Customer Demand Satisfaction in Production Systems: a Due-Time Performance Approach*

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Abstract

The problem of customer demand satisfaction in production systems with unreliable machines and finite Finished Goods Buffers (FGB) is addressed. The measure of customer demand satisfaction, referred to as the Due-Time Performance (DTP), is characterized by the probability to ship to the customer a required number of parts during a fixed time interval. A method for DTP calculation is developed and applied within a case study.

1 INTRODUCTION

1.1 Problem Addressed

This paper addresses the problem of customer demand satisfaction in supply chain management. Each supplier is assumed to be a production system with unreliable machines. Each customer requires the supplier to ship him a required number of parts during a fixed interval of time. This arrangement is a typical customer-supplier relation in the automotive industry.

If the machines were absolutely reliable, perfect customer demand satisfaction would be easy to achieve. In reality, however, the machines are never absolutely reliable and experience unscheduled downtime due to random breakdowns. In this situation, the number of parts (units or subsystems) produced by a production system is a random variable. Its distribution characterizes both production volume and production variability. The production volume is typically defined as the expected value of this random variable. In the situation described above, the production variability can be defined as the probability to ship to the customer a required number of parts during a fixed time interval. We refer to this probability as the Due-Time Performance (DTP).

In this paper a method for DTP calculation is developed. To this end, Subsection 2.1 provides the problem formulation. Section 2 describes the idea of the approach to DTP calculations developed in this paper. Section 3 is devoted to DTP calculations for serial and assembly lines. The application is described in Section 4. Due to space limitations, the literature review, all proofs, and numerous details are omitted and can be found in [1].

1.2 Problem Formulation

The structure of production systems analyzed in this work is shown in Figure 1.1 (serial line) and Figure 1.2 (assembly line), where the circles represent the machines and rectangles are the buffers. Assumptions, formulated in Subsections 1.2.1 and 1.2.2, define the serial and the assembly line, respectively. All assumptions are classified into four groups, pertaining to machines, buffers, interactions between the machines, and the demand.

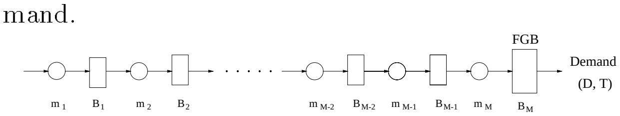

Figure 1.1: Serial Production Line with FGB

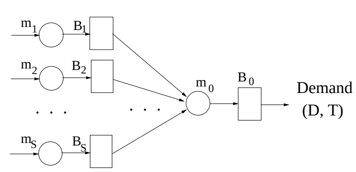

Figure 1.2: Assembly Line with FGB

1.2.1 Serial lines

Machines:

(i) Each machine, m_i, $i = 1, \cdots, M$, requires a fixed unit of time to process a part. This unit is referred to as the *cycle time*. All machines have identical cycle time. The time axis is slotted with the slot duration equal to the cycle time.

(ii) During a cycle time, each machine can be in either "up" or "down". When up, the machine could process a part. When down, no processing takes place.

(iii) The status of the machines, i.e., up or down, is

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determined by the process of Bernoulli trials. In other words, it is assumed that during each slot, machine m_i, $i = 1, \cdots, M$, is up with probability p_i and down with probability $1 - p_i$; the status of the machine is determined at the beginning of each cycle, independent of the status of this machine in the previous cycle.

Buffers:

(iv). Each buffer B_i, $i = 1, \cdots, M$, has capacity $0 \leq N_i < \infty$, $i = 1, \cdots, M$. Buffers $B_1, \cdots, B_{M-1}$ are called in-process buffers. Buffer B_M is the finished goods buffer.

Starvation rule:

(v) If B_i, $i = 1, \cdots, M-1$, is empty at the beginning of the time slot, then m_{i+1}, $i = 1, \cdots, M-1$, is starved during this time slot. The first machine is never starved.

