Sliding-mode Observer For Uncertain Systems Part II: Nonlinear Systems Case

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Abstract

A new sliding mode observer based on Wallcot and Zak's observer for linear uncertain systems was proposed in Part I of this article. The proposed observer works under much less conservative conditions than the ones previously proposed. In this article, the observer design methodology that was proposed for linear systems in the first part of this paper is extended to a general class of nonlinear uncertain systems. Numerical examples are used to illustrate the validity of the proposed observer design strategy.

Keyword: Sliding mode observer; Uncertain systems; Nonlinear systems; Nonlinear unknown input observer.

1 Introduction

A common approach to the problem of observer design for nonlinear systems has been to extend the linear Luenberger observer or Kalman Filter design approach to nonlinear systems. In this respect, one design approach deals with nonlinear systems for which observers with linearizable error dynamics can be designed (see e.g. [2], [14], [15], [17]). In several works (see eg. [4], [7],[9], [16]) systems which are composed of a linear unforced part and a nonlinear state dependent controlled part are considered. In such cases, the nonlinearity is usually accounted for via its Lipschitz constant in the observer design strategy.

In [13], under some uniform detectability conditions, the authors have proposed a constant gain observer for a class of nonlinear systems without inputs. This condition is usually not easy to verify and as a result, the gain of the observer can not be explicitly calculated. In [10], an observer is given for a class of nonlinear systems which are not necessarily control affine. However, the gain of the proposed observer is not easily computable. In recent works [6, 11], observers based on some ideas from the high gain approach, whose gain could easily be designed, were proposed for multivariable nonlinear systems.

Nonlinear unknown input observers (NUIO) were subjects of few studies with a main motivation of applying such an observer to model based fault detection and isolation (FDI) problems. In this realm, [19] extended the method of linear unknown input observer to a class of nonlinear systems, where if existence conditions are satisfied, a nonlinear transformation is used to produce a reduced-order model which is unaffected by unknown inputs. Then a nonlinear observer is constructed for this transformed model. Similarly, [21] considered the nonlinear systems which can be transformed into output injection form.

A different class of observer design methodology for nonlinear systems is that based on variable structure systems (VSS) theory, such as sliding mode observers (SMO). [8] extended Utkin SMO for linear systems to nonlinear systems of the form in

$$\begin{aligned} \dot{x} &= f(x) + B(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

This extension was also applied to nonlinear systems in triangular input form in [1], output and output derivative injection form in [5], and general nonlinear observerable systems with measurements that are linearly related to the state vector (i.e. $y = Cx$) in [18, 20]. In [22] adaptive sliding mode observers were used for FDI in a class of nonlinear uncertain systems. In this article, unknown input decoupled nonlinear observers from the viewpoint of sliding mode are developed.

2 Sliding Mode Observer For Nonlinear Uncertain systems

The multivariable nonlinear systems we consider are described in state space form by equations of the fol-

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lowing form

$$\begin{array}{ll} \dot{x} = & f(x) + B(x,u) + \sum_{i=1}^{m} g_i(x) d_i(x,u,t) \\ y_1 = & h_1(x) \\ \ldots & \ldots \\ y_p = & h_p(x) \end{array} \tag{2}$$

in which $x \in M$, a C^∞ connected manifold of dimension n, $f(x), B(x,u), G(x) = [g_1(x), ..., g_m(x)]$ are smooth vector fields on M, and $h_i(x), i = 1, ..., p$ are smooth functions from M in R. the term $d(x,u,t)$ represents the uncertainty due to modeling error or component/actuator faults, which is unknown. In what follows, local coordinates are generally used. When global properties are considered, notions are simplified by assuming that M accepts a global coordinate system.

Assumption 1 *We assume that $p \geq m$, and the first m outputs have relative degree $\{q_1, ..., q_m\}$ corresponding to $G(x)$ at each point $x_0 \in M$. This means*

$$L_{g_j} L_f^k h_i(x) = 0$$

for all $j = 1, ..., m$, for all $k < q_i - 1$, for all $i = 1, ..., m$, and for all x in M. Further, the $m \times m$ matrix

$$A(x) = \begin{pmatrix} L_{g_1} L_f^{q_1-1} h_1(x) & \ldots & L_{g_m} L_f^{q_1-1} h_1(x) \\ L_{g_1} L_f^{q_2-1} h_2(x) & \ldots & L_{g_m} L_f^{q_2-1} h_2(x) \\ \ldots & \ldots & \ldots \\ L_{g_1} L_f^{q_m-1} h_m(x) & \ldots & L_{g_m} L_f^{q_m-1} h_m(x) \end{pmatrix}$$

is nonsingular at each point $x_0 \in M$.

