Improvement of Servo Performance via Nonlinear Feedback Control in Hard Disk Drive Servo Systems

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Abstract

In hard disk servo systems, two or more types of controllers are used for track seek, track follow and track settling modes. Due to this nature there is a need to consider the problem of mode switching among these controllers. In this paper, a composite nonlinear feedback controller is used to perform the above functions. The goal of this paper is to propose a unified controller to perform different modes of operation in HDD servo system with the saturation constraints on control signal. This is by the design of composite nonlinear feedback control law based on a nominal linear controller. In the face of actuator saturation, this control law not only increases the speed of closed-loop response but also improves the settling performance.

Keywords: Servo Systems, Modelling, Nonlinear Control, state-feedback.

1 Introduction

Nowadays, the areal density of hard disk drives (HDDs) is increasing by 70% compound annual growth rate and their average data access times are becoming less than 8 msec. The hard disk drive that can accomplish a seek maneuver in minimum time is likely to command a significant share of the sales market.

The two main requirements of HDD servo systems are a) high-speed head positioning in seeking, and b) highly-accurate head positioning in tracking. To meet both requirments, almost all HDDs have two or more control structures. In a conventional controller-switching method, a velocity controller is employed in seeking and a positioning feedback controller such as a PID controller is used in tracking [6].

Track seeking is a time optimal control problem [5]. In the past, a number of arguments have been put forwarded to discount the use of a Time-Optimal Control (TOC) law for a magnetic disk drive storage system. The classic time optimal design, *bang-bang* control, is not practical because with even the smallest system process or measurement noise, the control signal will chatter between the maximum and minimum value. Proximate Time Optimal Servo (PTOS) proposed by Workman (1997), is an algorithm which was derived from bang-bang control and can be made arbitarily close to time optimal. In this method maximum acceleration and deceleration are applied to the limit of a power amplifier and then switched to a tracking controller. To ensure smooth hand over from seeking mode to tracking mode the parameters of both the controllers are designed to meet the constraints imposed by PTOS. Due to these constraints there is a limit to decrease the seek time. Hence there is a need to find a new control technique to decrease the access time.

This paper proposes the use of composite nonlinear control law based on a nominal linear controller proposed by Lin et al, 1998; to hard disk drive servo systems to decrease the access time. In classical linear control theory performance specifications are given in terms of a unit step response. To ensure a quick rise time, the damping ratio should not be too large. On the other hand , if the damping ratio is too small, there will be overshoot and 'ringing'. A compromise is usually made by choosing the damping ratio as a certain fixed number [4]. The composite nonlinear control proposed in [4] takes the advantage of both the quick response of systems with a small damping ratio and the small overshoot of systems with a large damping ratio. This is achieved with two components in the control law namely, 1) nonlinear and 2) linear parts. The nonlinear component is used to increase the damping ratio as the system output approaches the target value. The linear component is used to settle the system at the desired value. The control law obtained by this method gives room to supply an extra energy to compensate for the disturbances. This advantage was utilized to add an internal-loop compensator to eliminate the disturbances. The results are compared with the conventional PTOS method.

This paper first briefly describes a head-positioning sys-

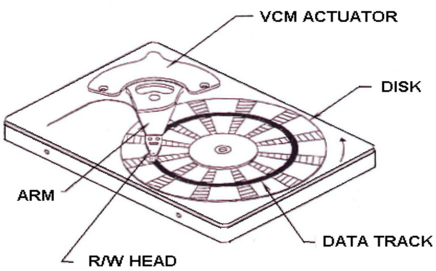


Figure 1: The hard disk drive with VCM actuator servo system.

tem of disk drives. Also, a plant model was derived to implement the proposed method. This is done in Section 2. The main part of this paper is in Section 3, where the Composite Nonlinear Feedback Control is explained. In Section 4, the design of the internal-loop compensator using H_∞ optimization is explained. Simulation and experimental results are shown in Section 5, which clearly shows around 30% reduction in seek time than in conventional PTOS. Finally, we draw some concluding remarks in Section 6.

