Identification of systems with direction-dependent dynamics

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Keywords: System identification; Direction-dependent dynamics; Nonlinear systems; Wiener models; Perturbation signals; Pseudo-random signals

Abstract

In this paper, the identification of systems with direction-dependent dynamics by means of bilinear models and Wiener models is considered. It is shown that when such a system is perturbed by a pseudo-random binary signal based on a maximum-length sequence, distinctive patterns are observed in the cross-correlation function between the system input and the system output. These patterns are not present when other kinds of pseudo-random binary signals are used. The patterns obtained for bilinear models and Wiener models are similar, and both depend on the characteristic polynomial of the maximum-length sequence used. For the case in which the dynamics involved are first-order, analytical results are obtained which allow the patterns to be compared in detail. The results expected when the pseudo-random signals used are inverse-repeat are also described. It is concluded that both kinds of model are suitable for use in this application, provided that the model parameters are appropriately chosen.

1. Introduction

The identification of systems with direction-dependent dynamics was first considered by Briggs and Godfrey [1], and the results were later applied by Godfrey and Moore to the identification of the dynamics of a gas turbine [2]. More recently, a new MATLAB routine for the generation of perturbation signals has been applied to the identification of such systems [3]. In each case, identification was carried out using pseudo-random binary perturbation signals, and it was assumed that the system could be identified as a bilinear model comprising one linear part when the direction of the system output was positive and another linear part when the direction of the system output was negative. Analytical solutions were obtained for the case in which the linear parts were first-order, so that the directions of the system input and output were always the same.

In practice, many nonlinear systems exhibit dynamics that depend on the direction of the system output, or the system input, or both, and such systems may be modelled by one of the many derivatives of the Volterra functional series [4]. In a recent paper [5], it was shown that one of these derivatives, a Wiener model in which a dynamic linear subsystem precedes an instantaneous nonlinearity, could be used as an effective model for the identification of nonlinear systems by pseudo-random perturbation signal testing. Attention here is therefore focused on the use of such a model for direction-dependent dynamic systems when identified by pseudo-random perturbation signal testing, and the relationships with the bilinear model used for the same purpose.

2. Bilinear model

As shown in [1-3], pseudo-random binary signals based on maximum-length sequences [6] are the most suitable signals for identification with the bilinear model. When these signals are used to perturb the bilinear model, patterns are observed in the cross-correlation function between the input and output that are not present when other kinds of pseudo-random binary signals are used. As there are no obvious patterns in frequency response estimates obtained with any kind of pseudo-random binary signal, identification here will be confined to the time domain, with the impulse response estimate obtained through the input-output cross-correlation function.

For illustration, and subsequent comparison with a Wiener model, a bilinear model is used for which the transfer function is $\dfrac{1}{1+\mathrm{s}T_{\mathrm{U}}}$ when the direction of the system output is positive and $\dfrac{1}{1+\mathrm{s}T_{\mathrm{D}}}$ when the direction of the system output is negative. The model is perturbed by a pseudo-random binary signal $u(t)$ with period NT, which is generated using a constant clock-pulse interval of T s from a maximum-length binary sequence $s(i)$ with period N and characteristic polynomial $f(\mathrm{D})$ by converting the zero elements of $s(i)$ to -1 and the unity elements of $s(i)$ to $+1$. The cross-correlation function between the model input $u(t)$ and the model output $y(t)$ for the case in which

$T_U = 3T$, $T_D = 12T$, $f(D) = 1 + D + D^4 + D^6 + D^7$ and $N = 127$ is shown in Figure 1.

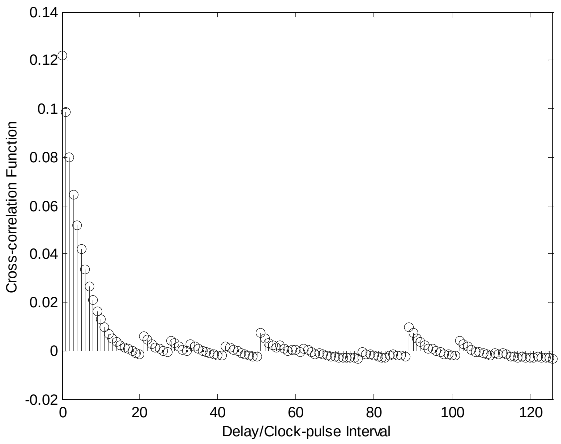

Figure 1. Input-output cross-correlation function for a bilinear model with $T_U = 3T$ and $T_D = 12T$.

