom}}{\partial x}, p)$ is given by [see (14)] :

$$\hat{\delta}(x, \frac{\partial V_{nom}}{\partial x}, p) := \frac{1}{2\gamma^2} \begin{pmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{pmatrix} \frac{\partial V_{nom}^T}{\partial x} \quad (23)$$

Two situations are then considered :

- In the first, the disturbance signals $w_i(t)$, for $i = 1, \dots, 3$ are given functions of time [see (??)].
- In the second, w is taken exactly equal to the worst case, that is

$$w(t) = w(x(t), p) := \hat{\delta}(x, \frac{\partial V_{nom}}{\partial x}, p) \quad [\text{see (23)}]$$

w is therefore a state dependent disturbances.

5 Simulations

In all the following simulations, the behaviour of the closed loop system is compared to the result of [14] when the same fixed parameters proposed in that paper are used. These results are used as a reference behaviour that enables the effect of different choices of z on the closed loop evolution to be demonstrated. This permits a validate of the design procedure.

It is important to note however that, since no effort is done here to tune the parameters of the feedback law proposed in [14], the following simulations cannot be used to compare the two robust feedback laws.

5.1 Time-varying state-independent disturbances

Here, equations (22) and (17) are considered. The following cases are simulated depending on the value of the penalty vector c used to define the regulated output z :

c	γ_{opt}	P_{opt}
(0.25, 0.25, 0.25, 0.25)	2.88	(2.3, 1.2, 6.7, 0.45)
(0.99, 0, 0, 0.01)	0.50	(1.2, 0.1, 8.9, 0.17)
(0, 0.99, 0, 0.01)	0.79	(0.1, 1.2, 3.4, 0.34)
(0, 0, 0.99, 0.01)	0.50	(0.1, 0.1, 4.5, 0.06)

Table 1: Results for the disturbance rejection-based tuning

‡ **Case 1:** $c := (0.25, 0.25, 0.25, 0.25)$

‡ **Case 2:** $c := (0.99, 0, 0, 0.01)$

‡ **Case 3:** $c := (0, 0.99, 0, 0.01)$

‡ **Case 4:** $c := (0, 0, 0.99, 0.01)$

Table 1 shows the results obtained by the algorithm proposed above in the different cases under consideration. The disturbance profiles are those used in [14], namely, $w_1(t) := e^{-0.5t} \sin(t)$, $w_2(t) := e^{-0.05t} \sin(0.7t)$ and $w_3(t) := e^{-0.5t} \sin(1.2t)$. The dynamic behaviour of the closed loop system in the four cases defined above are showed on Figures 2-5 while Figure 6 shows that the basic L_2 -robustness inequality (16) is satisfied.

The following remarks can be done when examining the above figures

- Figure 2 shows that the proposed feedback law is at least comparable to the one proposed in [14] since all the quantities seem to be equally or less perturbed. Recall that the comparisons done in [14] showed that many other non robust feedback laws exhibited very poor performances.
- Figures 3-5 show that the choice of z leads to the expected consequences on the closed loop behaviour. More precisely, the state components A (on Figure 3), Φ (Figure 4) and Ψ (Figure 5) are better decoupled from w when the vector c used to define z is given by $c = (0.99, 0.0, 0.0, 0.01)$, $c = (0.0, 0.99, 0.0, 0.01)$ and $c = (0.0, 0.0, 0.99, 0.01)$ respectively.
- Figure 6 shows that the basic robustness inequality is satisfied.
- In case 3 where only Φ is decoupled, the convergence of A becomes very slow. That is because in the spirit of the feedback law's design, it is basically the term $\Phi^2 + 2\Phi$ that accelerates the convergence of A . Since the reactivity of this term is decreased in accordance with the design's objective (decoupling Φ from w), the convergence of

A suffers. (Note that the natural convergence of A -in the neighbourhood of $\Phi = 0$ - is given by $\dot{A} \sim A^3$).

5.2 State dependent disturbances

Here, the case where the worst disturbance profile defined by (23) occurs is simulated. Two choices for p are tested corresponding to cases 1 and 4 of the preceding section. Figures 7 and 8 show the behaviour of the closed loop systems for the present controller and the one proposed in [14] in cases 1 and 4 respectively.

6 Conclusion

In this paper, a robust stabilizing feedback law is proposed to handle the control of rotating stall and surge in axial flow compressor. This is done by first design a globally asymptotically stabilizing feedback depending on four parameters. Then, a robustness-oriented algorithm is proposed to tune the above free parameter. The procedure is based on dynamic games theory and may be used to robustify any other parameterized stabilizing algorithm that can be found in the literature.

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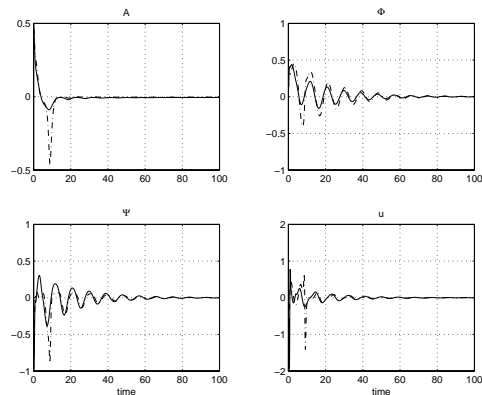


Figure 2: Disturbance rejection with L_2 -norm oriented feedback design. State independent disturbances. Under the feedback of [14] (- -). Under the proposed feedback (-). Case 1: $c = (0.25, 0.25, 0.25, 0.25)$

