Integrating Identification with Robust Control: a Mixed H_2/H_∞ Approach

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Abstract

This paper builds on previous work which developed a robust control algorithm which used statistical confidence bounds. The key idea was to change the nominal design so as to reduce the overall variability from an a-priori specified desired performance. However, due to the choice of cost function, the original design method did not necessarily guarantee stability. This paper addresses the latter problem in a mixed H_2/H_∞ framework.

1 Introduction

In the earlier work, a procedure was proposed for obtaining a better match between robust control and system identification by using statistical confidence bounds for robust control design. The current paper builds on previous work described in ([8] [5] [16]). The first step towards a better 'match' was to seek a cost function for robust design that was closely related to the conventional model information provided by system identification. Our choice was the mean squared error between the predicted actual performance (as measured by the closed loop sensitivity function) and some target performance. In order to obtain a simple solution, our design was based on two steps: a nominal control system was designed and this was then enhanced to achieve the target performance. The type of cost function used however, does not, by itself, produce a robust control system that has guaranteed stability. Instead a system is produced that minimises the variance from a nominal design. Here, we extend the earlier methods to cover the issue of stability by adding a constraint to the original quadratic cost function. This results in a form of mixed H_2/H_∞ optimisation problem. The existing literature in this area has focused on finding solutions from state space formulations (see for example,[4]). This type of problem has also been addressed with the aid of convex nonlinear programming [3]. What we propose here is a numerical approach, similar in spirit to that of nonlinear programming. We parameterise the robust controller using an orthonormal model ([11]). This converts the nonlinear optimisation problem into a simple linear quadratic problem with quadratic or linear inequality constraints. A classical algorithm proposed by Lawson (1961) (see [10]) is then used to convert this linear quadratic programming problem with quadratic constraints into an iterative weighted least squares problem. The convergence of this algorithm is yet to be proved, however, simulation studies have proven to be very satisfactory. A similar algorithm was used in [14] for designing a constrained model predictive controller.

2 Controller Validation

Here we present a technique for validating a robust controller. The validation procedure checks whether a given robust controller stabilizes all models within a given probabilistic uncertainty set containing the frequency response of the true system. This uncertainty set is deduced from the covariance matrices of the estimated frequency responses. The recent advances in this area include [1].

2.1 Statistical Confidence Bounds

We assume that we are given a nominal model with numerator $B_0(s)$ and denominator $A_0(s)$ together with a probabilistic description of the errors [7] [12] for $B_0(s)$ and $A_0(s)$ respectively. The true process is assumed to be $G(s)$ with numerator $B(s)$ and denominator $A(s)$, hence the error $\tilde{G}$ between the plant and the estimated nominal model can be described by

$$\tilde{G}(s) = \frac{A_0(s)(B(s) - B_0(s)) - B_0(s)(A(s) - A_0(s))}{A(s)A_0(s)} \tag{1}$$

This error will be approximated by

$$\tilde{G} \approx \frac{A_0(jw)(B - B_0) - B_0(jw)(A - A_0)}{A_0(jw)^2} \tag{2}$$

Standard identification algorithms [12] yield an estimated model satisfying: $E(B(jw) - B_0(jw)) = 0$ and

$E(A(jw) - A_0(jw)) = 0$. From Equation (2) it follows that $E(\tilde{G}(jw)) = 0$. Using (2), the variance of the model error takes the form:

$$E(|\tilde{G}(jw)|^2) \approx \frac{E(|\Delta(jw)|^2)}{|A_0(jw)^2|^2} \quad (3)$$

where $\Delta(jw)$ is defined by

$$\Delta = A_0(jw)(B - B_0) - B_0(jw)(A - A_0) \quad (4)$$

This uncertainty description (3) (variance error) has been used previously by us to reduce performance variability [16], and it will be used here as part of the design requirement.

