

MULTIRATE CONTROLLERS DESIGN BY RATE DECOMPOSITION

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Abstract. In many industrial control applications the control action updating can be faster than the output measurement, leading to multirate (MR) controllers. In this paper, the controller design is approached based on the characteristics of each available sampling rate. The controller is split into two parts acting at different sampling rates. By the lowest rate sub-controller the main points of the desired response are determined while the response envelope, approaching the continuous time response of the controlled system is completed by the action of the fast rate control part. Cancellation controllers, three-term controllers and PID controllers are designed using this approach and promising results are obtained.

1. INTRODUCTION

Data acquisition systems in industrial control applications have, in general, more time constraints than actuators, and as a result, different rates are used for control updating and output sampling. Some process variables are difficult to be directly measured and there are delays in the measurement process. This can be, for instance, in the case of chemical analyzers or off-line measurements devices where the sample should be prepared beforehand requiring some preparation time, like in cement or ceramics applications [1]. Also in robotics and manufacturing systems where the measurement is got through a visual feedback [2], the image processing will require a lot of time. In other cases, although it could be possible, it is not convenient to have so many measurements and the resources are used for other purposes. This is the case of the reader/writer header positioning in hard disk drives servo systems. In the modern embedded sector servo technique, the position signal is only available at the sectors, improving the traditional dedicated servo technique where an area was dedicated to store the servo position. This results in a larger storage capability without increasing the number of servo sectors [3].

In distributed computer controlled systems, as reported in Integrated Communication and Control Systems by Ray, [4] a number of control loops are sharing the same communication channels, and some unexpected delays may appear due to conflicts in the use of the common resource. In this case, if the measurement sampling period is large enough all the measurements can be processed, the network is not so busy and the control actions can be applied on time. This approach has been also applied in robotics applications [5].

Under these settings, the main difficulty is the design of the MR control algorithms. Many approaches have been reported in the literature, [6,7]. One simple and direct approach is proposed by [8]. A DT controller is redesigned using a dual-rate hold device and a DT equivalent controller of the continuous one at high frequency. The discretization of the controller is attached to one of them and interesting results are reported. In the other extreme of the complexity, the controller is defined to fulfil some general properties about robustness and ripple-free behavior. The solution, which is provided in closed form and difficult to interpret and tune, appears to be ideal.

The approach in this paper is to try to get advantage of the fast control action updating to achieve, from a slow measurement data sequence, similar results than those achievable with a faster controller but without requiring a faster output sampling. The paper is structured as follows. In the next section the problem is properly defined and a number of input/output discrete time (DT) models are reviewed. The design of cancellation controllers is considered as a basic design.

2. MR CONTROL AND SAMPLED-DATA MODELS

The basic MR control schema is shown in fig. 1, where the plant is represented by an n -th order single-input-single-output LTI continuous system (CT), with transfer function G_p . The controller output is updated at a period T through the fast hold device, H . The output is measured at period NT and compared to the reference $R(t)$, which is available at any time, the error being elaborated by the MR controller $\overline{G}_{R,N}^T$. To simplify the computation, and without loss of generality, N is assumed to be integer.

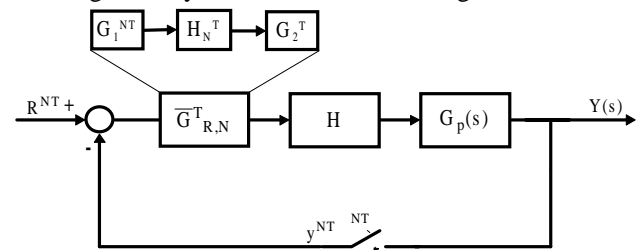


Fig. 1. Basic MR control loop.

The MR controller can be implemented as a periodic controller. In this paper, three parts, as depicted in figure 1, compose the MR controller. The low rate computed error

is first processed by the subcontroller G_1^{NT} and passed through a rate converter H_N^T . The second and faster subcontroller, G_2^T , finally provides the control input to the plant. The main issue now is how to design the dual-rate controller.

In order to deal with MR systems, some discrete-time models attached to a CT system $G_p(s)$, [9], are reviewed.

From the CT signal $y(t)$, a sequence of data taken at rate $1/T$ is represented by Y^T , and Y^{NT} for the slow rate. To convert sequences at different rates it is worth to recall the auxiliary operations [10]:

a) *expand*: adding zeros in the $N-1$ non-signal times of Y^{NT}

$$Ex[Y^{NT}] \equiv (Y^{NT})^T, \quad (1a)$$

that is a T -period sequence with zero values except at “ NT ” instants.

b) *skip*: extracting 1 every N samples of Y^T

$$Sk[Y^T] \equiv (Y^T)^{NT} = Y^{NT} \quad (1b)$$

that is a sequence formed by values of signal Y^T at multiple instants “ NT ”.