2 Dynamic Model of Disk Drives

Figure 1 shows a hard disk drive with a VCM actuator servo system. On the surface of a disk, there are thousands of data tracks. A magnetic head is supported by a suspension and a carriage, and it is suspended several microinches above the disk surface. The actuator, called a voice-coil motor (VCM), actuates the carriage and moves the head on a desired track. The mechanical part of the plant, that is, the controlled object, consists of the VCM, the carriage, the suspension, and the heads. The controlled variable is the head position. The plant model can be derived from the frequency response characterstics of the HDD servo system taken from an experiment. The frequency reponse characterstics of the HDD servo system is shown in Figure 2. It is clear from the response characterstics that the servo system has many resonance modes. The complete model may be of order 40 [3]. It is quite conventional in HDD industries to approximate the dynamics of the VCM actuator by a second order state space model as in the following state model

$$\begin{pmatrix} \dot{y} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & k_y \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ k_v \end{pmatrix} u \qquad (1)$$

where u is the actuator input (in Volts), y and v are the position (in tracks) and the velocity of the R/W head, k_y is the position measurement gain and $k_v = k_t/m$, with k_t being the current-force conversion coefficient. Thus, the transfer function from u to y of the VCM

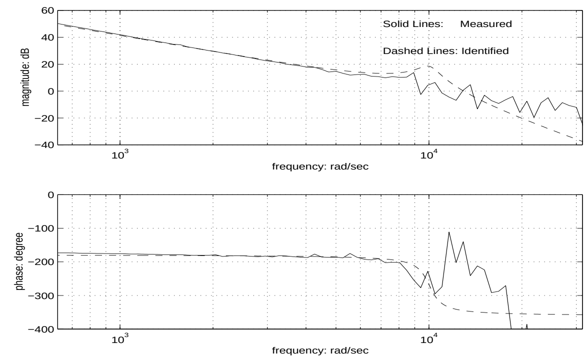


Figure 2: Frequency Response Characteristics of HDD.

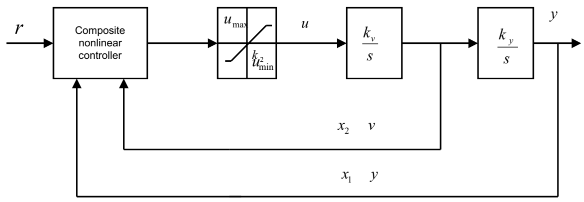


Figure 3: Composite Nonlinear Feedback Control.

model can be written as

$$G_{v1}(s) = \frac{k_v k_y}{s^2} \qquad (2)$$

However, if we consider the high frequency resonance modes also, a more realistic model for the VCM actuator would be

$$G_v(s) = \frac{k_v k_y}{s^2} \frac{k_d s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \qquad (3)$$

Using the measured data from the actual system (see Figure 2), we obtained a fourth order model for the actuator,

$$G_v(s) = \frac{1.832 \times 10^8 s + 1.17 \times 10^{16}}{s^2(s^2 + 1.382 \times 10^3 s + 1.005 \times 10^8)}. \qquad (4)$$

3 Composite Nonlinear Feedback Control

The schematic diagram of Composite Nonlinear Feedback Control is shown in Figure 4. It is clear from the modelling part that the plant (VCM) can be considered as a second order system. Hence the plant model in state space will have two state variables. Let us consider the head position(y) as the first state, the plant velocity(v) as the second state and the head position as the output variable to write the plant model in state space. i.e.,

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y \\ v \end{pmatrix} \qquad (5)$$

$$y = x_1 \qquad (6)$$

From Figure 4, we can write that,

$$\dot{y} = k_y v(t) \quad (7)$$
$$\dot{v} = k_v u(t) \quad (8)$$

Then, from equations (5)-(8) the state model with $x(0) = (0 \quad 0)' = x_0$ as the initial state and the constraint on the control $| u(t) | \leq u_{max}$ can be written as,

$$\dot{x}(t) = \begin{bmatrix} 0 & k_y \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ k_v \end{bmatrix} \text{sat}(u) \quad (9)$$

$$y(t) = [1 \quad 0] x(t) \quad (10)$$

where

$$\begin{aligned} \text{sat}(u) &= u_{max} && \text{if } u \geq u_{max} \\ &= u && \text{if } |u| < u_{max} \\ &= -u_{max} && \text{if } u \leq u_{max} \end{aligned}$$

In order to derive a composite nonlinear control law let the state model (9)-(10) be transformed into a controllable canonical form as

$$\begin{cases} \dot{x} = Ax + B\text{sat}(u) \\ y = Cx \end{cases} \quad (11)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } C = [\, c_1 \quad c_2 \,].$$

Now, the control law can be derived by following the three step alogrithm reported in [4]. Step 1 and 2 deals respectively with the design of the linear and the nonlinear feedback control laws. In step 3, these linear and nonlinear feedback laws are combined to form the desired nonlinear feedback law. These three steps are as follows.

