Robust Tracking Controllers of Uncertain Delay Systems

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Abstract

A new procedure for determining robust tracking control laws for a class of linear uncertain time-delay systems is presented in this paper. The system is controlled to track a dynamic input generated from a reference model. In the case that the system uncertainties satisfy the generalized matching conditions, by using the proposed linear controller, the ultimate bound of the tracking error can be made arbitrarily small and, therefore, the system practically tracks the reference model. In the case of mismatched uncertainties, we present a sufficient condition such that the designed controller guarantees ultimate boundedness of the tracking error. Compared with the existing results, our approach admits much larger system uncertainties in the control matrix. Numerical examples also show that the resultant linear robust tracking controllers are less conservative.

Keywords: Robust control, uncertain systems, tracking, time-delay systems, Riccati-type equation.

1. Introduction

Time delays exist in many engineering systems, such as chemical processes and long transmission lines in pneumatic systems. It may severely affect the system behaviours. So the control problem of uncertain time-delay systems has received considerable attention in recent years (see, for example, [1]~[3]). The topic of robust tracking of dynamic signals for linear uncertain time-delay systems was addressed in [4] and [5]. More recently, [6] has presented an interesting result on the design of robust controllers for a class of uncertain time-delay systems to track dynamic inputs generated from a non-delay reference model. However, in order to apply this result of [6], a strong condition must be imposed on the system uncertainty arising in the control matrix.

In this paper, we further investigate the design of robust tracking controllers for this class of linear uncertain time-delay systems. First, we take away the strong condition of [6] and allow the uncertainty existing in the control matrix to be twice as large as that in [6]. Then, by using the positive definite solution of a Riccati-type equation, a new procedure for determining linear robust tracking control laws is derived.

2. Systems and Assumptions

The uncertain time-delay system is described by

$$\begin{aligned} \dot{x}(t) &= [A+\Delta A(r(t))]x(t) + A_d(s(t))x(t-\tau) \\ &\quad + [B+\Delta B(v(t))]u(t) + W(\alpha(t))q(t) \\ y(t) &= C\,x(t) \\ x(t) &= \Phi(t) \quad , \quad t\in[-\tau, 0] \end{aligned} \tag{1}$$

where $x(t)\in R^n$, $u(t)\in R^m$, and $y(t)\in R^p$ are the state, control, and output vectors, respectively. $\tau\in R^+$ is the time delay and $W(\alpha(t))q(t)\in R^n$ is an additive disturbance. A, B, and C are constant matrices of appropriate dimensions. It is assumed that the right-hand side of (1) is continuous and satisfies enough smoothness conditions to ensure the existence and uniqueness of the solution through every initial condition $\Phi(t)$. The uncertainties $r(t)\in\Omega_1\subset R^k$, $s(t)\in\Omega_2\subset R^l$, $v(t)\in\Omega_3\subset R^s$, and $\alpha(t)\in\Omega_4\subset R^q$ are Lebesgue measurable and take values in known compact bounding sets Ω_1, Ω_2, Ω_3, and Ω_4, respectively. The pair (A,B) is assumed controllable and there exist continuous matrices $D(r)$, $E(s)$, $F(v)$, and $L(\alpha)$ of appropriate dimensions such that

$$\Delta A(r) = BD(r) + \Delta\tilde{A}(r) \quad ; \quad A_d(s) = BE(s) + \Delta\tilde{A}_d(s)$$
$$\Delta B(v) = BF(v) \quad ; \quad W(\alpha) = BL(\alpha) + \tilde{W}(\alpha) \tag{2}$$

Also

$$F(v) + F^T(v) + 2I > 0 \quad , \quad \text{for all} \quad v \in \Omega_3 \tag{3}$$

As indicated in [6], the matrices $BD(r)$, $BE(s)$, $BF(v)$, and $BL(\alpha)$ model the matched uncertainties, whereas the terms $\Delta\tilde{A}(r)$, $\Delta\tilde{A}_d(s)$, and $\tilde{W}(\alpha)$ represent the mismatched uncertainties. But, compared with [6], the matched conditions given above are in a generalized sense. It is noted that condition (3) above is weaker than condition (3) of [6]. One may also observe that we allow the uncertainty in the control matrix to be twice as large as that in [6].

For completeness, we restate the definition of uniformly ultimate boundedness in the following.