Assumption 2 *The distribution spanned by the vector fields $g_1(x), ..., g_m(x)$ is involutive.*

Lemma 1 *Given the system (2), if assumptions 1-2 are valid, then*

$$q_1 + \ldots + q_m \leq n$$

Set, for $i = 1, ..., m$,

$$\begin{array}{ll} \phi_1^i = & h_i(x) \\ \phi_2^i = & L_f h_i(x) \\ \ldots & \ldots \\ \phi_{q_i}^i = & L_f^{q_i-1} h_i(x) \end{array}$$

if $q = q_1 + \ldots + q_m$ is strictly less than n, it is always possible to find $n - q$ more functions $\phi_{q+1}(x), ..., \phi_n(x)$ such that the mapping

$$\Phi(x) = col(\phi_1^1(x), ..., \phi_{q_1}^1(x), ..., \phi_{q_m}^m(x), \phi_{q+1}(x), ..., \phi_n(x))$$

has a Jacobian matrix which is nonlinear at each $x_0 \in M$ and therefore qualifies as a local coordinate transformation in M. Moreover, it is always possible to choose $\phi_{q+1}(x), ..., \phi_n(x)$ in such a way that

$$Lg_j \phi_i(x) = 0$$

for all $i = q+1, ..., n, j = 1, ..., m$, and all x in M. Set

$$x_d^i = \begin{pmatrix} x_1^i \\ x_2^i \\ \ldots \\ x_{q_i}^i \end{pmatrix} = \begin{pmatrix} \phi_1^i(x) \\ \phi_2^i(x) \\ \ldots \\ \phi_{q_i}^i(x) \end{pmatrix}$$

for $i = 1, ..., m$, and $x_d = (x_d^1, ..., x_d^m)$,

$$x_o = \begin{pmatrix} x_o^1 \\ x_o^2 \\ \ldots \\ x_o^{n-q} \end{pmatrix} = \begin{pmatrix} \phi_{q+1}(x) \\ \phi_{q+2}(x) \\ \ldots \\ \phi_n(x) \end{pmatrix}$$

the equations under new coordinates can be written as

$$\begin{array}{ll} \dot{x}_1^i & = x_2^i + b_1^i(x_d, x_o, u) \\ \ldots & \ldots \\ \dot{x}_{q_i-1}^i & = x_{q_i}^i + b_{q_i-1}^i(x_d, x_o, u) \\ \dot{x}_{q_i}^i & = a_i(x_d, x_o) + b_{q_i}^i(x_d, x_o, u) + \sum_{j=1}^{m} c_{ij}(x_d, x_o) d_j \\ y_i & = x_1^i \end{array} \tag{3}$$

for $i = 1, ..., m$, and

$$\begin{array}{ll} \dot{x_o} & = q(x_d, x_o) + p(x_d, x_o, u) \\ y_{m+1} & = h_{m+1}(x_d, x_o) \\ \ldots & \ldots \\ y_p & = h_p(x_d, x_o) \end{array} \tag{4}$$

where

$$a_i(x_d, x_o) = L_f^{q_i} h_i(\Phi^{-1}(x_d, x_o)),$$

$$c_{ij} = L_{g_j} L_f^{q_i-1} h_i(\Phi^{-1}(x_d, x_o))$$

and

$$b_k^i(x_d, x_o, u) = \frac{\partial (L_f^{k-2} h_i)}{\partial x} B(\Phi^{-1}(x_d, x_o), u)$$

The above lemma is a trivial extension of Proposition 5.1.2 in [12]. Similar to the role of SCB transformation for linear systems, a nonlinear transformation $\Phi(x)$ decomposes the system (2) into two subsystems. Of the two subsystems, only x_d subsystem is effected by unknown inputs. Of course, this is not a complete decomposition because it is relies on Assumption 1 and 2. The development of a complete transformation for general nonlinear system will be studied in the future.

Next, we shall discuss the observer design for system (3)-(4).