Step 1 - Linear Feedback Design. Choose a linear feedback law

$$u_L = -Fx + Gr \quad (12)$$

where $F = [f_1 \quad f_2]$ is such that $A - BF$ is Hurwitz, the closed-loop system $C(sI - A + BF)^{-1}B$ has a small damping ratio, and $G = (-a_1 + f_1)/c_1$.

It has been proved that the linear feedback control law (12) would cause the system output to asymptotically track a step command of amplitude r, as long as the initial state $x(0)$ and r satisfy

$$\left(x_1(0) - \frac{r}{c_1}, x_2(0) \right) \in \{x : x^T P x \leq c\}, \text{and} \left| \frac{a_1}{c_1} r \right| \leq \Delta. \quad (13)$$

where $\Delta \in (0, 1)$, $P > 0$ is the solution of the Lyapunov equation,

$$(A - BF)^T P + P(A - BF) = -Q, \quad (14)$$

and $c > 0$ be the largest positive number such that

$$x \in \{x : x^T P x \leq c\} \Rightarrow |Fx| \leq 1 - \Delta. \quad (15)$$

Let P be partitioned as

$$P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}. \quad (16)$$

We can conclude from the quiescent initial conditions, viz, from the zero initial state, that any step of amplitude r, $|r| \leq \sqrt{c_1^2 c / p_1}$ and $\left| \frac{a_1}{c_1} r \right| \leq \Delta$, can be asymptotically tracked. By increasing Δ and/or decreasing $|p_1|$ through choice of f_1, f_2 and Q, we can increase the input command amplitude that can be asymptotically tracked. This change of f_1 and f_2 will of course affect the closed-loop damping ratio and hence the rising time.

Step 2 - Nonlinear Feedback Design. The nonlinear state feedback control law is given by

$$u_N = -\rho(x, r) B^T P \left(x - \begin{bmatrix} r/c_1 \\ 0 \end{bmatrix} \right) \quad (17)$$

where $\rho : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}_+$ is any nonnegative function locally Lipschitz in x, which is chosen to increase the damping ratio as the output approaches the command input.

Step 3 - Composition of Linear and Nonlinear Feedback. The final nonlinear feedback law is then composed as

$$u = u_L + u_N = [-F - \rho(x, r) B^T P] x + \frac{-a_1 + f_1 - \rho(x, r) p_2}{r} \quad (18)$$

With the nonlinear state feedback law as given by (18), the following theorem has been proved in [4] concerning the step response of the closed-loop system.

Theorem 3.1 *Consider the system given by (9)-(10). For any nonnegative function $\rho(x, r)$, locally Lipschitz in x, the nonlinear feedback law (18) will cause the system output to asymptotically track the step command input of amplitude r from an initial state $x(0)$, provided that $x(0)$ and r satisfy (13).*

Therefore, the composite control law (18) can be used for fast track seeking and track following operations. This control law was implemented on HDD servo system and the results were shown in the following sections.

4 Compensator Design in the H_∞ Framework

In Section 3 the design of servo controller was discussed. In order to achieve robustness with this controller

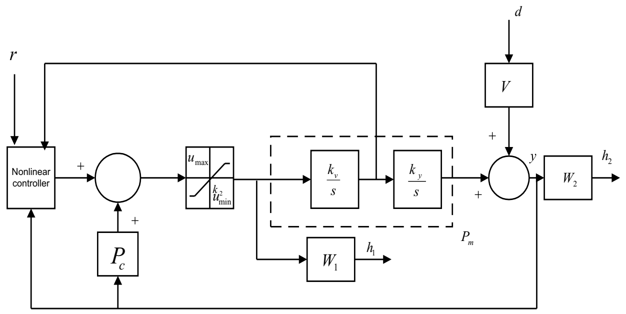


Figure 4: Nonlinear feedback controller Based on Compensator.

against disturbances, such as runout disturbances an internal-loop compensator can be used. In this section, such a compensator has been designed via H_∞ optimization techniques using the methods reported in [2].