Definition 1[6]: The solutions $x_t(t_0, \Phi)$ of (1) are said to be uniformly ultimately bounded with respect to the bound ε if, for each $\delta > 0$, there exists $T = T(\varepsilon, \delta) > 0$ independent of t_0 such that $\|x(t_0, \Phi(t))\| \le \varepsilon$ for all $t \ge t_0 + T$ when $|x_{t0}| < \delta$, where $|x_{t0}| \triangleq \sup \|x(t_0 + \theta)\|$, $-\tau \le \theta \le 0$.

The reference input $y_m(t)$ is the output of the reference model described by

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{4}$$

where $x_m(t) \in R^{n_m}$, and $y_m(t)$ has the same dimension as $y(t)$. Similarly to [6] and [7], it is further assumed that $\|x_m(t)\| \le M^*$ for all $t \ge 0$ and there exist matrices $G \in R^{n \times n_m}$ and $H \in R^{m \times n_m}$ such that

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} GA_m \\ C_m \end{bmatrix} \tag{5}$$

The solution of G and H from (5) is discussed in [7].

The objective of this paper is to find a linear robust controller such that the output of system (1) follows the output of model (4) as closely as possible.

3. Controller Design

In this section, by using the positive definite solution of a Riccati-type equation instead of the Lyapunov equation, we present a new procedure for determining the robust tracking controller for system (1).

The structure of the tracking controller to be determined in this paper is

$$u(t) = [K - \gamma B^T P]x(t) + [H - (K - \gamma B^T P)G]x_m(t) \tag{6}$$

It contains both a linear feedback control based on $x(t)$ and a linear feedforward control based on $x_m(t)$. The gain matrix K is chosen by using the pole placement or linear optimal regulator technique such that $A_1 = A + BK$ is an asymptotically stable matrix. The parameter $\gamma > 0$ is a scalar and the matrix P is the positive definite solution of the following Riccati-type equation [8]

$$A_1^T P + PA_1 + \xi PBB^T P + Q = 0 \tag{7}$$

where Q is a positive definite symmetric matrix and $\xi > 0$ is a scalar.

Let $z(t) \triangleq x(t) - Gx_m(t)$. From (1), (4) and (5), we have the tracking error

$$e(t) \triangleq y(t) - y_m(t) = Cz(t) \tag{8}$$

Further, letting $\omega(t) \triangleq u(t) - Hx_m(t)$, the governing equation of $z(t)$ can be written as

$$\begin{aligned} \dot{z}(t) &= [A + \Delta A(r)]z(t) + A_d(s)z(t-\tau) \\ &\quad + [B + \Delta B(v)]\omega(t) + h(r,s,v,\alpha,x_m) \end{aligned} \tag{9}$$

where

$$\begin{aligned} h(r,s,v,\alpha,x_m) &= [\Delta A(r)G + \Delta B(v)H]x_m(t) \\ &\quad + A_d(s)Gx_m(t-\tau) + W(\alpha)q(t) \end{aligned} \tag{10}$$

(10) can be further divided into matched and mismatched parts as follows

$$h(r,s,v,\alpha,,x_m) = Bf(r,s,v,\alpha,,x_m) + g(r,s,v,\alpha,,x_m) \tag{11}$$

where

$$\begin{aligned} f(r,s,v,\alpha,x_m) &= [D(r)G + F(v)H]x_m(t) \\ &\quad + E(s)Gx_m(t-\tau) + L(\alpha)q(t) \end{aligned} \tag{12}$$

$$\begin{aligned} g(r,s,v,\alpha,x_m) &= \Delta\tilde{A}(r)Gx_m(t) \\ &\quad + \Delta\tilde{A}_d(s)Gx_m(t-\tau) + \tilde{W}(\alpha)q(t) \end{aligned} \tag{13}$$

In the first place, we assume that the system uncertainties are fully matched, i.e.

$$\Delta\tilde{A}(r) = 0 \, , \quad \Delta\tilde{A}_d(s) = 0 \, , \quad \tilde{W}(\alpha) = 0 \tag{14}$$

in (2) and (13). In this case, the following theorem shows that the linear controller (6) is guaranteed to exist such that practical tracking of the reference model (4) is achieved.