2.1 Observability of Subsystems

If a linear system is observable, for any control input the initial state can be reconstructed. This property is in general not true for nonlinear systems, and the observability of nonlinear systems is associated with inputs. The observability of nonlinear subsystem (3)-(4) is great interest to us. Generally, unknown inputs can make a nonlinear system to become unobservable, just the same as known inputs. For known input signals, one can avoid applying those "bad inputs". However, the unknown inputs which may be the result of a failure or certain other disturbances are beyond our control. Therefore, observability for all unknown inputs is in general necessary in order to design a nonlinear unknown input observer, unless it can be guaranteed a priori that the unknown inputs do not belong to the class of bad inputs. Based on the work in [10], we have following lemma that shows the conditions such that the observability of x_d subsystem is independent of unknown inputs.

Lemma 2 *Let $\overline{x_d^i} = \{x_d^1, ..., x_d^{i-1}, x_d^{i+1}, ..., x_d^m\}$. Assume each x_d^i subsystem in (3) has its input term in the following form*

$$\begin{array}{ll} b_1^i(x_d, x_o, u) & = b_1^i(x_1^i, x_2^i, \overline{x_d^i}, x_o, u) \\ b_2^i(x_d, x_o, u) & = b_2^i(x_1^i, x_2^i, x_3^i, \overline{x_d^i}, x_o, u) \\ ... & ... \\ b_{q_i-1}^i(x_d, x_o, u) & = b_{q_i-1}^i(x_d, x_o, u) \\ b_{q_i}^i(x_d, x_o, u) & = b_{q_i}^i(x_d, x_o, u) \end{array} \tag{5}$$

and the functions

$$1 + \frac{\partial b_j^i}{\partial x_{j+1}^i} \neq 0, j = 1, ..., q_i - 1$$

and state $x_o, \overline{x_d^i}$ is considered as inputs for x_d^i subsystem. Then x_d^i subsystem is uniformly observable for all inputs $u, d, \overline{x_d^i}$ and x_o.

An open problem which does not effect the development here is whether x_o subsystem is observable if and only if the original system (2) is observable.

2.2 Sliding observer for subsystem with unknown inputs

We shall first build an unknown input independent observer for x_d subsystem.

Theorem 1 *If each x_d^i subsystem in (3) has its input term in the following form*

$$\begin{array}{ll} b_1^i(x_d, x_o, u) & = b_1^i(y, u) \\ b_2^i(x_d, x_o, u) & = b_2^i(x_2^i, y, u) \\ ... & ... \\ b_{q_i-1}^i(x_d, x_o, u) & = b_{q_i-1}^i(x_2^i, ..., x_{q_i-1}^i, y, u) \\ b_{q_i}^i(x_d, x_o, u) & = b_{q_i}^i(x_d, x_o, u) \end{array} \tag{6}$$

there exists a choice of $\lambda_j^i, i = 1, ..., m, j = 1, ..., q_i$ such that following sliding mode observer can be built to estimate states ξ,

$$\begin{array}{lll} \dot{\hat{x_1^i}} & = & \hat{x_2^i} + b_1^i(y, u) + \lambda_1^i sign(y_i - \hat{x_1^i}) \\ \dot{\hat{x_2^i}} & = & \hat{x_3^i} + b_2^i(\hat{x_2^i}, y, u) + \lambda_2^i sign(\overline{e_2^i}) \\ ... & . & ... \\ \dot{\hat{x}}_{q_i-1}^i & = & \hat{x}_{q_i}^i + b_{q_i-1}^i(\hat{x_2^i}, ..., \hat{x}_{q_i-1}^i, y, u) + \\ & & \lambda_{q_i-1}^i sign(\overline{e_{q_i-1}^i}) \\ \dot{\hat{x}}_{q_i}^i & = & a_i(\hat{x_d}, \hat{x_o}) + b_{q_i}^i(\hat{x_d}, \hat{x_o}, u) + \lambda_{q_i}^i sign(\overline{e_{q_i}^i}) \end{array} \tag{7}$$

where $\overline{e_j^i}, j = 1, ..., q_i$ are given by

$$\overline{e_j^i} = (\lambda_{j-1}^i sign(\overline{e_{j-1}^i}))_{eq} \tag{8}$$

as in equation (26) in Part I of this article.

Observer (7) is a simple extension of work in [1], where sliding mode observer for single output triangular input form nonlinear systems is studied. The proof is similar to Theorem 2 in Part I of this paper and is omitted here. The above form is more general than the output injection or triangular input form, but more conservative than the form (6), which assures uniform observability.