Blockage rule:

(vi) If B_i, $i = 1, \cdots, M$, is full at the beginning of a time slot and m_{i+1}, $i = 1, \cdots, M-1$ does not take a part from B_i at the beginning of this slot, then m_i, $i = 1, \cdots, M$, is blocked during this time slot.

Demand:

(vii). From the point of view of the demand, the time axis is divided into "epochs", each consisting of T time slots.

(viii). At the end of each epoch, a shipment of D parts has to be available for the customer. If PR is the production rate of the system, then

$$D \leq T \cdot PR. \tag{1.1}$$

Demand satisfaction policy:

(ix). At the beginning of epoch i, parts are removed from the FGB in the amount of $\min(H(i-1), D)$, where $H(i-1)$ is the number of parts in the FGB at the end of $(i-1)$-th epoch. If $H(i-1) \geq D$, the shipment is complete; if $H(i-1) < D$, the balance of the shipment, i.e., $D - H(i-1)$ parts, is to be produced by m_M during the shipping period T. Parts produced are immediately removed from the FGB and prepared for shipment, until the shipment is complete, i.e., D parts are available. If the shipment is complete before the end of the epoch, the system continues operating, but with the parts being accumulated in the FGB, either until the end of the epoch or until the last machine, m_M, is blocked, whichever occurs first. If the shipment is not complete by the end of the epoch, an incomplete shipment is sent to the customer. No backlog is allowed.

Assumptions (i)-(ix) define the serial production line under consideration. In the time scale of the epoch and in an appropriately defined state space, the system (i)-(ix) is a stationary ergodic Markov chain. We refer to its steady state as the "normal system operation".

Let $\bar{t}_i$ be the number of parts produced by machine m_M in epoch i during the normal system operation. Then DTP can by expressed as

$$DTP = Pr\Big(H(i-1) + \bar{t}_i \geq D\Big). \tag{1.2}$$

In the framework of model (i)-(ix), DTP is a function of all system parameters. In other words,

$$DTP = DTP(p, N, N_M, D, T), \tag{1.3}$$

where N_M, T, and D are defined in (iv), (vii), and (viii), respectively, p and N are vectors of the machines and in-process buffers parameters,

$$p = [p_1, \cdots, p_M], \qquad N = [N_1, \cdots, N_{M-1}].$$

Problem 1: Given production system (i)-(ix), develop a method for evaluating DTP as a function of system's parameters.

1.2.2 Assembly lines

Machines:

Assumptions (i) and (ii) remain the same. Assumption (iii) is modified as follows:

(iii′). During each slot machine m_k, $k = 0, 1, \cdots, S$, is up with probability p_k and down with probability $1 - p_k$. Machine m_k, $k = 1, \cdots, S$, is referred to as the k-th component machine; m_0 is referred to as the assembly machine.

Buffers:

(iv′). The system has in-process buffers, B_k, $k = 1, \cdots, S$. Buffer B_0 is the finished goods buffer. For all the in-process and finished goods buffers, $0 \leq N_i < \infty$.

Starvation rule:

(v′). Assembly machine m_0 is starved during the time slot, if at least one of the buffers B_k, $k = 1, \cdots, S$, is empty at the beginning of the time slot.

Blockage rule:

(vi′) If B_k, $k = 1, \cdots, S$, is full at the beginning of a time slot and m_0 does not take a part from B_k at the beginning of this slot, then m_k, $k = 1, \cdots, S$, is blocked during this time slot.

Demand:

Remains the same as in (vii), (viii).

Demand satisfaction policy:

Remains the same as in (ix).

In production system defined by (i), (ii), (iii′)-(vi′), (vii)-(ix), the expression for DTP remains the same as in (1.2), and

$$DTP = DTP(p, N, N_M, D, T), \tag{1.4}$$

where N_M, T, and D are defined in (iv), (vii), and (viii), respectively, p and N are vectors of the machines and in-process buffers parameters,

$$p = [p_1, \cdots, p_S, p_0], \quad N = [N_1, \cdots, N_S], \quad N_M = N_0.$$

No results on calculating this function are available in the literature. The problem, then, to be addressed in the paper is:

Problem 2: Given the production system, (i), (ii), (iii′)-(vi′), (vii)-(ix), develop a method for evaluating DTP as a function of system's parameters.