Figure 4 shows a schematic diagram of the combined internal-loop compensator and non-linear state feedback controller. Let P_m be the plant model of HDD and let its transfer function be given by

$$G_m(s) = C_m \left(sI - A_m\right)^{-1} B_m + D_m \tag{19}$$

with $(A_m,\ B_m)$ being stabilizable and $(A_m,\ C_m)$ being detectable[3] . Let P_C be the internally stabilizing controller i.e., the internal-loop compensator designed through H_∞ optimization techniques and let its transfer function be

$$G_c(s) = C_c \left(sI - A_c\right)^{-1} B_c + D_c \tag{20}$$

and the minimal realizations of the weighting functions V, W_1 and W_2 be respectively given as

$$V(s) = C_v \left(sI - A_v\right)^{-1} B_v + D_v \tag{21}$$

$$W_1(s) = C_{w1} \left(sI - A_{w1}\right)^{-1} B_{w1} + D_{w1} \tag{22}$$

and

$$W_2(s) = C_{w2} \left(sI - A_{w2}\right)^{-1} B_{w2} + D_{w2} \tag{23}$$

The sensitivity function S and the complementary sensitivity function T are respectively definded as

$$S := \left(I + G_m G_c\right)^{-1} \tag{24}$$

and

$$T := G_c \left(I + G_m G_c\right)^{-1}. \tag{25}$$

It is required to find an internally stabilizing control law P_c such that the H_∞-norm of the closed-loop system from the disturbance d to the controlled output $h = (h_1', h_2')'$, i.e.,

$$\left\| \begin{matrix} -W_1 T V \\ W_2 S V \end{matrix} \right\|_\infty = \left\| \begin{matrix} W_1 T V \\ W_2 S V \end{matrix} \right\|_\infty \tag{26}$$

is minimized. In this paper, this problem was solved using Linear System and Control Toolbox reported in [1] and the full-order observer based compensator was found in the following form,

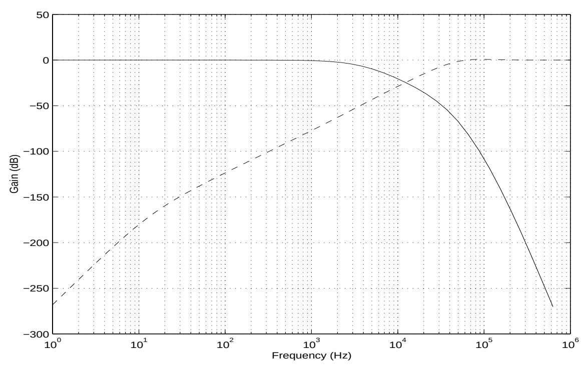


Figure 5: Sensitivity (S) and Complementary sensitivity (T) function.

$$P_c : \begin{cases} \dot{v} = A_c v + B_c y \\ u = C_c v + D_c y \end{cases} \tag{27}$$

where v is the state vector of the compensator. It is to be noted that the choices of the weighting functions V, W_1 and W_2 are subject to the design specifications of the overall system. In this paper, the weighting functions V, W_1 and W_2 were choosen based on the nature of the runout disturbance i.e assuming the runout disturbance is equivalent to a 100 Hz sinusoidal signal, complementary and sensitivity transfer function requirements, as shown in Figure 5 and they are given by

$$V(s) = \frac{628.3}{s + 628.3}, \tag{28}$$

$$W_1(s) = \frac{9100s}{s^2 + 9900s + 1e6}, \tag{29}$$

$$W_2(s) = \frac{5000}{s + 5000}. \tag{30}$$

Thus, the compensator was obtained as in the form of equation (27), and its eigen values were

$$-1.98 \times 10^4 \quad -564.44 \quad -2.20 \ \text{ and} -6.8 \times 10^3. \tag{31}$$

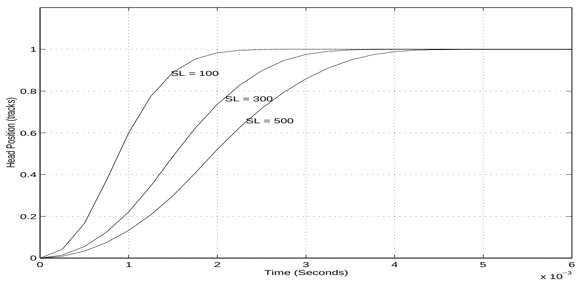
Therefore, the eigen values of the internal-loop were

$$-5.49 \times 10^3 \pm 8.34 \times 10^3 i, \ \ -1.12 \times 10^4, \ \ \text{and} -628.3 \tag{32}$$