Remark 1 In observer (7), $\hat{x_o}$ is the estimate of x_o, which will be further discussed later. However, as long as both $\hat{x_o}$ and x_o are bounded, observer (7) will always converge, no matter how large $\|\hat{x_o} - x_o\|$ may be. In the other words, letting $\hat{x_o} = 0$ does not effect its convergence property. The reason for this desirable property is that x_o is only confined to the last equation.

Remark 2 Similar to linear systems case in part I, it is not always necessary to design the proposed sliding mode observer based on the transformed system model, noting that it is much more difficult to find the suitable nonlinear transformation than linear transform. For nonlinear systems with linear output equation, we suggest to check the possibility of designing based on original system equation. The basic rule is to make sure unknown inputs will not appear in the equivalent control signal. Our example illustrate this intuitive design procedure.

Remark 3 It should be noted that if $q_i = 1, i = 1, ..., m$, above theorem does not put any special constraint on the input term. This condition corresponds to condition $rank(CG) = rank(G)$ for linear systems. $q_i = 1, i = 1, ..., m$ means all states of subsystem x_d are measurable. An observer for such a system may be unnecessary for controller synthesis, but will be useful in fault detection and isolation (FDI) applications.

Remark 4 The nonlinear unknown input observer design method reported in [19], transforms the system (2) by $z = T(x)$, where $\frac{\partial T(x)}{\partial x} G(x) = 0$. Then an observer for the reduced order z subsystem is built. The original state is obtained through $x = \Phi(z, y^*)$, where y^* is obtained from a nonlinear transformation $y^* = S(y)$. It stated that the inverse function $x = \Phi(z, y^*)$ exists, if and only if

$$rank \begin{pmatrix} \frac{\partial T(x)}{\partial x} \\ \frac{\partial S(y)}{\partial x} \end{pmatrix} = n; \tag{9}$$

$$lim_{\|x\| \to \infty} \|(T(x) \quad S(y))^T\| = \infty \tag{10}$$

However, the question as to under what conditions there exists $S(y)$ that satisfy conditions (9-10) is still unresolved. In [19] it is stated that $p > m$ is a necessary condition which at least in theory is not true. Our above analysis clearly implies that $p_i = 1, i = 1, ..., m$ and x_o subsystem being detectable, are sufficient for existence of nonlinear unknown inputs observer.

Remark 5 The m observers for $x_d^i (i = 1, ..., m)$ subsystems can run in parallel, although in each observer, e_j^i converges to zero if all the e_k^i with $k < j$ have already converged to zero. If we allow states x_d^i to be estimated after x_d^k with $k < i$ have been estimated correctly, then the input term can be following a more general form,

$$\begin{array}{ll} b_1^i(x_d, x_o, u) & = b_1^i(y, x_d^1, ..., x_d^{i-1}, u) \\ b_2^i(x_d, x_o, u) & = b_2^i(x_2^i, y, x_d^1, ..., x_d^{i-1}, u) \\ ... & ... \\ b_{q_i-1}^i(x_d, x_o, u) & = b_{q_i-1}^i(\xi_2^i, ..., \xi_{q_i-1}^i, y, \xi^1, ..., \xi^{i-1}, u) \\ b_{q_i}^i(x_d, x_o, u) & = b_{q_i}^i(x_d, x_o, u) \end{array}$$

Further, if x_o subsystem is independent of x_d and can be estimated correctly by certain observer, then it is not a problem for x_o to appear in all input terms of x_d subsystem.

Remark 6 Although sliding mode concept provides a nice framework for general class of linear and nonlinear uncertain systems, due to the inherent property of sliding mode there exists two basic drawbacks for practical applications. First, although the bound of unknown input is known, the estimation error bound is not known *a priori*. This makes the selection of the gain difficult. If the gain is too large, observer will exhibit excessive chattering before reaching sliding mode. If the gain is too small, observer may never be able to reach sliding mode and converge to real state value. Secondly, it is difficult to choose a suitable time to use the equivalent control signal. The equivalent control signal should be used only if its corresponding estimation error is near zero, or in sliding mode. However, except for those states which are measured, we do not know if a state estimation is in sliding mode or not. If the equivalent control is used too early, peaking phenomena is unavoidable. If the equivalent control is used too late, a correct estimation in time can not be achieved which is unacceptable for high quality control.