Theorem 1: Consider the uncertain time-delay system (1) subject to conditions (2), (3) and (14). Assume the tracked reference model is given by (4). Let

$$\bar{f} = \max\{\|f\| : r \in \Omega_1,\, s \in \Omega_2,\, v \in \Omega_3,\, \alpha \in \Omega_4,\, \|x_m(t)\| \le M^*\} \quad (15)$$
$$\bar{E} = \max\{\|E(s)\| : s \in \Omega_2\}$$

If

$$F(v) + F^T(v) + 2I \ge \delta I, \quad \text{for some } \delta > 0 \text{ and all } v \in \Omega_3 \quad (16)$$

then the linear controller given by (6) can ensure the output tracking error to be uniformly ultimately bounded with respect to the bound

$$\varepsilon = \|C\|\bar{f}\sqrt{\frac{1+\varepsilon_1}{\mu}}\sqrt{\frac{\lambda_M(P)+\tau}{\underline{\lambda}_m(\tilde{Q})\lambda_m(P)}} \quad (17)$$

where $\varepsilon_1 > 0$ is an arbitrarily small constant, $\mu > 0$ is an arbitrary constant chosen by the designer and

$$\underline{\lambda}_m(\tilde{Q}) \triangleq \min\{\lambda_m(Q - D^T(r)D(r) - K^T F^T(v)F(v)K - I) : r \in \Omega_1, v \in \Omega_3\} \quad (18)$$

Here $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively. The parameters in (6) and (7) are chosen as follows:

$$Q > D^T(r)D(r) + K^T F^T(v)F(v)K + I, \quad \text{for all } r \in \Omega_1, v \in \Omega_3 \quad (19)$$

and $\xi > 0$ is a sufficiently small scalar such that the solution $P > 0$ to (7) exists [8]. The positive scalar γ is chosen to satisfy that

$$\gamma \ge \frac{\mu + \bar{E}^2 + 2 - \xi}{\delta} \quad (20)$$

Moreover, the tracking error bound ε in (17) can be made arbitrarily small by increasing the parameters μ and γ, simultaneously, such that (20) still holds. As a result, practical tracking is achieved.

Remark 1: If a solution cannot be found to satisfy (5), a different reference model (4) must be selected.

Remark 2: Let $\bar{D} = \{\max\|D(r)\| : r \in \Omega_1\}$, $\bar{F} = \{\max\|F(v)\| : v \in \Omega_3\}$. We can see that

$$D^T(r)D(r) \le \bar{D}^2 I, \quad \text{for all } r \in \Omega_1 \quad (21)$$

$$F^T(v)F(v) \le \bar{F}^2 I, \quad \text{for all } v \in \Omega_3 \quad (22)$$

Therefore, one may choose

$$Q > (\bar{D}^2 + 1)I + \bar{F}^2 K^T K \quad (23)$$

to ensure condition (19).

Next, we extend the result of Theorem 1 to the general case, namely, the system contains both matched and mismatched uncertainties.

For this case, the following theorem presents a sufficient condition such that ε-tracking of the reference model (4) is achieved.

Theorem 2: Consider the uncertain time-delay system (1) subject to conditions (2) and (3). The tracked reference model is given by (4). Suppose that condition (14) is not satisfied. Let

$$\bar{P}_g = \max\{\|Pg\| : r \in \Omega_1,\, s \in \Omega_2,\, v \in \Omega_3,\, \alpha \in \Omega_4,\, \|x_m(t)\| \le M^*\} \quad (24)$$

where $P > 0$ is the solution of (7) and g is defined in (13). If

$$\underline{\lambda}_m(\hat{Q}) > 0 \quad (25)$$

where

$$\underline{\lambda}_m(\hat{Q}) \triangleq \min\{\lambda_m(Q - D^T(r)D(r) - K^T F^T(v)F(v)K - I) : r \in \Omega_1, v \in \Omega_3\} - \lambda_M(P)[2M_1 + \lambda_M(P)M_2(2\|B\|\bar{E} + M_2)] \quad (26)$$

and

$$M_1 \triangleq \max\{\|\Delta\tilde{A}(r)\| : r \in \Omega_1\},$$
$$M_2 \triangleq \max\{\|\Delta\tilde{A}_d(s)\| : s \in \Omega_2\} \quad (27)$$

then the output tracking error of system (1) driven by the controller (6), which is determined in the same way as in Theorem 1, is uniformly ultimately bounded with respect to the bound

$$\varepsilon = \|C\|\sqrt{\frac{1}{\underline{\lambda}_m(\hat{Q}) - \theta}\left(\frac{\bar{f}^2}{\mu} + \frac{\bar{P}_g^2}{\theta}\right)}\sqrt{(1+\varepsilon_1)\frac{\lambda_M(P)+\tau}{\lambda_m(P)}} \quad (28)$$

where $\varepsilon_1 > 0$ is an arbitrarily small constant, $\theta > 0$ is a scalar chosen by the designer such that

$$\underline{\lambda}_m(\hat{Q}) - \theta > 0. \quad (29)$$

Remark 3: The scalar $\theta > 0$ subject to (29) may be optimized by means of numerical search such that a less conservative tracking error (28) is achieved.