To derive a bound on the unknown process frequency response, we consider the covariance matrix, $P(w)$, for $\hat{G}(jw)$ given by

$$P(w) = E\{\begin{bmatrix} Re(\tilde{G}(jw)) \\ Im(\tilde{G}(jw)) \end{bmatrix} \begin{bmatrix} Re(\tilde{G}(jw)) & Im(\tilde{G}(jw)) \end{bmatrix}\}$$

This covariance matrix can be estimated from standard identification algorithms [15], where undermodelling is explicitly considered. Assuming that the estimated covariances matrices, $P(w_k)$, $k = 0, 1, 2, \ldots, M$ are available, we can construct confidence regions on the Nyquist plot. In particular, if the errors have a normal distribution, then the quantity

$$\begin{bmatrix} Re(\tilde{G}(jw_k)) & Im(\tilde{G}(jw_k)) \end{bmatrix} P(w_k)^{-1} \begin{bmatrix} Re(\tilde{G}(jw_k)) \\ Im(\tilde{G}(jw_k)) \end{bmatrix}$$

will have a Chi-squared distribution on two degrees of freedom. Hence, using Bayes rule, it follows that we can find a constant β such that the following ellipsoid:

$$\begin{bmatrix} Re(\tilde{G}) & Im(\tilde{G}) \end{bmatrix} P(w_k)^{-1} \begin{bmatrix} Re(\tilde{G}) \\ Im(\tilde{G}) \end{bmatrix} < \beta^2 \quad (5)$$

centred on the nominal frequency estimate $\hat{G}(jw_k)$ contains a desired fraction (γ) of the values of $\tilde{G}(jw)$.

2.2 Controller Validation

Suppose that a controller is designed based on the nominal model to give nominal closed-loop stability. The question whether this controller will stabilize the real plant with a prescribed probability γ, is answered using the following controller validation procedure.

Theorem 1 *Closed-loop stability is guaranteed with prescribed probability γ when the designed controller*

$$C(s) = \frac{M(s)}{L(s)}$$

is applied to the true plant G, if the estimated model $G_0(s)$ has the same number of unstable poles as the true plant $G(s)$ and, for all, w either of the following conditions is satisfied

$$|\frac{C(jw)}{1 + C(jw)G_0(jw)}|^2 \times \beta^2 \lambda_{max} P(w) < 1 \quad (6)$$

$$|\frac{C(jw)}{1 + C(jw)G_0(jw)}|^2 \times \beta^2 E(|\tilde{G}|^2) < 1 \quad (7)$$

Proof:
Under the conditions given in theorem, it is known that the closed-loop system is guaranteed stable if, for all w

$$|\frac{C(jw)}{1 + C(jw)G_0(jw)}||\tilde{G}(jw)| < 1 \quad (8)$$

where $\tilde{G} = G - \hat{G}$ is the specific model error for $\hat{G}$. Note that (8) is satisfied if, for all, w

$$|\frac{C(jw)}{1 + C(jw)G_0(jw)}|^2|\tilde{G}(jw)|^2 < 1 \quad (9)$$

Using equation (5) it is seen that with probability γ

$$\begin{aligned} & ([Re(\tilde{G})]^2 + [Im(\tilde{G})]^2) \times \lambda_{min}(P(w))^{-1} \\ < & \begin{bmatrix} Re(\tilde{G}) & Im(\tilde{G})) \end{bmatrix} P(w)^{-1} \begin{bmatrix} Re(\tilde{G}) \\ Im(\tilde{G}) \end{bmatrix} \\ < & \beta^2 \quad (10) \end{aligned}$$

Hence with probability γ,

$$\begin{aligned} & |\tilde{G}(jw)|^2 < \beta^2 \frac{1}{\lambda_{min}(P(w))^{-1}} \\ = & \beta^2 \lambda_{max}(P(w)) \quad (11) \\ \leq & \beta^2 trace\{P(w)\} \\ = & \beta^2 E(|\tilde{G}(jw)|^2) \quad (12) \end{aligned}$$

The result follows by substituting (12) or (11) into (9).