The discontinuities in the input-output cross-correlation function form a distinctive pattern that depends on the characteristic polynomial $f(D)$ of the sequence $s(i)$. In this case, the major part of the pattern is formed by discontinuities occurring at 89T, 51T, 21T, 102T, 28T, 42T, 33T, etc., and the minor part is formed by discontinuities occurring at 59T, 113T, 85T, 95T, 118T, 107T, etc.

As the linear parts of the bilinear model are first-order, and its input is a binary signal, the output signal $y(t)$ may be obtained as an infinite series [1-3], from which the input-output cross-correlation function may also be obtained as an infinite series. Analytical expressions for the terms in this series involve the following parameters:

$$a = 0.5(\exp(-T / T_U) + \exp(-T / T_D)) \quad (1)$$

where $0 < a < 1$

$$b = 0.5(\exp(-T / T_U) - \exp(-T / T_D)) \quad (2)$$

where $-1 < b < 1$

$$c = (1 - \exp(-T / T_U))(1 - \exp(-T / T_D)) \quad (3)$$

where $0 < c < 1$

The four most significant terms are:

Zero-order term

$$\phi_0 = -\frac{1}{N} \quad (4)$$

This is simply a constant bias.

First-order term

$$\phi_1(iT) = \frac{N+1}{N} \frac{c}{1-a} a^i \qquad i = 0, 1, 2, \ldots \quad (5)$$

This provides an estimate of the linear behaviour of the model.

Second-order term

$$\phi_2(iT) = -\frac{N+1}{N} \frac{cb}{1-a} \sum_{\text{all } r} a^{r-1+i-i_r}$$

$$i = i_r, i_r + 1, i_r + 2, \ldots \quad (6)$$

where

$$s_i \oplus s_{i-r} = s_{i-i_r} \qquad r = 1, 2, 3, \ldots \quad (7)$$

This accounts for second-order discontinuities.

Third-order term

$$\phi_3(iT) = \frac{N+1}{N} \frac{cb^2}{1-a} \sum_{all\ q,r} a^{r-2+i-i_{q,r}}$$

$$i = i_{q,r}, i_{q,r} + 1, i_{q,r} + 2, \ldots \quad (8)$$

where

$$s_i \oplus s_{i-q} \oplus s_{i-r} = s_{i-i_{q,r}} \qquad q = 1, 2, 3, \ldots$$

$$r = 2, 3, 4, \ldots \quad (9)$$

This accounts for third-order discontinuities.

The zero-order term is negative, the first- and third-order terms are positive, but the second-order term is either positive when b is negative, that is when $T_U < T_D$, or negative when b is positive, that is when $T_U > T_D$. An illustration of the former case, with $T_U = 3T$ and $T_D = 12T$, is that shown in Figure 1, and an illustration of the latter case, with $T_U = 12T$ and $T_D = 3T$, is that shown in Figure 2.

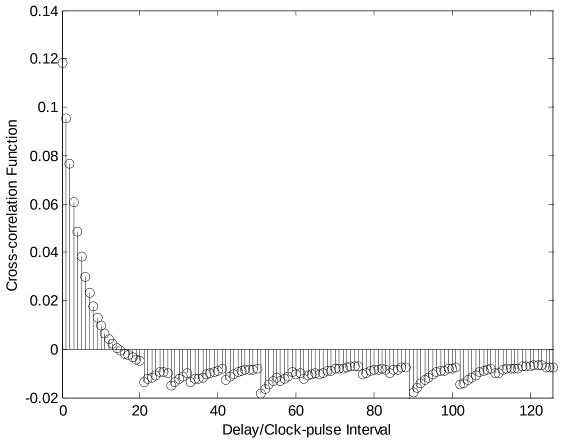

Figure 2. Input-output cross-correlation function for a bilinear model with $T_U = 12T$ and $T_D = 3T$.