The approach to the solutions of Problems 1 and 2 is described next.

2 APPROACH

Production systems with FGBs were addressed in [2], where DTP for one-machine systems was calculated and a lower bound of DTP for serial lines with many machines was derived. Although results of [2] may be sufficient for the simplest situations (e.g., single machine lines or bound-based approach), analysis and design of many production systems require additional possibilities of DTP calculation. This is especially true for assembly systems, the DTP of which has not been studied before. The main difficulty in developing a method for calculating DTP in multi-machine line is due to the fact that the behavior of a line within an epoch (i.e., in the time scale of the machine cycle time) is not described by a stationary random process. For instance, the probability that the last machine, m_M (in serial line) or m_0 (in assembly line) is blocked during a time slot j, $j = 1, \cdots, T$, clearly depends on j and is a monotonically increasing function of j. This makes exact calculations of DTP all but impossible. Therefore, a simplification is necessary. The simplification used in this work is based on two ideas: stationarization and subsequent recursive iterations. These ideas are described below.

2.1 Stationarization

Consider a one-machine line with the FGB capacity N. Assume that the probability that the machine, m, is blocked (in the manufacturing sense, i.e., m is up and FGB is full) during a time slot j, $j = 1, \cdots, T$, is known. Denoted this probability as $P_b(j)$. Then the probability, P_{b_k}, that m is blocked (in the manufacturing sense) during an epoch exactly k_b times, $k_b = 1, \cdots, T$, can be expressed as

$$P_{b_k} = \begin{cases} P_b(T - k_b + 1) - P_b(T - k_b), & 1 \leq k_b \leq T - 1, \\ 0, & k_b = T. \end{cases} \tag{2.1}$$

Thus, the expected number of blockages in an epoch, $\overline{k}_b$, is

$$\overline{k}_b = \sum_{k_b=0}^{T} k_b P_{b_k} = \sum_{k_b=1}^{T-1} k_b P_{b_k}. \tag{2.2}$$

Using this expression, the approximate probability that FGB is full during any time slot j, i.e., the probability of machine blockage in the communication sense, is

$$\widehat{cb} = \frac{\overline{k}_b}{Tp}. \tag{2.3}$$

We use these expressions as the stationarized probabilities of the appropriate events.

To evaluate $\widehat{cb}$, the distribution $P_b(j)$ is necessary. It can be calculated as follows:

Introduce the probabilities:

$$\begin{aligned} q &= 1 - p, \\ h(i, j) &= \text{Prob}\{i \text{ parts in FGB at beginning of slot } j \\ &\qquad \text{in an epoch}\}, i = 0, \cdots, N, j = 1, \cdots, T, \\ h_0(0, j, k) &= \text{Prob}\{0 \text{ parts in FGB at beginning of} \\ &\qquad \text{slot } j \text{ and } k \text{ parts have been shipped in an} \\ &\qquad \text{epoch}\}, j = 1, \cdots, T, k = 0, \cdots, D. \end{aligned}$$

Clearly, under assumptions (i)-(ix) and $M = 1$, function $h(i, j)$ is defined recursively by

$$h(i, j) = \begin{cases} h(1, j-1)q + h_0(0, j-1, D)p, & i = 1, \\ h(i, j-1)q + h(i-1, j-1)p, & 1 < i < N, \\ h(N, j-1) + h(N-1, j-1)p, & i = N, \end{cases} \tag{2.4}$$

where

$$h_0(0, j, k) = \begin{cases} z_0 q^{j-1}, & k = 0, \\ h_0(0, j-1, k)q & \\ \quad +h_0(0, j-1, k-1)p, & k > 0, \end{cases} \tag{2.5}$$