Remark 1 • *In practice, assuming that the frequency response of the plant transfer function is smooth over the whole frequency region, (7) can be checked over a given frequency range pointwise. However, in theory, this is an infinite dimensional problem on the frequency domain. To avoid this search, linear matrix inequality (LMI) tools ([2]) can be used to validate the robust controller, see for example ([1]).*

3 The H_2 Cost Function for Performance Robustness

The results presented in this section are a summary of our previous work in this area; a detailed description is available in ([8] [16]).

Based on the nominal model $G_0(jw)$, we assume that a design is carried out which leads to acceptable performance. Let us say that this has been achieved with a nominal controller C_0 and that the corresponding nominal sensitivity function is S_0. Of course, the true plant is assumed to satisfy (3) and the value S_0 will thus not be achieved in practice. Hence there will be some variability of the achieved sensitivity S from S_0. Our strategy is to modify a nominal controller so as to minimize the expected mean square variation of the actual system performance from the a-priori given desired performance as measured by S_0. We assume that we design the nominal controller $C_0(s) = \frac{M(s)}{L(s)}$ to stabilize the nominal model $G_0(s) = \frac{B_0(s)}{A_0(s)}$ using a pole-assignment method so that the following Diophantine equation is satisfied $A_0(s)L(s) + B_0(s)M(s) = E(s)$ where $E(s)$ is stable, and the degrees of $P(s)$, $L(s)$ and $E(s)$ are properly chosen. The nominal sensitivity function $S_0(s)$ for this design is then given by $S_0(s) = \frac{A_0(s)L(s)}{E(s)}$. The achieved sensitivity, S_1 using C_0 applied to the true plant, G, is given by $S_1 = \frac{1}{1+C_0G}$. The key idea in the proposed robust design is to adjust the controller (i.e. replace C_0 by some controller C, leading to a sensitivity S_2) so that the average 'distance' between the sensitivity S_2 and S_0 is minimized. This problem is addressed by using the alternative Youla parameterisation. Using this parameterisation, the class of all controllers that stabilize the nominal model $G_0(s)$ is

$$C(s) = \frac{M(s) + A_0(s)\tilde{Q}(s)}{L(s) - B_0(s)\tilde{Q}(s)} \quad (13)$$

where $\tilde{Q}(s)$ is stable. For notational convenience, we also introduce $Q(s)$ defined by

$$\begin{aligned} Q(s) &= \frac{C(s)}{1 + C(s)G_0(s)} \\ &= \frac{A_0(s)M(s)}{E(s)} + \frac{A_0(s)^2}{E(s)}\tilde{Q}(s) \end{aligned} \quad (14)$$

Note that the sensitivity function S_2 obtained using $C(s)$ applied to the true plant is expressed as $S_2(s) = \frac{1}{1+C(s)G(s)} = \frac{1-Q(s)G_0(s)}{1+Q(s)\tilde{G}(s)}$. Here S_2 is expressed in terms of the uncertainty $\tilde{G}(s)$. Observe that S_2 and S_0 denote respectively the sensitivity achieved when the plant is G and the controller is parameterized by Q, and the sensitivity when the plant is G_0 and the controller is parameterized by Q_0. Thus the error to be minimized with respect to $\tilde{Q}$ is

$$S_2 - S_0 = \frac{1 - Q(s)G_0(s)}{1 + Q(s)\tilde{G}(s)} - (1 - Q_0(s)G_0(s)) \quad (15)$$

This is a nonlinear function of $Q(s)$. We thus choose a suitable weighting function on the error to achieve linearisation. A suitable candidate for the weighting function is $W_1(s) = 1 + Q(s)\tilde{G}(s)$. Selecting the L_2 norm as the measure of error, we define the following loss function to be minimized with respect to $\tilde{Q}$

$$\begin{aligned} J &= \int_{-\infty}^{\infty} E\left(|W_1(jw)(S_2(jw) - S_0(jw)|^2\right) dw \\ &= \int_{-\infty}^{\infty} |\frac{A_0B_0}{E}|^2|\tilde{Q}|^2 dw \\ &+ \int_{-\infty}^{\infty} |\frac{LM}{E^2} + \frac{S_0}{E}\tilde{Q}|^2 E(|\Delta|^2) dw \end{aligned} \quad (16)$$

where we have used the assumption that $E(\Delta) = 0$. This cost function is quadratic in $\tilde{Q}$ leading to a simple design procedure.