First, there is a single rate model for each one of the sampling periods. The fast sampling DT model (FSDT) is defined by:

$$\frac{Y(z)}{U(z)} = G(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=1}^n b_{i,T} z^{-i}}{1 + \sum_{i=1}^n a_{i,T} z^{-i}} = \frac{Y^T}{U^T} \quad (2)$$

where z is the T unit-delay shift operator. For future developments, the notation $A(z) \stackrel{\Delta}{=} A^T$ is also used, denoting a polynomial on z (or, as before, a T -spaced sequence). The DT transfer function poles are denoted by $\alpha_{i,T}$. That is:

$$A(z) = \prod_{i=1}^n (z - \alpha_{i,T}). \quad (3)$$

The slow sampling DT model (SSDT) is given by:

$$\frac{Y(z^N)}{U(z^N)} = G_N(z) = \frac{B_N(z^N)}{A_N(z^N)} = \frac{\sum_{i=1}^n b_{i,NT} z^{-iN}}{1 + \sum_{i=1}^n a_{i,NT} z^{-iN}} = \frac{Y^{NT}}{U^{NT}} \quad (4)$$

Again, a simpler notation will be $A_N(z^N) \stackrel{\Delta}{=} A^{NT}$, denoting a polynomial on z^N (or an NT -spaced sequence). In this case the poles are denoted by $\alpha_{i,NT}$. That is:

$$A_N(z^N) = \prod_{i=1}^n (z^N - \alpha_{i,NT}). \quad (5)$$

Note that, dealing with the same CT system, $\alpha_{i,NT} = \alpha_{i,T}^N; \forall i=1, \dots, n$. Thus, the following useful relationship is derived:

$$W_A(z) = \frac{\prod_{i=1}^n (z^N - \alpha_{i,NT})}{\prod_{i=1}^n (z - \alpha_{i,T})}$$

$$W_A(z) = \prod_{i=1}^n (z^{N-1} + \alpha_{i,T} z^{N-2} + \dots + \alpha_{i,T}^{N-1}) \stackrel{\Delta}{=} W_A^T = \frac{A^{NT}}{A^T} \quad (6)$$

Similar polynomials $W(\cdot)$ can be obtained for any polynomial, other than A .

Two very mild technical assumptions to avoid aliasing, [6], are taken:

a) $G(z)$ has no zero at $z=1$.

b) If μ is a pole of $G_N(z)$, then $\mu e^{2\pi jk/N}$ is not a pole of $G_N(z)$.

It is also possible to introduce dual-rate operators (DRDT) able to express the relationship between two differently time-spaced sequences. First, for input updating faster than output measurement sampling, the operator is:

$$\frac{Y(z^N)}{U(z)} = \bar{G}(z) = \frac{\bar{B}(z, N)}{A(z^N)} = \frac{\sum_{i=1}^{nN} \bar{b}_{i,T} z^{-i}}{1 + \sum_{i=1}^n a_{i,NT} z^{-iN}} = \frac{\bar{B}^T}{A^{NT}} = \frac{Y^{NT}}{U^T} \quad (7)$$

A special feature of this DRDT operator is the following:

the numerator parameters of \bar{B}^T are distributed into n groups of N coefficients in such a way that the sum of each of these groups leads to the slow single-rate numerator coefficient. So, it is possible to obtain the SSDT model from the FSDT model (or from the DRDT model) [9].

As the polynomial $(A^{NT})^T$, (see (1a)), has zeroes in the T instants not multiple to NT , the following relationships are obtained

$$(A^{NT})^T Y^{NT} = A^{NT} Y^{NT} \quad (8a)$$

$$(A^{NT})^T Y^T = A^{NT} Y^T \quad (8b)$$

$$(A^{NT} Y^T)^{NT} = A^{NT} Y^{NT} \quad (8c)$$

$$(A^T (Y^{NT})^T)^{NT} = (A^T)^{NT} Y^{NT} \quad (8d)$$

Thus, the DRDT operator (7) is also given by:

$$\bar{G}^T = \frac{\bar{B}^T}{(A^{NT})^T} \quad (9)$$

Based on the definition of the polynomial $W(z)$, it is also simple to express:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{B^T}{A^T} = \frac{B^T}{A^T} \frac{W_A^T}{W_A^T} = \frac{\bar{B}^T}{(A^{NT})^T} = \frac{Y^T}{U^T} \quad (10)$$

That is, $(A^{NT})^T Y^T = \bar{B}^T U^T$.