with initial conditions

$$h(i, 1) = \begin{cases} z_{D+i}, & i > 0, \quad N > D, \\ \sum_{j=0}^{D} z_j, & i = 0, \quad N > D, \\ 0, & i > 0, \quad N \leq D, \\ 1, & i = 0, \quad N \leq D, \end{cases} \tag{2.6}$$

and

$$h_0(0, 1, k) = \begin{cases} z_k, & 0 \leq k \leq D, N > D, \\ z_k, & 0 \leq k \leq N, N \leq D, \\ 0, & N + 1 \leq k \leq D, N < D, \end{cases} \tag{2.7}$$

and z_k, $k = 0, 1, \cdots, N$, is calculated according to [2]. Therefore, the probability that m is blocked during slot j is:

$$P_b(j) = h(N,j)p. \tag{2.8}$$

Thus, $\widehat{cb}$ can be calculated as follows:

$$\widehat{cb} = \frac{1}{T}\sum_{j=2}^{T}\Big[h(N,j) - h(N,j-1)\Big](T-j+1). \tag{2.9}$$

To summarize, it is convenient to introduce an operator of stationarization, Φ_1, defined as:

$$\begin{aligned}\widehat{cb} &= \Phi_1(p, N, T, D) \\ &:= \frac{1}{T}\sum_{j=2}^{T}\Big[h(N,j) - h(N,j-1)\Big](T-j+1),\end{aligned} \tag{2.10}$$

where $h(N,j)$, $j = 1, \cdots, T$, is calculated according to (2.4)-(2.7). Expression (2.10) is used in Section 3 for DTP calculation.

2.2 Iterations

Consider the serial or assembly line shown in Figures 1.1 and 1.2. To adopt the above stationarization procedure, assume that the probability that the last machine m_M (for the serial line) or m_0 (for the assembly line) is not starved in the communication sense is known. Denote an estimate of this probability as $\widehat{p}_{ns}$. Modify p_M (respectively, p_0) by multiplying it by $\widehat{p}_{ns}$ and denote the modified machine as m'_M (respectively, m'_0). Using this machine and procedure (2.4)-(2.7) and (2.10), calculate the stationarized probability, $\widehat{cb}$. Introduce now another fictitious machine, m''_M, defined by $p_M(1-\widehat{cb})$ (respectively, $p_0(1-\widehat{cb})$). Using this machine and deleting FGB, calculate a new $\widehat{p}_{ns}$ by employing the recursive procedure for performance analysis of serial lines developed in [3] (respectively, in [4] for assembly lines). Having this probability, repeat the process described above anew.

In other words, in the case of serial lines, the iterations have the form:

$$\begin{aligned}
p'_M(n+1) &= p_M\widehat{p}_{ns}(n), \\
\widehat{cb}(n+1) &= \Phi_1\Big(p'_M(n+1), N_M, T, D\Big), \\
p''_M(n+1) &= p_M[1-\widehat{cb}(n+1)], \\
\widehat{p}_{ns}(n+1) &= \Phi_2\Big(p_1, \cdots, p_{M-1}, p''_M(n+1), \\
&\qquad N_1, \cdots, N_{M-1}\Big), \\
n = 0, 1, 2, \cdots, &\quad \widehat{p}_{ns}(0) = 1,
\end{aligned} \tag{2.11}$$

where Φ_1 is the stationarization operator defined in (2.10) and Φ_2 is the operator that represents the mapping from p''_M to $\widehat{p}_{ns}$. This operator is described in Section 3.