4 Mixed H_2/H_∞ Approach

We consider the stable case only in the sequel. The results can be readily extended to include the unstable case by slightly changing the formulation. The essential idea of the robust controller design with guaranteed stability (at a given probabilistic confidence level) is to minimize the original cost

$$J = \int_{-\infty}^{\infty} (|G_0\tilde{Q}|^2 + |S_0^{'}Q_0 + S_0\tilde{Q}|E(|\tilde{G}|^2))dw \quad (17)$$

subject to the robust stability constraint

$$|Q_0(jw) + \tilde{Q}|^2 \times \beta^2 \lambda_{max}(P(w)) < 1 - \epsilon \quad (18)$$

for all $w \geq 0$, where ϵ is a small constant. The solution to this problem is obtained using either a quadratic programming algorithm or a weighted least squares algorithm as shown in the following subsections.

4.1 Least Squares Solution using Orthonormal Functions

Before we tackle the constrained optimization problem, this subsection shows that the solution of the original cost function can be obtained using a simple least squares algorithm when $\tilde{Q}$ is parameterised using an orthonormal function model. Let us assume that there is an integrator contained in the nominal controller. We then write $\tilde{Q} = s\tilde{Q}_{int}$, where $\tilde{Q}_{int}$ is strictly proper. Since $\tilde{Q}$ must be stable, we choose to represent $\tilde{Q}_{int}$ using an othonormal network model [11]: $\tilde{Q}(s)_{int} = \sum_{i=1}^{N} c_i L_i(s)$ where $L_i(s)$, $i = 1, 2, \ldots, N$, are suitably chosen orthonormal filters, which satisfies the orthonormal properties $\frac{1}{2\pi}\int_{-\infty}^{\infty} |L_i(jw)|^2 dw = 1$ and $\frac{1}{2\pi}\int_{-\infty}^{\infty} L_i(jw)L_k(jw)dw = 0$ where $k \neq i$.

A particular choice for $L_i(s)$ is the set of Laguerre filters where for some $p > 0$, $L_i(s) = \sqrt{2p}\frac{(s-p)^{i-1}}{(s+p)^i}$. Alternatively one could use Kautz networks [9] which allow complex poles.

Proposition 1 *The original cost function J defined by equation (17) is equivalent to the quadratic cost function*

$$J^{'} = \int_0^\infty 2\alpha^T[Re(\Omega)Re(\Omega)^T + Im(\Omega)Im(\Omega)^T]M(w)\alpha + 4[Re(R)Re(\Omega)^T + Im(R)Im(\Omega)^T]\alpha dw + C \quad (19)$$

where C is a constant, $\alpha = [\ c_1 \quad c_2 \quad \ldots \quad c_N\ ]^T$; $\Omega(jw)^T = [L_1(jw) \quad L_2(jw) \quad \ldots \quad L_N(jw)]$; $M(w) = |G_0(jw)|^2 + |S_0(jw)|^2 E(|\tilde{G}|^2)$; and $R(jw) = S_0^* S_0^{'} Q_0 E(|\tilde{G}|^2)$ ($S_0^{'} = sS^{'}$).

Proof: The results are obtained from an extension to equation (17).

Theorem 2 *The parameter vector α, that minimizes the cost function $J^{'}$, is given by*

$$\alpha = -[\int_0^\infty (Re(\Omega)Re(\Omega)^T + Im(\Omega)Im(\Omega)^T)M(w)dw]^{-1} \int_0^\infty (Re(R)Re(\Omega) + Im(R)Im(\Omega))dw \quad (20)$$

Proof: The result follows by noting that the cost function (19) is a quadratic function of α, and its minimum is thus achieved for α as in equation (20).