4. Illustrative Example

Consider the uncertain time-delay system (1) with

$$A=\begin{bmatrix}0&1&0\\0&1&2\\-1&-2&0\end{bmatrix},\quad B=\begin{bmatrix}0\\0.1\\1\end{bmatrix},\quad C=\begin{bmatrix}1&0&0\end{bmatrix} \quad (30)$$

and the matched uncertainties and time delay satisfying

$$\Delta A(r)=BD(r),\ \ D(r)=\begin{bmatrix}r_1&0&r_2\end{bmatrix},$$
$$|r_1|\le 0.15\ ,\quad |r_2|\le 0.2 \quad (31)$$

$$A_d(s)=BE(s),\ \ E(s)=\begin{bmatrix}0&s_1&s_2\end{bmatrix},$$
$$|s_1|\le 0.1\ ,\ |s_2|\le 0.15\ ,\ \tau=0.1 \quad (32)$$

$$\Delta B(v)=BF(v),\ F(v)=v,\ |v|\le 0.5 \quad (33)$$

$$W(\alpha)=0 \quad (34)$$

The reference model is given by (4) with

$$A_m=\begin{bmatrix}0&1\\-1&0\end{bmatrix},\quad C_m=\begin{bmatrix}1&0\end{bmatrix} \quad (35a)$$

and

$$x_m(t)=\begin{bmatrix}1&0\end{bmatrix}\ ,\quad t\in[-\tau,0] \quad (35b)$$

The requirement for the tracking error is $\varepsilon\le 0.15$.

We can apply Theorem 1 presented above to determine the linear robust tracking controller (6) such that the system output robustly tracks the model.

First, we choose the matrix

$$K=\begin{bmatrix}-3.2649&-7.0158&-5.2984\end{bmatrix} \quad (36)$$

such that $\lambda(A+BF)=\{-1,\ -2\pm 2j\}$, as shown in [6].

Then, using the technique of [7] to solve (5) yields

$$G=\begin{bmatrix}1&0\\0&1\\-0.5786&-0.5711\end{bmatrix},\quad H=\begin{bmatrix}1.5711&1.4214\end{bmatrix} \quad (37)$$

Further, one can obtain that $\overline{D}=0.25$, $\overline{E}=0.1803$, $\overline{F}=0.5$, $\delta=1$, $\|x_m(t)\|\le M^*=1$ for all $t\ge 0$ and $\bar{f}=1.2666$.

In the light of Theorem 1 and Remark 2, we choose

$$Q=(20+\overline{D}^2+1)I+\overline{F}^2K^TK\ ,\ \ \xi=0.01 \quad (38)$$

for (7) and obtain

$$P=\begin{bmatrix}34.5374&15.3290&1.6199\\15.3290&32.4845&4.6607\\1.6199&4.6607&3.9616\end{bmatrix} \quad (39)$$

where $\lambda_M(P)=49.2965$ and $\lambda_m(P)=3.1913$. According to the tracking error requirement $\varepsilon\le 0.15$ and by choosing the parameter $\varepsilon_1=0.01$, one can get the required parameters $\mu=55.728$ and $\gamma=57.751$ based on (17) and (20), respectively. Consequently, we obtain the controller (6) as follows

$$\begin{aligned}u(t)=&\begin{bmatrix}-185.3410&-463.7758&-261.0027\end{bmatrix}x\\&+\begin{bmatrix}35.8960&316.1386\end{bmatrix}x_m(t)\end{aligned} \quad (40)$$

which ensures the tracking error $\varepsilon\le 0.15$. Furthermore, it is concluded from Theorem 1 that the tracking error in this example can be made arbitrarily small.

Using controller (40) and the following uncertainty parameter values: $r_1=0.15$, $r_2=0.2$, $s_1=0.1$, $s_2=0.15$ and $v=-0.5$, which can roughly be regarded as the worst case, we have given the simulation results as shown in Fig. 1. Clearly, the tracking error is within the predicted bound.