The discontinuities in Figures 1 and 2 clearly start at the same points. The starting point $i_r T$ of the r-th second-order discontinuity is determined by dividing the polynomial $1+D^r$ by the characteristic polynomial f(D) until the single-term remainder D^{i_r} is obtained [7]. For example, the starting point 89T of the first discontinuity is obtained by dividing $1+D$ by $1+D+D^4+D^6+D^7$ until the single-term remainder D^{89} is obtained. Similarly, the starting point $i_{q,r}$ of the q, r-th third-order discontinuity is determined by dividing the polynomial $1+D^q+D^r$ by the characteristic polynomial f(D) until the single-term remainder $D^{i_{q,r}}$ is obtained [7]. For example, the starting point 59T of the first discontinuity is obtained by dividing $1+D+D^2$ by $1+D+D^4+D^6+D^7$ until the single-term remainder D^{59} is obtained.

3. Wiener model

The Wiener model used for comparison is shown in Figure 3. The input signal $u(t)$ is the same pseudo-random binary signal as before. For illustration, and comparison with the bilinear models in Section 2, the Wiener model parameters are $K_0 = 1.43$ when $b < 0$ and $K_0 = -0.11$ when $b > 0$, $K_1 = 0.69$, $K_2 = 0.88$, $K_3 = 0.99$ and $T_c = 5$, as will be justified later. Then when $b < 0$ the input-output cross-correlation function is shown in Figure 4, and when $b > 0$ the input-output cross-correlation is shown in Figure 5.

The patterns formed by the discontinuities in Figures 1 and 4 are strikingly similar, as are those in Figures 2 and 5, essentially because the discontinuities start at the same points.

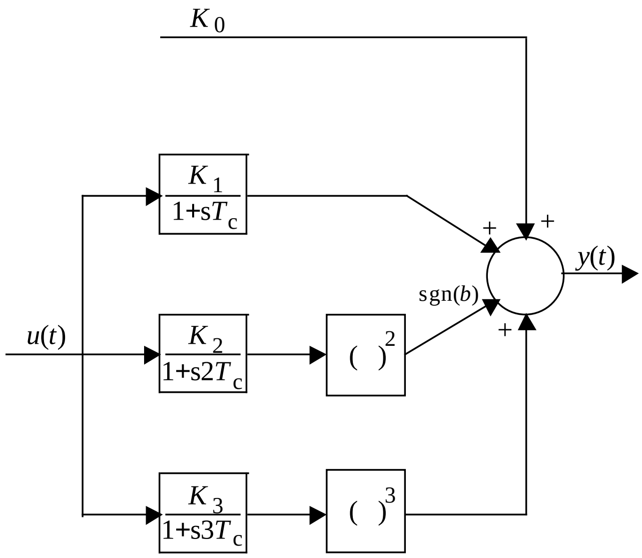

Figure 3. Wiener model

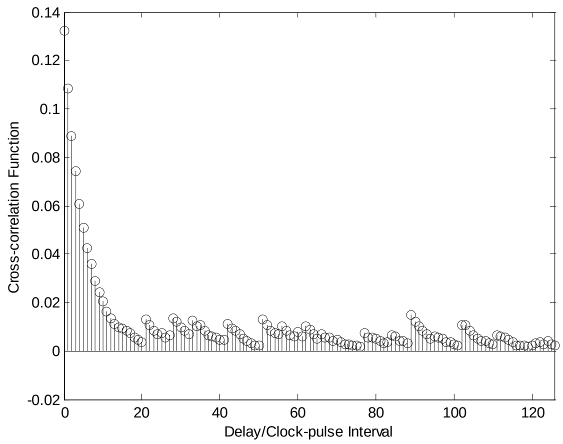

Figure 4. Input-output cross-correlation function for the Wiener model when $b < 0$.

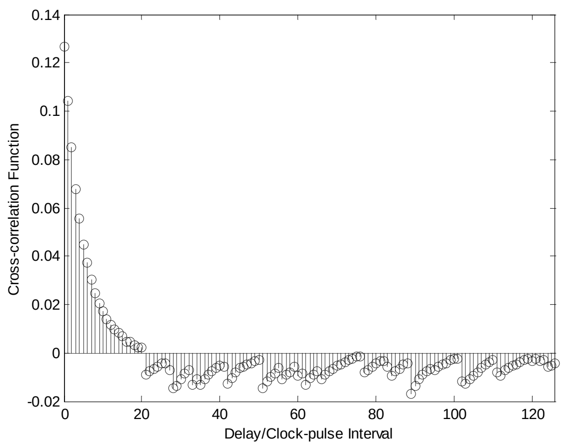

Figure 5. Input-output cross-correlation function for the Wiener model when $b > 0$.