2.3 Observer for subsystem without unknown inputs

The x_o subsystem in (4) is free of unknown inputs. Therefore, an observer can be designed using any of existing techniques, including the standard sliding mode observer. Although, no uniform design methodology for general nonlinear systems exists.

3 Illustrative Example

Example 1 A three-phase current motor model [3] is described by following nonlinear equations,

$$\dot{x} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + B(x)u + g(x)d$$

$$= \begin{bmatrix} x_2 \\ -A_1 x_2 - A_2 x_3 sin x_1 - A_3 sin 2x_1 \\ -D_1 x_3 + D_2 cos x_1 \end{bmatrix} x +$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where $x = [x_1, x_2, x_3]^T$, and x_1, x_2 and x_3 denote the motor's rotor angle, speed deviation and field flux linkage, respectively. The known inputs are u_1(nominal mechanical power input) and u_2(field voltage) and unknown inputs are

$$d_1 = \Delta A_1 x_2 + \Delta A_2 x_3 sin x_1 + \Delta A_3 sin 2x_1$$

which represents uncertainties of parameters A_1, A_2 and A_3, and

$$d_2 = \Delta D_1 x_3 + \Delta D_2 cos x_1$$

which represents uncertainties of parameters D_1, and D_2. All changes are induced by the operating temperature, or component incipient faults. The unknown inputs d_1 or d_2 may be small enough to be neglected in different operation conditions. To illustrate the robust SMO design, we consider several cases.

Case 1: Both d_1 and d_2 are nonzero

In this case, it is noted that x_1, x_2 is a sub-system in the triangular form of (3) if x_1 is measured, namely $y_1 = x_1$, which will make x_2 to be observed. If we have $y_2 = x3$, then all states will be estimated by following observer,

$$\begin{array}{rcl} \dot{\hat{x_1}} & = & \hat{x_2} + \lambda_1 sign(y_1 - \hat{x_1}) \\ \dot{\hat{x_2}} & = & -A_1 \hat{x_2} - A_2 y_2 sin y_1 - A_3 sin 2y_1 + \\ & & \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x_1})_{eq}) \\ \dot{\hat{x_3}} & = & -D_1 \hat{x_3} + D_2 cos y_1 + \lambda_1 sign(y_2 - \hat{x_3}) \end{array}$$

Case 2: d_1 is nonzero, d_2 is zero

In this case, x_3 is a unknown input free subsystem. Fortunately, it is also detectable because $D_1 > 0$. We can build the following observer,

$$\begin{aligned}
\dot{\hat{x_1}} &= \hat{x_2} + \lambda_1 sign(y_1 - \hat{x_1}) \\
\dot{\hat{x_2}} &= -A_1\hat{x_2} - A_2\hat{x_3}siny_1 - A_3 sin2y_1 + \\
&\quad \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x_1})_{eq}) \\
\dot{\hat{x_3}} &= -D_1\hat{x_3} + D_2 cosy_1
\end{aligned}$$

Case 3: d_1 is zero, d_2 is nonzero

In this case, with only one measurement for x_1, the system can be transformed into triangular form and all states can be estimated. Note $g(x) = [0 \quad 0 \quad 1]^T$, the relative degree of output $y = h(x) = x_1$ corresponding to $g(x)$ can be calculated as

$$\frac{\partial h_1}{\partial x} = (0 \quad 0 \quad 1), L_g h_1(x) = 0, L_f h_1(x) = f_1(x) = x_2$$

$$\frac{\partial (L_f h_1)}{\partial x} = \frac{\partial f_1}{\partial x} = (0 \quad 1 \quad 0),$$

$$L_g L_f h_1(x) = 0; L_f^2 h_1(x) = f_2(x)$$

$$\frac{\partial (L_f^2 h_1)}{\partial x} = (-A_2 x_3 cosx_1 - A_3 cos2x_1 \quad -A_1 \quad -A_2 sinx_1),$$

$$L_{g_1} L_f^2 h_1(x) = -A_2 sinx_1$$

Note that $L_g L_f^2 h_1(x) \neq 0$ if $x_1 \neq k\pi$, thus relative degree is $r_1 = 3$ at point $x_1 \neq k\pi$. This means that we shall be able to find a transformation only locally, away from any point such that $x_1 = k\pi$. The transformation is

$$\begin{aligned}
\xi_1 = \phi_1(x) &= h_1(x) = x_1 \\
\xi_2 = \phi_2(x) &= L_f h_1(x) = x_2 \\
\xi_3 = \phi_3(x) &= L_f^2 h_1(x) = f_2(x)
\end{aligned}$$