For assembly lines, the iterations remain the same with the only difference that subscript M is changed to 0, and operator Φ_2 to Φ_3 (described in Section 3):

$$\begin{aligned}
p'_0 &= p_0\widehat{p}_{ns}(n), \\
\widehat{cb}(n+1) &= \Phi_1\Big(p'_0(n+1), N_0, T, D\Big), \\
p''_0(n+1) &= p_0[1-\widehat{cb}(n+1)], \\
\widehat{p}_{ns}(n+1) &= \Phi_3\Big(p_1, \cdots, p_S, p''_0(n+1), N_1, \cdots, N_S\Big)
\end{aligned} \tag{2.12}$$

If procedure (2.11) (respectively, (2.12)) is convergent and the following limit exists,

$$\lim_{n\to\infty}\widehat{p}_{ns}(n) := \widehat{p}_{ns},$$

the DTP of a M-machine serial line (respectively, $(S+1)$-machine assembly line) can be evaluated, according to [2], as the DTP of a one-machine system defined by $p_M\widehat{p}_{ns}$ (respectively, $p_0\widehat{p}_{ns}$) and FGB of capacity N_M (respectively, N_0), i.e.,

$$\widehat{DTP}_M = DTP_1(p_M\widehat{p}_{ns}, N_M, D, T), \tag{2.13}$$

$\Big($respectively,

$$\widehat{DTP}_{S+1} = DTP_1(p_0\widehat{p}_{ns}, N_0, D, T)\Big), \tag{2.14}$$

where the calculation of DTP_1 was derived in [2]. In Section 3, operators Φ_2 and Φ_3 are described, convergence of the iterations are considered, and the accuracy of DTP calculations is investigated.

3 DTP CALCULATION

3.1 Operator Φ_2

Operator Φ_2 can be defined through the aggregation procedure for performance analysis of serial lines developed in [3]. Indeed, assume that $p''_M(n+1)$, calculated according to (2.11), is known. Consider the serial line consisting of M machines with parameters $p_1, \cdots, p_{M-1}, p''_M(n+1)$ and $M-1$ in-process buffers $N_1, \cdots, N_{M-1}$. According to [3], its performance can be analyzed using the following convergent recursive procedure (see [3] for details):

$$\begin{aligned}
p_i^b(s+1) &= p_i\Big[1 - Q\Big(p_{i+1}^b(s+1), p_i^f(s), N_i\Big)\Big], \\
&\qquad 1 \le i \le M-1, \\
p_i^f(s+1) &= p_i\Big[1 - Q\Big(p_{i-1}^f(s+1), p_i^b(s+1), N_{i-1}\Big)\Big], \\
&\qquad 2 \le i \le M,
\end{aligned} \tag{3.1}$$

with boundary conditions

$$p_1^f(s) = p_1, \quad p_M^b(s) = p''_M(n+1), \quad s = 1, 2, 3, \cdots,$$

and initial conditions

$$p_i^f(0) = p_i, \qquad i = 1, \cdots, M,$$

where

$$Q(x,y,N) = \begin{cases} \frac{(1-x)(1-\alpha)}{1-\frac{x}{y}\alpha^N}, & x \neq y, \\ \frac{1-x}{N+1-x}, & x = y, \end{cases} \quad (3.2)$$

$$\alpha = \frac{x(1-y)}{y(1-x)}. \quad (3.3)$$

In terms of the steady states of this procedure, i.e.,

$$\lim_{s\to\infty} p_i^f(s) := p_i^f(n+1), \quad i = 1,\cdots,M,$$
$$\lim_{s\to\infty} p_i^b(s) := p_i^b(n+1), \quad i = 1,\cdots,M, \quad (3.4)$$

the probability that buffer B_{M-1} is not empty can be evaluated as

$$\widehat{p}_{ns}(n+1) = p_M^f(n+1)/p_M. \quad (3.5)$$

Thus, equations (3.1)-(3.5) represent the mapping from $p_M''(n+1)$ to $\widehat{p}_{ns}(n+1)$, denoted in (2.11) as Φ_2.