In the case of a Wiener model, an analytical expression may be obtained for the cross-correlation function, whatever the order of the linear parts. As shown by Barker and Obidegwu [8], each nonlinear path in the model contributes terms that corrupt the cross-correlation function obtained for the linear path alone, and the occurrences of discontinuities in the total cross-correlation function resulting from these terms may be accounted for by Equations 7 and 9. The cross-correlation function in this case comprises four terms, which are:

Zero-order term

$$\phi_0 = \frac{1}{N}(K_0 - K_1 + [\mathrm{sgn}(b)]K_2^2 - K_3^3) \tag{10}$$

From Equations 4 and 10, the zero-order terms in both models are the same if

$$K_0 - K_1 + [\mathrm{sgn}(b)]K_2^2 - K_3^3 = -1 \tag{11}$$

K_0 is therefore determined after the other parameters.

First-order term

$$\phi_1(iT) = \frac{N+1}{N}(1-\exp(-T/T_c))(K_1 + kK_3^3)\exp(-iT/T_c) \qquad i = 0, 1, 2, \ldots \tag{12}$$

where kK_3^3 is small compared with K_1, reflecting the fact that there is some distortion of the linear estimate, in the form of a small change in gain, due to the third-order path in the model [8]. By a suitable choice of parameters, the first-order terms in both models can be made similar. In particular, from Equations 5 and 12, the decrement is the same in both models if

$$T_c = -T/\ln(a) \tag{13}$$

Then from Equations 5, 12 and 13, there is reasonable agreement between the first-order terms in both models if

$$K_1 = \frac{c}{(1-a)^2} \tag{14}$$

Second-order term

$$\phi_2(iT) = [\mathrm{sgn}(-b)]\frac{N+1}{N}2(K_2(1-\exp(-T/2T_c)))^2 \times \sum_{\text{all } r}\exp(-(r/2+i-i_r)T/T_c) \qquad i = i_r, i_r+1, i_r+2, \ldots \tag{15}$$

where r and i_r are given in Equation 7.

From Equations 6, 13 and 15, all the second-order discontinuities in both models have the same decrement, but it is not possible to make every discontinuity in Equation 6 identical to the corresponding discontinuity in Equation 15. If the first discontinuities, which are the largest, are equated, then there is reasonable agreement between the second-order terms in both models if

$$K_2 = \frac{1}{1-a^{1/2}}\left(\frac{c|b|}{2(1-a)a^{1/2}}\right)^{1/2} \tag{16}$$

Third-order term

$$\phi_3(iT) = \frac{N+1}{N}6(K_3(1-\exp(-T/3T_c)))^3 \times \sum_{\text{all } q,r}\exp(-((q+r)/3+i-i_r)T/T_c) \qquad i = i_{q,r}, i_{q,r}+1, i_{q,r}+2, \ldots \tag{17}$$

where q, r and $i_{q,r}$ are given in Equation 9.

From Equations 8, 13 and 17, the third-order discontinuities in both models all have the same decrement, but it is not possible to make every discontinuity in Equation 8 identical to the corresponding discontinuity in Equation 17. If the first discontinuities, which are the largest, are equated, then there is reasonable agreement between the third-order terms in both models if

$$K_3 = \frac{1}{1-a^{1/3}}\left(\frac{cb^2}{6(1-a)a}\right)^{1/3} \tag{18}$$

The values $T_U = 3$ and $T_D = 12$, or $T_U = 12$ and $T_D = 3$, in Equations 1, 2, 3, 11, 13, 14, 16 and 18 gives the values of the Wiener model parameters K_0, K_1, K_2, K_3 and T_c.

4. Pseudo-random signals

When pseudo-random signals other than those based on maximum-length sequences are used, the cross-correlation functions do not contain the patterns noted above, but they do contain substantial distortions. Although the distortions appear to be random, they are in fact repeatable, and they arise from the high-order autocorrelation functions of the pseudo-random signals, which do not have the desirable properties of those based on maximum-length sequences [7]. Inverse-repeat pseudo-random binary signals, generated from maximum-length binary sequences by inverting alternate members, and inverse-repeat pseudo-random ternary signals, generated from maximum-length ternary sequences, may however be used. In both cases, the cross-correlation functions contain no even-order terms, so only odd-order discontinuities occur.