Another dual rate system operator can be considered if the output sampling is faster than the input updating. In this case, a vector of transfer functions can be established, such as proposed in [8]

$$\begin{bmatrix} Y_0(z^N) \\ \vdots \\ Y_{N-1}(z^N) \end{bmatrix} = \begin{bmatrix} G_{p,0}(z^N) \\ \vdots \\ G_{p,N-1}(z^N) \end{bmatrix} \cdot U(z^N) \quad (11)$$

where the NT sequence $Y_i(z^N)$ is formed by the i -shifted elements of the fast sequence Y^T , leading explicitly to a periodic model. Using the expand operator, (1a), the simplest and useful example is the basic slow-to-fast rate converter, for step-like sequences. It has the transfer function:

$$H_N^T = \frac{1 - z^{-N}}{1 - z^{-1}} = \frac{Y^T}{(U^{NT})^T} \quad (12)$$

2.1 Closed-loop MR controlled system

Lemma 1. With the notation above, the process output in the MR controlled system, fig.1, is expressed by:

$$\bar{Y}^T = \frac{G_p^T \bar{G}_{R,N}^T}{1 + [(G_p^T \bar{G}_{R,N}^T)^{NT}]} (R^T)^{NT} = \bar{M}_N^T (R^T)^{NT} \quad (13)$$

where \bar{Y}^T is the fast sequence of the MR-controlled process and \bar{M}_N^T is a dual rate operator, providing the output sampled at the faster rate, $1/T$, the input being updated at the lower rate, $1/NT$.

Proof. Taken into account (8), the full model for the system represented in fig. 1 may be detailed as:

$$\bar{Y}^T = G_p^T U^T; \quad U^T = \bar{G}_{R,N}^T E^{NT}; \quad (14)$$

The slow error sequence is given by

$$E^{NT} = (E^T)^{NT} = (R^T - \bar{Y}^T)^{NT} = (R^T)^{NT} - (\bar{Y}^T)^{NT}$$

$$E^{NT} = (R^T)^{NT} - (G_p^T \bar{G}_{R,N}^T E^{NT})^{NT} = (R^T)^{NT} - (G_p^T \bar{G}_{R,N}^T)^{NT} E^{NT}$$

$$E^{NT} = \frac{1}{1 + (G_p^T \bar{G}_{R,N}^T)^{NT}} (R^T)^{NT} \quad (15)$$

Thus, leading to (13).

Based on these models, the dual rate implementation of classical controllers is considered. As previously mentioned, a fast controller is assumed to be good, but limitations in the output-sampling rate only allow for a lower output availability rate.

3. MULTIRATE CONTROLLER DESIGN

The main issue now is how to design the MR controller. For a CT process, $G_p(s)$, a CT controller $G_R(s)$ is designed to achieve a closed-loop transfer function $M(s)$. If a single rate operation (every S/H operating at the same frequency) for the implementation of the controller is assumed, that is for a fast or slow controller G_R^T , or G_R^{NT} , the controlled process, as depicted in fig. 1, is given by:

$$M^T = \frac{Y^T}{U^T} = \frac{G_p^T G_R^T}{1 + G_p^T G_R^T}; \quad \text{or} \quad M^{NT} = \frac{Y^{NT}}{U^{NT}} = \frac{G_p^{NT} G_R^{NT}}{1 + G_p^{NT} G_R^{NT}} \quad (16)$$

respectively. The DT controllers, G_R^T or G_R^{NT} , can be designed either to achieve the DT behavior of $M(s)$, or by discretizing the CT controller $G_R(s)$. In any case, the larger the sampling period the worse the intersampling behavior of the output is.

Now, the goal is to design a MR controller to achieve the fast controlled system performances based on slow output measurement sequence. The result can be expressed as follows.