4.2 Quadratic Programming Approach to the Constrained Solution

Commercial software packages are available for quadratic programming (qp). We show here how these packages can be used to add the stability constraint (18) to the optimisation problem discussed above. The following proposition shows how the original constrained stability problem can be converted into a standard qp problem.

Proposition 2 *The constraint*

$$|Q_0(jw) + \tilde{Q}|^2 \times \beta^2 \lambda_{max}(P(jw)) \leq 1 - \epsilon \quad (21)$$

is guaranteed for all w if

$$-\frac{\sqrt{1-\epsilon}}{\beta\sqrt{2}\sqrt{\lambda_{max}(P(jw))}} + Re(Q_0(jw)) \leq w \times Im(\Omega)^T\alpha \leq \frac{\sqrt{1-\epsilon}}{\sqrt{2}\beta\sqrt{\lambda_{max}(P(jw))}} + Re(Q_0(jw)) \quad (22)$$

and

$$-\frac{\sqrt{1-\epsilon}}{\sqrt{2}\beta\sqrt{\lambda_{max}(P(jw))}} - Im(Q_0(jw)) \leq w \times Re(\Omega)^T\alpha \leq \frac{\sqrt{1-\epsilon}}{\sqrt{2}\beta\sqrt{\lambda_{max}(P(jw))}} - Im(Q_0(jw)) \quad (23)$$

Proof: From (21) we have

$$|Q_0(jw) + \tilde{Q}|^2 \leq \frac{1-\epsilon}{\beta^2\lambda_{max}(P(jw))} \quad (24)$$

which is equivalent to

$$\begin{aligned} & |Re(Q_0) + jIm(Q_0) + jwRe(\Omega)^T\alpha - wIm(\Omega)^T\alpha|^2 \\ = & (Re(Q_0) - wIm(\Omega)^T\alpha)^2 + (Im(Q_0) + wRe(\Omega)^T\alpha)^2 \\ \leq & \frac{1-\epsilon}{\beta^2\lambda_{max}(P(jw))} \end{aligned} \quad (25)$$

Equation (25) defines a circle with radius $\sqrt{\frac{1-\epsilon}{\beta^2\lambda_{max}(P(jw))}}$. A maximum rectangular area that is contained inside this circle is defined by equations (22) and (23). Then the result is obtained by replacing the circular area with the rectangular area which replaces the quadratic constraints by the linear constraints. We then have:

Theorem 3 *The optimal solution α that minimizes a discretized form of the cost function (17) subject to the constraint (18) at the sampling frequency points w_0, w_1, …, w_M is the solution of the quadratic programme:*

$$\begin{aligned} J_2 = & 2\alpha^T[\sum_{k=0}^{M}(Re(\Omega)Re(\Omega)^T \\ + & Im(\Omega)Im(\Omega)^T)M(jw_k)\Delta w]\alpha \\ + & 4\sum_{k=0}^{M}(Re(R(w_k))Re(\Omega^T) \\ + & Im(R(w_k))Im(\Omega)^T)\Delta w\alpha \end{aligned} \quad (26)$$

with the set of linear constraints defined by (22) and (23) for $k = 0, 1, 2, \ldots$.

4.3 Weighted Least Squares Approach

An alternative to qp is next described, based on an iterative procedure. This algorithm achieves the same computational objective as quadratic programming, yet appears to be better numerically.

Background to Lawson's Algorithm

For a given sequence $\{y(k)\}$ and a regressor vector $\phi(k)$ ($\phi(k)$ has dimension of $n \times 1$ and $k = 1, 2, \ldots, M$), we seek a parameter vector $\hat{\theta}$ such that

$$\hat{\theta} = argmin\{sup_k|y(k) - \phi(k)^T\theta|\} \quad (27)$$

Lawson(1961)[10] proposed an algorithm to convert this L_∞ problem to an L_2 problem which can be readily solved using an iterative ordinary least squares algorithm.