3.2 Operator Φ_3

For the system shown in Figure 1.2, operator Φ_3 of (2.12) can be defined through the recursive procedure introduced in [4]. As it follows from [4], estimate $\widehat{X}_i(0)$ of the probability that buffers B_i, $i = 1,\cdots,S$, is not empty can be calculated using the following recursive procedure ([4]):

$$\widehat{X}_i(0,s+1) = Q\Big(p_i, p_0''(n+1)\prod_{j=1}^{i-1}[1-\widehat{X}_j(0,s+1)] \prod_{j=i+1}^{S}[1-\widehat{X}_j(0,s)], N_i\Big), \quad (3.6)$$
$$i = 1,\cdots,S, \quad s = 0,1,2,\cdots,$$

with initial conditions

$$\widehat{X}_i(0,0) = 0, \quad i = 1,\cdots,S,$$

and $Q(x,y,N)$ defined in (3.2).

In terms of the steady state of this procedure, i.e., in terms of

$$\lim_{s\to\infty} \widehat{X}_i(0,s) := \widehat{X}_i(0), \quad i = 1,\cdots,S, \quad (3.7)$$

the probability that the assembly machine is not starved is evaluated as

$$\widehat{p}_{ns}(n+1) = \prod_{i=1}^{S}[1-\widehat{X}_i(0)]. \quad (3.8)$$

Thus, equations (3.6)-(3.8) represent the mapping from $p_0''(n+1)$ to $\widehat{p}_{ns}(n+1)$, denoted as Φ_3 in (2.12).

3.3 Convergence

To prove the convergence of recursive procedure (2.11), (3.1)-(3.5) (respectively, (2.12), (3.6)-(3.8)), it is important to know whether function $\widehat{cb}$, defined by (2.4)-(2.10), is monotonic with respect to p. Intuitively, this property clearly takes place. However, a rigorous proof of this fact seems to be all but impossible. Therefore, although in every example numerically analyzed this function was found to be monotonically increasing, we introduce this property as a hypothesis:

Hypothesis 3.1 *For system (i)-(ix) with $M = 1$, the stationarized probability of blockage in the communication sense, $\widehat{cb}$, defined by (2.4)-(2.7) and (2.10), is a monotonically increasing function of p.*

Theorem 3.1 *Under Hypothesis 3.1, iteration procedure (2.11), (3.1)-(3.5) (respectively, (2.12), (3.6)-(3.8)) is convergent, i.e., the following limits exist:*

$$\lim_{n\to\infty} \widehat{cb}(n) := \widehat{cb}, \qquad \lim_{n\to\infty} \widehat{p}_{ns}(n) := \widehat{p}_{ns}. \quad (3.9)$$

Thus, the $\widehat{DTP}_M$ of a serial production line (respectively, $\widehat{DTP}_{S+1}$ of an assembly line) shown in Figure 1.1 (respectively, Figure 1.2) can be calculated using (2.13) (respectively, (2.14)) with $\widehat{p}_{ns}$ defined by (3.9).

3.4 Accuracy

The accuracy of $\widehat{DTP}_M$ and $\widehat{DT}_{S+1}$ has been evaluated using discrete event simulation. [1] presents dozens of systems defined by Subsection 1.2 with various machine, buffer, demand, and epoch parameters. As it can be seen from [1], the estimates result in a relatively high precision and the accuracy is well within 5%.

4 CASE STUDY: DTP ANALYSIS OF AN INJECTION MOLDING - ASSEMBLY SYSTEM AT AN AUTOMOTIVE COMPONENT PLANT

4.1 System Description and Physical Model

The layout of the injection molding - assembly production system under consideration is shown in Figure 4.1. It consists of 13 presses which mold 7 part-types and 3 identical assembly lines. The storage between the injection molding and the assembly serves as the in-process buffer, the storage after the assembly serves as a finished goods buffer.