Theorem 1. Given a CT process $G_p(s) = B_p(s)/A_p(s)$, and a reference model $M(s)$ for the controlled system, assuming a control updating rate $1/T$, the output being sampled at rate $1/NT$, fig.1, define the dual-rate controller $\bar{G}_{R,N}^T = G_2^T H_N^T G_1^{NT}$ where:

$$\text{- the fast part is given by} \quad G_2^T = \frac{M^T}{G_p^T} \quad (17a)$$

$$\text{- the slow part is given by} \quad G_1^{NT} = \frac{1}{1 - M^{NT}} \quad (17b)$$

$$\text{- and H is a rate converter} \quad H_N^T = \frac{1 - z^{-N}}{1 - z^{-1}} \quad (17c)$$

- 1) For step changes in the reference, it achieves the same DT performances than the fast controller, but using a slow sampling rate for the output.
- 2) If M^T is obtained through (16), the intersampling ripple can be avoided, but the response does not match the CT response.
- 3) Similar results can be derived for other reference inputs with a generalized rate converter, $H_{\lambda,N}^T$.

Proof. The output of the dual-rate controlled process \bar{Y}^T is given by (13). To be the same than the output of the fast controlled process implies:

$$\bar{Y}^T = \frac{G_p^T \bar{G}_{R,N}^T}{1 + [(G_p^T \bar{G}_{R,N}^T)^{NT}]} (R^T)^{NT} = G_p^T \bar{G}_{R,N}^T \frac{1}{1 + (G_p^T \bar{G}_{R,N}^T)^{NT}} (R^T)^{NT} = M^T R^T = Y^T \quad (18)$$

To match the slow controlled output sequence it should be:

$$(\bar{Y}^T)^{NT} = Y^{NT} = \frac{G_p^{NT} G_R^{NT}}{1 + G_p^{NT} G_R^{NT}} R^{NT} = M^{NT} R^{NT} \quad (19)$$

Applying (8c) to (18), it yields

$$(\bar{Y}^T)^{NT} = \left[\frac{G_p^T \bar{G}_{R,N}^T}{1 + [(G_p^T \bar{G}_{R,N}^T)^{NT}]} (R^T)^{NT} \right]^{NT} = \left[\frac{G_p^T \bar{G}_{R,N}^T}{1 + [(G_p^T \bar{G}_{R,N}^T)^{NT}]} \right]^{NT} (R^T)^{NT}$$

that is

$$(\bar{Y}^T)^{NT} = (G_p^T \bar{G}_{R,N}^T)^{NT} \frac{1}{1 + (G_p^T \bar{G}_{R,N}^T)^{NT}} (R^T)^{NT} = M^{NT} R^{NT} = Y^{NT} \quad (20)$$

From (1b), $(R^T)^{NT} = R^{NT}$. Thus, from (20)

$$1 + (G_p^T \bar{G}_{R,N}^T)^{NT} = \frac{1}{1 - M^{NT}} \quad (21)$$

and back to (18)

$$G_p^T \bar{G}_{R,N}^T (1 - M^{NT}) (R^T)^{NT} = M^T R^T$$

the dual-rate controller is given by:

$$\bar{G}_{R,N}^T = \frac{M^T}{G_p^T} \frac{R^T}{(R^T)^{NT}} \frac{1}{1 - M^{NT}} \quad (22)$$

The rate converter (17c), for stepwise references, $r(t) = \text{const.}$, is given by

$$H_N^T = \frac{R^T}{(R^T)^{NT}} = \frac{1 - z^{-N}}{1 - z^{-1}} \quad (23)$$

This completes part 1) of the theorem. To proof part 2), from (16),

$$\frac{M^T}{G_p^T} = \frac{G_R^T}{1 + G_p^T G_R^T} \quad (24)$$

and thus, if the closed-loop model is chosen in this way, the dual-rate controller does not cancel the numerator of the process transfer function.

Finally, for a ramp input, that is $r(t) = t$, the rate converter (23) is given by

$$H_{1,N}^T = \frac{R^T}{(R^T)^{NT}} = \frac{1}{N} \left(\frac{1-z^{-N}}{1-z^{-1}} \right)^2 \quad (25)$$

introducing a delay of $N-1$ units, as can be easily checked.

A general expression for the rate converter (23) can be derived for a generic polynomial input. This theorem allows for a simple design of dual-rate controllers, easy to be implemented.

4. APPLICATION DESIGNS

The proposed design approach can be used with different settings, always based on a desired closed-loop transfer function model and some sort of cancellation. The main design option is how to choose the closed-loop transfer function. It is well known that ripple and hidden oscillation may occur if the selection is not appropriated. In the following, some options are considered. In any case, this design methodology gives a rule of thumb for other industrial controllers.