For the mth iteration, define the error $e(k)^m = y(k) - \phi(k)^T\theta^m$. A set of weights $W(k)^{m+1}$ is then calculated according to

$$W(k)^{m+1} = \frac{W(k)^m|e(k)^m|}{\sum_{k=1}^{M} W(k)^m|e(k)^m|} \quad (28)$$

then θ^{m+1} was found as

$$\hat{\theta}^{m+1} = argmin\{\sum_{k=1}^{M} W(k)^m [y(k) - \phi(k)^T \theta^{m+1}]^2\} \quad (29)$$

It was shown in [13] that θ^m converges to θ^* where

$$\theta^* = argmin_\theta\{sup_k |y(k) - \phi(k)^T \theta|\} \quad (30)$$

Application to H_2/H_∞ Problem
Dividing both sides of equation (18) by $1-\epsilon$ gives

$$\frac{1}{1-\epsilon}|Q_0(jw) + \tilde{Q}|^2 \times \beta^2 \lambda_{max}(P(w)) < 1 \quad (31)$$

It can be shown that the minimum of the integral equation

$$\int_{-\infty}^{\infty} \frac{W(w)}{1-\epsilon} |(Q_0(jw) + \tilde{Q})|^2 \times \beta^2 \lambda_{max}(P(w)) dw \quad (32)$$

is given by

$$\begin{aligned} \alpha &= -[\int_0^\infty W(w)\lambda_{max}(P(w))w^2[Re(\Omega)Re(\Omega)^T \\ &+ Im(\Omega)Im(\Omega)^T]dw]^{-1} \\ &\times \int_0^\infty W(w)\lambda_{max}(P(jw))w[Im(Q_0)Re(\Omega) \\ &- Re(Q_0)Im(\Omega)]dw \end{aligned} \quad (33)$$

Define the frequency error at w_k, $k = 0, 1, 2, \ldots, M$ as

$$|e(w_k)| = \sqrt{\frac{\lambda_{max}(P(jw_k))}{1-\epsilon}} \beta |Q_0(jw_k) + \tilde{Q}(jw_k)| \quad (34)$$

Calculate the least squares solution for α as

$$\begin{aligned} \alpha &= -[\sum_{k=0}^{M}(Re(\Omega(w_k)Re(\Omega(w_k))^T \\ &+ Im(\Omega(w_k))Im(\Omega(w_k))^T)M(jw_k)\Delta w]^{-1} \\ &\times \sum_{k=0}^{M}(Re(R(w_k))Re(\Omega(w_k)) \\ &+ Im(R(w_k))Im(\Omega(w_k)))\Delta w \end{aligned} \quad (35)$$

Calculate the absolute frequency error $|e(w_k)|$ for $k = 1, 2, \ldots, M$;
Update the weights

$$W(w_k)^{m+1} = \frac{W(w_k)^m |e(w_k)^m|}{\sum_{k=0}^{M} W(w_k)^m |e(w_k)^m|} \quad (36)$$

and

$$V^{m+1} = \frac{V^m}{\sum_{k=0}^{M} W(w_k)^m |e(w_k)^m|} \quad (37)$$

Find the α that minimizes the augmented cost function

$$\begin{aligned} J^{m+1} &= \alpha^T \sum_{k=0}^{M}[W^m \frac{\beta^2}{1-\epsilon}\lambda_{max}(P)w_k^2[Re(\Omega)Re(\Omega)^T \\ &+ Im(\Omega)Im(\Omega)^T]\Delta w]\alpha \\ &+ 2\sum_{k=0}^{M}\frac{\beta^2}{1-\epsilon}W^m\lambda_{max}(P)w_k[Im(Q_0)Re(\Omega)^T \\ &- Re(Q_0)Im(\Omega)^T]\alpha \\ &+ V^m\alpha^T[\sum_{k=0}^{M}(Re(\Omega)Re(\Omega)^T \\ &+ Im(\Omega)Im(\Omega)^T)M(jw_k)\Delta w]\alpha \\ &+ 2\sum_{k=0}^{M}(Re(R)Re(\Omega)^T) + Im(R)Im(\Omega)^T)\Delta w\alpha \end{aligned} \quad (38)$$

This computational procedure is iteratively performed until the estimates converge.