The Jacobian matrix of the transformation thus defined

$$\frac{\partial \Phi}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -A_2 x_3 cosx_1 - A_3 cos2x_1 & -A_1 & -A_2 sinx_1 \end{bmatrix}$$

is nonsingular for all $x_1 \neq k\pi$, and the inverse transformation is given by

$$\begin{aligned}
x_1 &= \xi_1 \\
x_2 &= \xi_2 \\
x_3 &= z(\xi) = \frac{\xi_3 + A_1\xi_2 + A_3 sin2\xi_1}{-A_2 sin\xi_1}
\end{aligned}$$

In these new coordinates the system is described by

$$\begin{aligned}
\dot{\xi_1} &= \xi_2 \\
\dot{\xi_2} &= \xi_3 \\
\dot{\xi_3} &= -A_1\xi_3 - \xi_2(A_2 z(\xi) cos\xi_1 + 2A_3 cos2\xi_1) - \\
&\quad A_2 sin\xi_1(-D_1 z(\xi) + D_2 cos\xi_1) \\
y_1 &= \xi_1
\end{aligned}$$

Actually, the SMO can be done without above complicated transformation calculation. The observer is

$$\begin{aligned}
\dot{\hat{x_1}} &= \hat{x_2} + \lambda_1 sign(y_1 - \hat{x_1}) \\
\dot{\hat{x_2}} &= -A_1\hat{x_2} - A_2\hat{x_3}siny_1 - A_3 sin2y_1 + \\
&\quad \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x_1})_{eq}) \\
\dot{\hat{x_3}} &= -D_1\hat{x_3} + D_2 cosy_1 + \lambda_3 sign(\overline{e_3})
\end{aligned}$$

note the equivalent control signal based on the second equation is

$$(\lambda_2 sign(\overline{e_2}))_{eq} = -A_2 e_3 siny_1$$

thus

$$\overline{e_3} = \frac{(\lambda_2 sign(\overline{e_2}))_{eq}}{-A_2 siny_1} = e_3$$

Obviously, it is true only if $\sin y_1 \neq 0$. The parameters in the model have the value, $A_1 = 0.2703, A_2 = 12.01, A_3 = -48.04, D_1 = 0.3222, D_2 = 1.9$, and $\Delta D_2 = 0.6$. The control input $u_1 = 36.19, u_2 = 1.9333$. The gain $\lambda_1 = \lambda_2 = \lambda_3 = 200$. The initial state is assumed to be $x_0 = \{0.88, 0.0, 6.5\}$, and initial value of observer is $\xi_0 = \{0.0, 20\}$ and $\eta_0 = 0.8$. The simulation result for case 3 is shown in Figure 1-3.

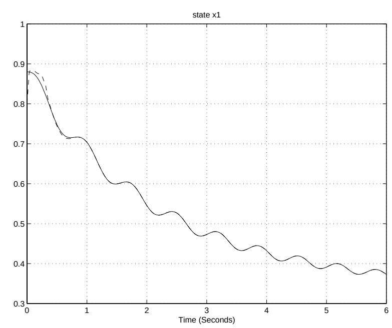

Figure 1: Estimation of state x_1

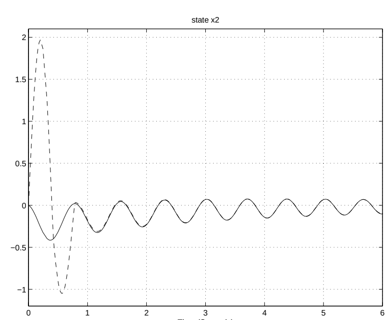

Figure 2: Estimation of state x_2

4 Conclusion

In this second part of a two parts article on the design of sliding mode observers, we extended our proposed observer to certain class of nonlinear uncertain systems. It is of future research interest to focus on sliding mode

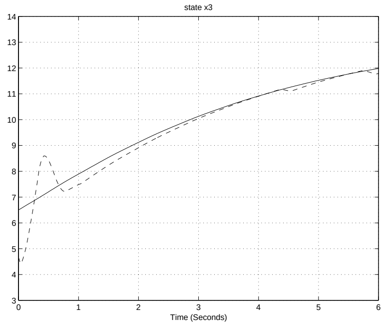

Figure 3: Estimation of state x_3

observer for more general classes of nonlinear systems, as well as application of this class of observers to the problem of fault detection and isolation. It is also a subject of future research interest to apply the observer design methodology proposed here to the problem of observer based fault detection and isolation (FDI).