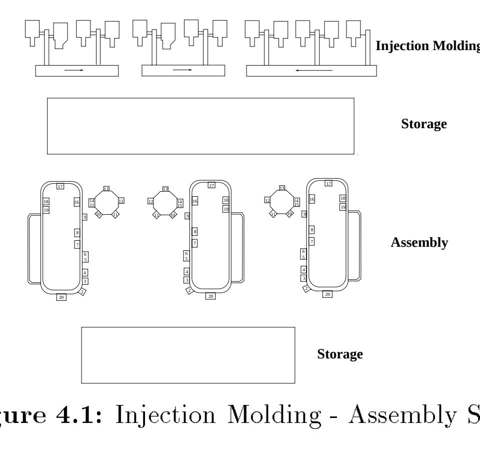

Figure 4.1: Injection Molding - Assembly System

Each mold can be placed on any of the 13 presses, depending on availability and schedule. The assembly uses each of the 7 part-types to assemble the final product. Thus, the physical model of the system can be represented as shown in Figure 4.2.

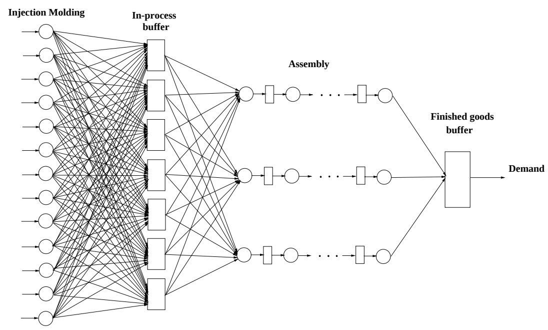

Figure 4.2: Physical Model of Injection Molding - Assembly System

The goal of this project was to select the capacity of the FGB so that $DTP \geq 0.99$, based on the shipping schedule defined by $T = 12$, $D = 9$.

4.2 Virtual Model

Direct analysis of the model in Figure 4.2 is quite complicated. Therefore, a simplification is necessary. The simplification used in this work can be described as follows:

From the point of view of the in-process storage, injection molding part of the system consists of seven machines, each producing a single part-type. The parts are placed in the storage and then drawn by the assembly, one part of each part-type to assembly the product. This simplified model, referred to as the virtual model, is shown in Figure 4.3.

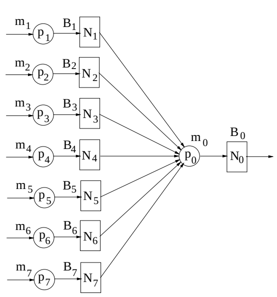

Figure 4.3: Virtual Model of Injection Molding - Assembly System

Based on a two-month performance data, the parameters of the virtual model have been identified as shown in Table 4.1.

Table 4.1: Parameters Identified

V. M.	1	2	3	4	5	6	7	0
p_i	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.79
N_i	26	27	30	15	18	45	45	N_0

The subsequent analysis is carried out in terms of this model.

4.3 Design of FGB Capacity

Using the recursive procedure (2.12), (2.14), (3.6)-(3.8) and the parameters of Table 4.1, we calculate DTP of the virtual model for various values of N_0, the capacity of FGB. The results are shown in Figure 4.4 and Table 4.2. As it can be seen, for $N_0 = 9$, $DTP > 0.997$ and subsequent increase of N_0 results in a very small improvement of DTP. Therefore, the capacity of N_0 has been recommended as one shipment size (i.e., $N_0 = 9$). This recommendation was accepted by the plant management.

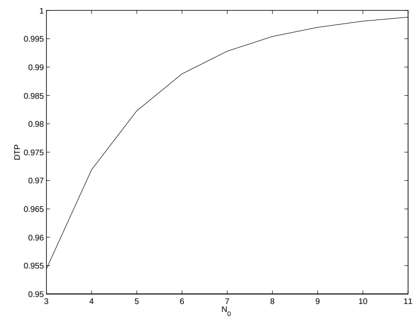

Figure 4.4: DTP of Injection Molding - Assembly System

Table 4.2: Due-Time Performance of Injection Molding - Assembly System

N_0	3	4	5	6	7
DTP	0.9544	0.9719	0.9823	0.9888	0.9928

N_0	8	9	10	11
DTP	0.9954	0.9970	0.9981	0.9988

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