4.4 Simulation Examples

This example is based on a pilot plant heat exchanger. A robust control system was designed to minimize the nominal cost function [6]. The approach used previously was to fit part of the H_2 cost function using a Laguerre model.

The estimated model for the heat exchanger was

$$G_0(s) = \frac{-0.7156s + 0.8829}{(s+1)^2} \quad (39)$$

The nominal Q parameter, Q_0, was selected as

$$Q_0 = \frac{4(s+1)^2}{0.8829(s+2)^2} \quad (40)$$

The covariance matrices $P(w)$ were calculated every 0.01 (rad/sec) (i.e. $\Delta w = 0.01$). β^2 was taken as 5.99 to give a probability $\gamma = 0.95$. A third order Laguerre model was used to describe $\tilde{Q}_{int}$ with the desired poles selected at $p = -2.5$. We artificially 'enlarge' the true confidence bounds by increasing the covariance matrices, originally obtained from the experimental data, by a factor of 4, so that the original design algorithm produces a robust controller that does not guarantee stability for the family of plants, see Figure 1. With the mixed H_2/H_∞ approach, the robust controller guarantees the stability for the family of the plants as shown in Figure 2. Note that the controller has been modified so as to ensure that (18) is now satisfied.

5 Conclusion

This paper has described a mixed $H2/H_\infty$ approach to robust controller design using statistical confidence bounds. A straight forward procedure was first discussed for controller validation when the uncertainty is obtained from identification. By parameterising the robust controller using an orthonormal network model, a robust controller is designed to stabilize the family of plants with a prescribed probability. The resulting H_2/H_∞ problem is solved using an iterative weighted least squares approach.

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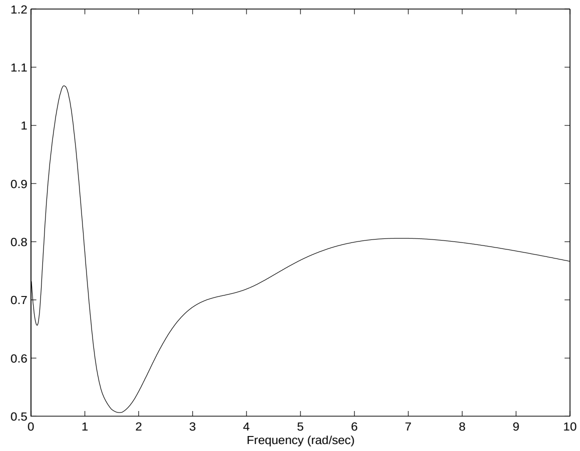

Figure 1: Left hand side of equation (18) (unconstrained solution).

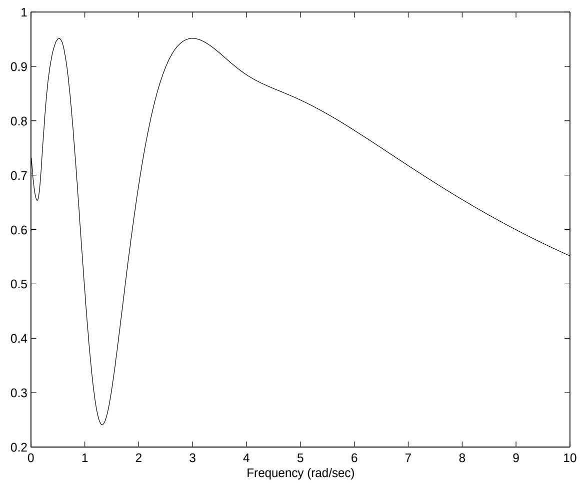

Figure 2: Left hand side of equation (18)(constrained solution)