Figures 6 and 7 show the input-output cross-correlation functions for the bilinear model and the Wiener model respectively when the pseudo-random binary input signal $u(t)$ used in Sections 2 and 3 is replaced by its inverse-repeat form.

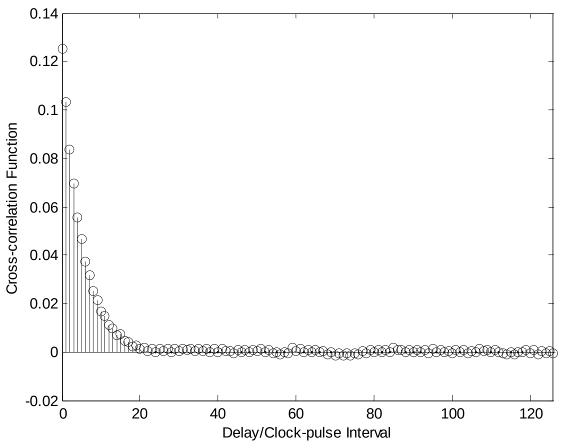

Figure 6. Input-output cross-correlation function for a bilinear model with an inverse-repeat pseudo-random binary input signal when $T_{\mathrm{U}} = 3T$ and $T_{\mathrm{D}} = 12T$, or $T_{\mathrm{U}} = 12T$ and $T_{\mathrm{D}} = 3T$.

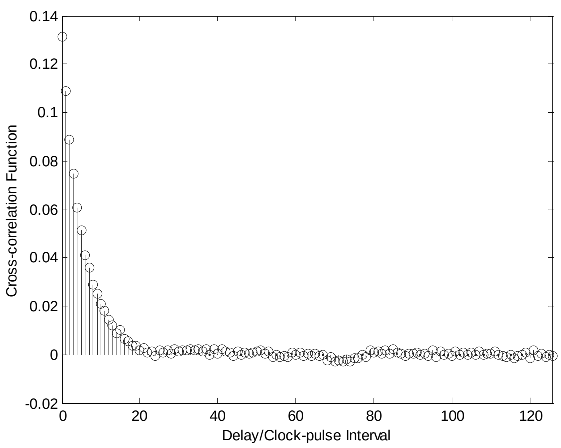

Figure 7. Input-output cross-correlation function for the Wiener model with an inverse-repeat pseudo-random binary input signal.

Comparing the cross-correlation functions in Figures 6 and 7 with the corresponding cross-correlation functions in Figures 1, 2, 4 and 5, it is seen that the zero- and second-order terms have been removed, and with the latter the discontinuities occurring at 89*T*, 51*T*, 21*T*, 102*T*, 28*T*, 42*T*, 33*T*, etc. The first- and third-order terms remain, and with the latter the discontinuities occurring at 59*T*, 113*T*, 85*T*, 95*T*, 118*T*, 107*T*, etc. The comparisons also serve to show the relative contributions of the second-and third-order discontinuities in this case.

The use of inverse-repeat pseudo-random signals in this application is advantageous, particularly if the estimation of the dynamics is carried out in the frequency domain. As such signals contain only odd harmonics, separating the odd and even harmonics in the system output allows the odd and even nonlinearities to be identified separately [9]. It is evident from the comparisons above that systems exhibiting direction-dependent dynamics contain substantial second-order nonlinearities, the effects of which will corrupt the linear estimates if not removed in this way, but which it is also necessary to identify to obtain the system models.

5. Conclusions

When systems that exhibit direction-dependent dynamics are identified using pseudo-random binary signals based on maximum-length sequences, distinctive patterns are observed in the cross-correlation function between the system input and the system output. The patterns are dependent on the characteristic polynomial of the maximum-length sequence, and are absent when other kinds of pseudo-random binary signals are used. Similar patterns are obtained for both bilinear and Wiener models, with discontinuities in the cross-correlation function occurring at the same points. Analytical results may be obtained for cases in which the dynamics are first-order, allowing the patterns to be compared in detail. Either kind of model may be used in this application, provided that the model parameters are appropriately chosen.

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