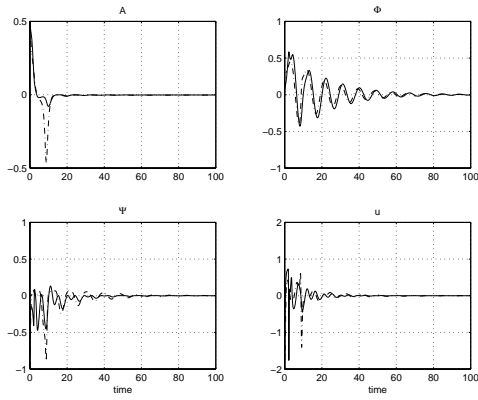


Figure 3: Disturbance rejection with L_2 -norm oriented feedback design. State independent disturbances. Under the feedback of [14] (- -). Under the proposed feedback (-). Case 2: $c = (0.99, 0.0, 0.0, 0.01)$

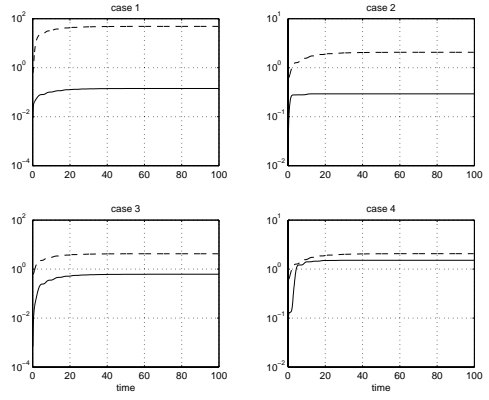


Figure 6: Disturbance rejection with L_2 -norm oriented feedback design. Verification of the basic design inequality $z^T z \leq V(x_0) + \gamma^2 \int_0^t w^T w d\tau$. $[V(x_0) + \gamma^2 \int_0^t w^T w d\tau]$ (- -). $z^T z$ (-).

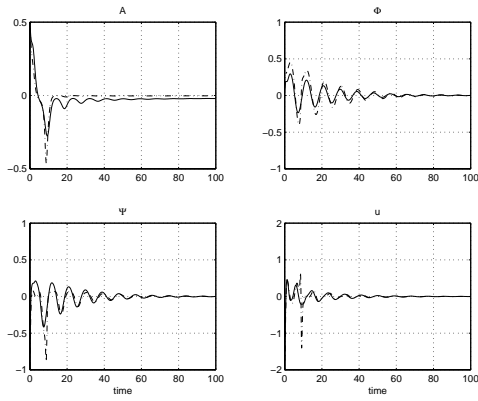


Figure 4: Disturbance rejection with L_2 -norm oriented feedback design. State independent disturbances. Under the feedback of [14] (- -). Under the proposed feedback (-). Case 3: $c = (0.0, 0.99, 0.0, 0.01)$

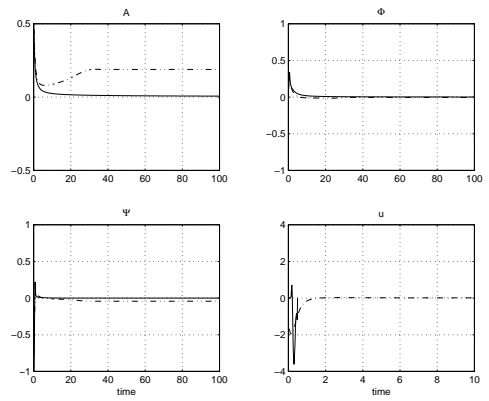


Figure 7: Disturbance rejection with L_2 -norm oriented feedback design. State-dependent disturbances. Under the feedback of [14] (- -). Under the proposed feedback (-). Case 1: $c = (0.25, 0.25, 0.25, 0.25)$.

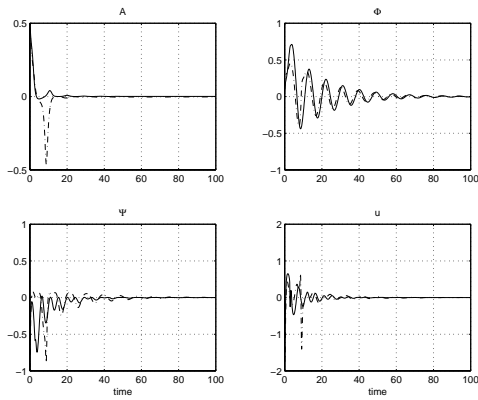


Figure 5: Disturbance rejection with L_2 -norm oriented feedback design. State independent disturbances. Under the feedback of [14] (- -). Under the proposed feedback (-). Case 4: $c = (0.0, 0.0, 0.99, 0.01)$

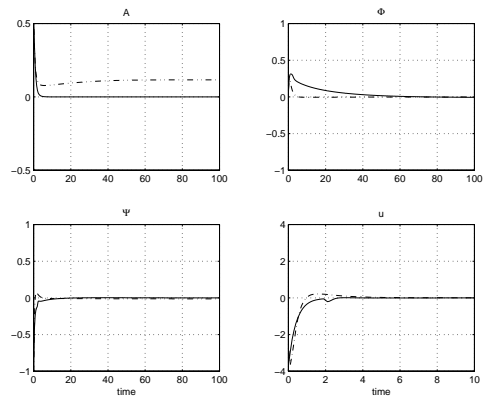


Figure 8: Disturbance rejection with L_2 -norm oriented feedback design. State-dependent disturbances. Under the feedback of [14] (- -). Under the proposed feedback (-). Case 4: $c = (0.0, 0.0, 0.99, 0.01)$.