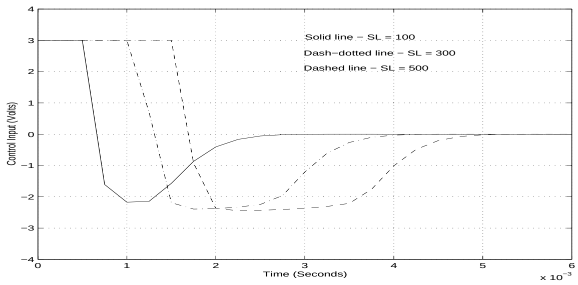
Since all the eigenvalues are in the left-half of the s-plane and the compensator was designed using H_∞ optimization, the above internal-loop compensator P_C can be used along with the nonlinear state-feedback time-optimal controller.

5 Summary of Simulation and Experimental Results

The above nonlinear feedback control law and the internal-loop compensator were simulated on a typical



(a) Output Response



(b) Control Signal

Figure 6: Normalized (to Full Scale)response for wide range of Seek Length(SL) in PTOS.

3.5" hard disk model whose frequency response characterstics were shown in Figure 2. The discretized form of controller, compensator and plant with sampling frequency 16 kHz were used in experiment and simulation. The results were then compared with conventional PTOS. The discretized plant model used in simulation is

$$x(k+1) = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 0.2344 & 0.2344 \end{pmatrix} x(k) \quad (33)$$

Following the design procedure in Section 3, we choose

$$f_1 = -0.2950 \quad f_2 = 0.3274 \quad \text{and } G = 0.0692$$

which resulted in the closed-loop damping ratio of 0.8. The solution of discrete Lyapunov equation was found as

$$P = \begin{pmatrix} -79.9985 & -77.4959 \\ -77.4959 & -78.9985 \end{pmatrix} \quad (34)$$

The Linear feedback control law is then given by

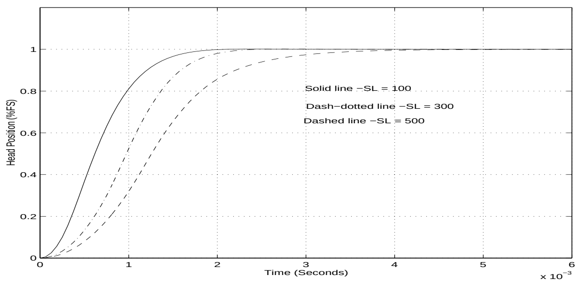
$$u_L = 0.2950x_1(k) - 0.3274x_2(k) + 0.0692r \quad (35)$$

Finally choosing the function $\rho(x, r)$ as

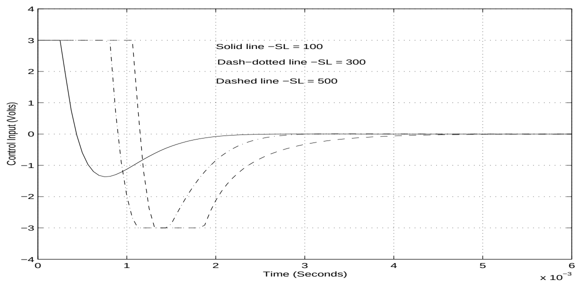
$$\rho(x, r) = 10e^{-|y-r|} \quad (36)$$

the composite nonlinear feedback law is explicitly given by

$$\begin{aligned} u = \quad & (0.2950 + 77.4959\rho(x, r))x_1(k) \\ & -(0.3274 - 78.9985\rho(x, r))x_2(k) \\ & +(0.0692 + 4.266\rho(x, r))r \end{aligned} \quad (37)$$



(a) Output Response



(b) Control Signal

Figure 7: Normalized (to Full Scale)response for wide range of Seek Length(SL) in Composite nonlinear control.

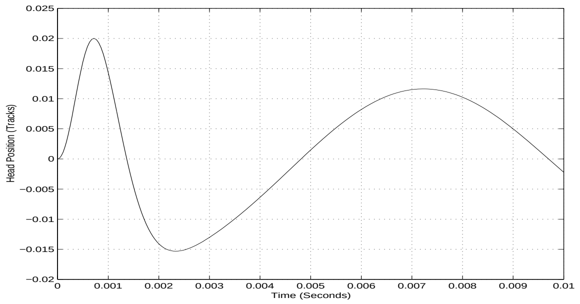
The Figure 6 shows the normalized output responses and control signals applied for various track seek lengths by the use of the conventional PTOS method. It is clear from the control histroy that the deceleration stage is very slow which causes large time to settle at the required track and also there is no extra control input available for disturbance compensation.