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References

[1] Barbot, J.P., T. Boukhobza, and M. Djemai (1996), Sliding mode observer for triangular input form,*35th IEEE Conf. on Decision and control*, pp. 1489-1490.

[2] Bestle, D. and M. Zeitz, (1983), Canonical form observer design for nonlinear time-variable systems, *Int. J. Control*, Vol. 38, No. 2, pp. 419-431.

[3] Birk, J. and M. Zeitz (1988), Extended Luenberger observer for nonlinear multivariable systems,*Int. J. Control*, Vol 47, No. 6, pp. 1823-1836.

[4] Bornard, G., and H. Hammouri, (1991), A high gain observer for a class of uniformly observable systems, *Proceedings of the IEEE CDC*, Brighton, GB.

[5] Boukhobza, T., M. Djemai and J.P. Barbot, (1996), Nonlinear sliding observer for systems in output and output derivative injection form, *IFAC 13th Worlf Congress,*, pp. 299-304.

[6] Busawon, K., and M. Saif (1999), A state observer for nonlinear systems, *IEEE Trans. on Automatic Control*, Vol. 44, No. 11, pp. 2098-2103.

[7] Busawon, K., H. Hammouri, and G.Bornard, (1997), An observer for a class of nonlinear systems, *internal report–LAGEP*, University of Lyon I.

[8] Drakunov, S.V. (1992), Sliding mode observers based on equivalent control method, *Proc. of IEEE 31st CDC*, Tucson, Arizona, pp. 2368-2369.

[9] Gauthier, J.P., H. Hammouri and S. Othman, (1992), A simple observer for nonlinear systems-Application to bioreactors, *IEEE Trans. on Aut. Control*, Vol. 37, No. 6.

[10] Gauthier, J.P. and I.A.K. Kupta, (1994), Observability and observers for nonlinear systems, *SIAM J. Control and Optimization,*, Vol. 32, No. 4, pp. 975-994.

[11] Hou, M., K.K. Busawon, and M. Saif, (2000), Observer Design for Nonlinear Systems via Injective Mapping, to appear in *IEEE Transactions on Automatic Control*.

[12] Isidori,A., (1995) *Nonlinear Control Systems*, 3rd Edition, Springer Verlag.

[13] Kou, S.R., D.L. Elliot and T.J. Tarn, (1975), Exponential observers for nonlinear dynamic systems, *Information and Control*, Vol. 29, pp. 204-216.

[14] Krener A.J., and A. Isidori, (1983), Linearization by output injection and nonlinear observers, *Systems and Control Letters*, Vol. 3, pp. 47-52.

[15] Krener A.J., and W. Respondek, (1985), Nonlinear observers with linearizable error dynamics, *SIAM J. Control and Optimization*,Vol. 23, No. 2, pp. 197-216.

[16] Raghavan S., and J.K. Hedrick, (1994), Observer design for a class of nonlinear systems, *Int. J. Control*, Vol. 59, No. 2, pp. 515-528.

[17] Rudolph, J., and M. Zeitz, (1994), A block triangular nonlinear observer normal form, *Systems and Control Letters*, Vol. 23, pp. 1-8.

[18] Slotine, J.J. E., J.K. Hedrick, and E.A. Misawa, (1987), On sliding Observers for nonlinear systems, *ASME J. Dyn. System Measurement Control,* Vol. 109, pp. 245-252.

[19] Seliger, R. and P.M. Frank, (1991), Fault Diagnosis by disturbance decoupled nonlinear observers, *Proc. of IEEE CDC*, pp. 2248-2253.

[20] Wang, G.B., S.S. Peng and H.P. Huang, (1997), A sliding observer for nonlinear process control, *Chemical Engineering Science,*, Vol. 52, No. 5, pp. 787-805.

[21] Yang, H. and M. Saif (1996), Monitoring and Diagnostics of a Class of Nonlinear Systems Using a Nonlinear Unknown Input Observer, *Proc. of IEEE CCA*, pp. 1006-1011.

[22] Yang, H., and M. Saif, (1995), Fault Detection in a Class of Nonlinear Systems via Adaptive Sliding Observer Design, *Proceedings of the IEEE Conference on Systems, Man and Cybernetics*, Vancouver, BC, pp. 2199-2204.