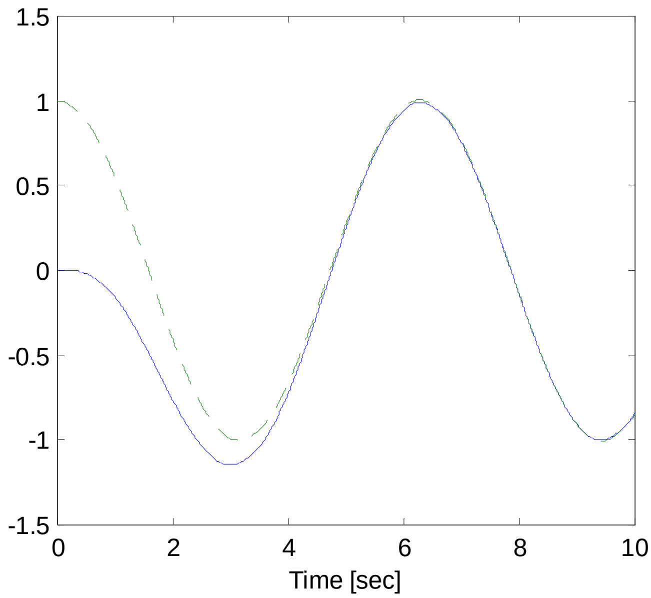

Fig. 1. Tracking Responses
— system output ; --- model output

It is noted that this example does not satisfy condition (3) of [6]. Therefore, the results of [6] are not applicable here. On the other hand, however, if assuming that there exists no uncertainty in the input control matrix, i.e., $\Delta B(v) = 0$, the example reduces to one conducted in [6] where the controller

$$\begin{aligned} u(t) = &[-14.4199 \quad -34.2423 \quad -18.8748]x(t) \\ &+ [5.07 \quad 24.8844]x_m(t) \end{aligned} \tag{41}$$

is desired. Based on the parameters given in [6], calculating (19) of [6] yields the tracking error bound $\varepsilon = 0.20$ (We were unable to reproduce the tracking error $\varepsilon \le 0.15$ as claimed in [6]).

For comparison, we use Theorem 1 presented above to design the robust tracking controller for the case of $\Delta B(v) = 0$ in the following.

It is easy to determine that $\overline{D} = 0.25$, $\overline{E} = 0.1803$, $\overline{F} = 0$, $\delta = 2$, $\|x_m(t)\| \le M^* = 1$ for all $t \ge 0$ and $\bar{f} = 0.2072$. In the light of Theorem 1 and Remark 2, we choose

$$Q = (15 + \overline{D}^2 + 1)I + \overline{F}^2 K^T K\ ,\ \xi = 0.01 \tag{42}$$

for (7) and obtain

$$P = \begin{bmatrix} 25.9928 & 11.1103 & 1.0380 \\ 11.1103 & 23.3991 & 2.9129 \\ 1.0380 & 2.9129 & 2.3305 \end{bmatrix} \tag{43}$$

where $\lambda_M(P) = 36.1001$ and $\lambda_m(P) = 1.9234$. According to the tracking error requirement $\varepsilon \le 0.15$ and by choosing the parameter $\varepsilon_1 = 0.01$, one can get the required parameters $\mu = 2.4185$ and $\gamma = 2.2205$ based on (17) and (20), respectively. Consequently, we obtain the controller (6) as follows

$$\begin{aligned} u(t) = &[-8.0368 \quad -18.6796 \quad -11.1201]x(t) \\ &+ [3.1738 \quad 13.7503]x_m(t) \end{aligned} \tag{44}$$

which ensures the tracking error $\varepsilon \le 0.15$.

Comparing (44) with (41), one can see that the controller resulting from our Theorem 1 not only achieves a smaller tracking error, but also has much ‘smaller’ control gains. This clearly shows that Theorem 1 presented above not only admits larger system uncertainties than Theorem 2 of [6], but also is more efficient than Theorem 2 of [6] in the case that condition (3) of [6] is satisfied and Theorem 2 of [6] is applicable.

5. Conclusions

In this paper, by using a Riccati-type equation, we have presented a new approach to the design of linear robust controllers for a class of uncertain time-delay systems to track dynamic inputs generated from a reference model. In the case of matched uncertainties, the resulting controller can make the ultimate bound of the tracking error arbitrarily small. As a consequence, the system practically tracks the model. In the case of mismatched uncertainties, a sufficient condition is presented such that the controller ensures ultimate boundedness of the tracking error. Numerical examples and simulation results have shown the effectiveness of the approach presented in this paper.

Acknowledgment

M. J. Er acknowledges the generous support provided by the Association of Commonwealth Universities for the work presented.

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