The Figure 7 shows the normalised response with the combined nonlinear controller and compensator for various track seeks with runout distubance approximated as a sine wave of frequency 100 Hz & amplitude 0.5 tracks. These results shows that the composite nonlinear control law has outperformed the PTOS based HDD servo system .

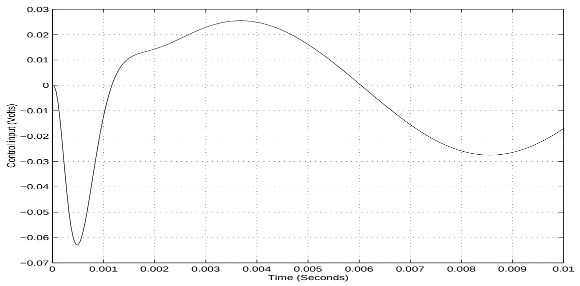
In order to show the performance of the internal-loop compensator alone to the runout disturbances that appears in HDD servo systems, simulations were done for zero track reference and with disturbance 0.5 tracks with frequency of 100 Hz. These results were shown in Figure 8. The following table ie Table 1, shows the

Table 1: Seek Time from Simulation and Implementation Results.

TSL	ST(msec)		% decrease	ST from IR
	PTOS	CNC		using CNC
100	2.15	1.90	11.6	1.95
300	3.41	2.30	32.5	2.85
500	4.45	3.90	12.3	4.10



(a) Output Response



(b) Control Signal

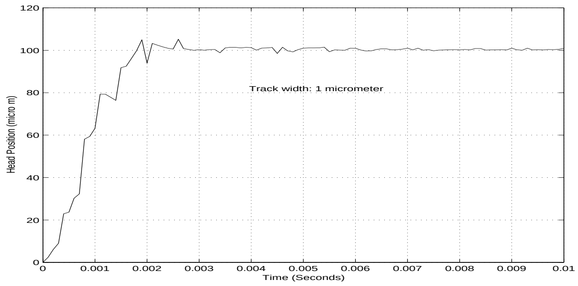
Figure 8: Response for 100Hz & 0.5 tracks runout disturbance.

percentage decrease in seek time in simulation with the composite nonlinear controller (CNC) over the PTOS method. Also, it shows the seek time from the implementation results (IR) using composite nonlinear controller.

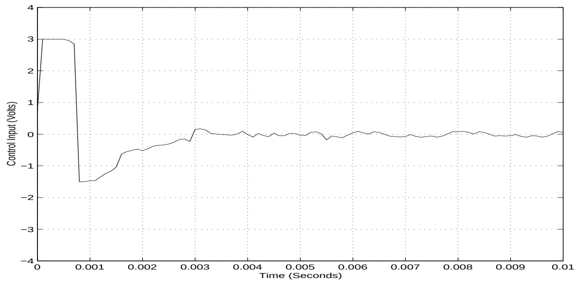
The implementation results were obtained from the experiment on a typical 3.5-inch hard disk drive. Figure 9 shows the implemetation results for 100 track seek using the composite nonlinear controller. Our controller was implemented on an open hard disk drive with a TMS320 digital signal processor (DSP) with a sampling rate of 16 kHz. The R/W head position was measured using a Laser Doppler Vibrometer (LDV) and the track width was assumed as $1\mu m$ and hence the track density was assumed as 25,000 TPI. The experimental results were in line with simulation results.

6 Concluding Remarks

A method to improve the seek time of hard disk drive servo system has been proposed in this paper. The proposed method used nonlinear state feedback control which increases the damping ratio as the head approaches the desired track. This resulted in decrease in the seek time. Also, an internal-loop compensator was designed via H_∞ optimization suppressed the runout disturbances and improved the robustness. The experimental results shows that the proposed method has less seek time than the conventional method. The problem



(a) Output Response



(b) Control Signal

Figure 9: Experimental results for 100 track seek with nonlinear feedback control.

of mode switching in HDD servo systems was eliminated with the use of the composite nonlinear controllers.

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