4.1 PID Controllers

Given a CT process $G(s)$, a CT PID controller $G_R(s)$ is designed. If a discretized PID is applied [11], the controlled process dynamic performances are degraded if a slow sampling period is chosen. Assume as acceptable the DT behavior for a sampling period T , with the fast controller $G_{R,1}^T$, but too poor if it is taken as NT , $G_{R,2}^{NT}$. The question in the MR setting is: if the input can be updated at rate $1/T$, is it possible to apply the PID control with similar performances if the output is measured any NT sec?

In this case, the fast part of the MR controller, $G_{R,2}^T$, adapts the process dynamics in such a way that the new process can be controlled by a slow DT new PID, $G_{R,1}^{NT}$.

As an example, consider the process:

$$G(s) = \frac{1.5}{(s+0.5)(s+1.5)}$$

An acceptable CT PID controller is given by

$$u(t) = K_p \left[e(t) + T_d \dot{e}(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau \right]$$

for: $K_p = 8$ $T_d = 0.2$ $T_i = 3.2$ A DT PID controller approximation for T , is given by

$$G_R(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}},$$

$$q_0 = K_p \left(1 + \frac{T_d}{T} \right) \quad q_1 = -K_p \left(1 + \frac{T_d}{T_i} + 2 \frac{T_d}{T} \right) \quad q_2 = K_p \left(\frac{T_d}{T} \right)$$

For different sampling periods the controlled system response is plotted in fig. 2, becoming unstable for $T=0.5$ sec.

A dual-rate controller, with $T=0.1$, $N=3$, is designed and implemented. The first solution is to implement a

cancellation PID controller arranging a fast and a slow part given by:

$$G_{R,1}^{NT} = \frac{1}{1 - M^{NT}}; \quad G_{R,2}^T = \frac{M^T}{G_p^T}$$

where M^{NT} and M^T are the SSDT and FSDT equivalent of $M(s)$. The response matches the points of the CT controlled system, as expressed by Theorem 1, but the ripple appears, as shown in fig. 3.

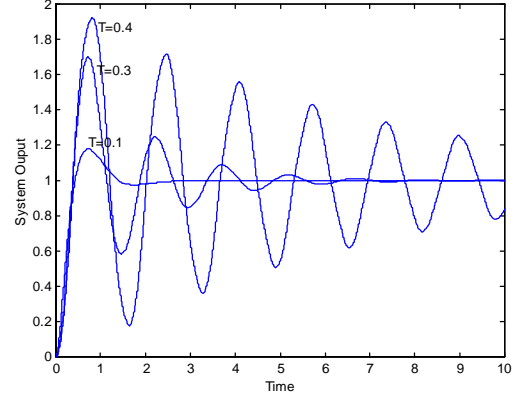


Fig. 2. PID control

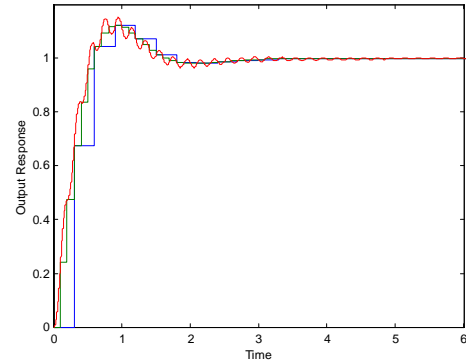


Fig. 3. PID multirate cancellation control

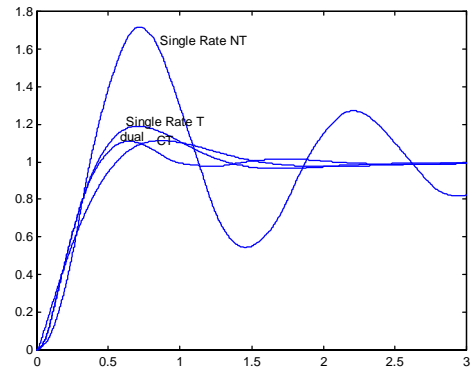


Fig. 4. PID dual rate control

To avoid it, the fast part given by $G_{R,2}^T = \frac{M_c^T}{G_p^T}$, where

M_c^T is the DT transfer function computed from the fast DT process and controller transfer functions, (24), that is,

$$G_{R,2}^T = \frac{M_c^T}{G_p^T} = \frac{G_R^T}{1 + G_R^T G_p^T}$$

The excellent results are shown in fig. 4. The initial response of the dual rate controlled plant, using the perfect plant model, follows that achieved with the fast rate controller. But, at time $t=NT=0.3$ sec. a new measurement is taken and the response is improved.

4.2 Finite settling time Controller

In robotics applications it would be very useful to achieve model reference tracking with minimum time response, the control magnitude being under certain limits. If the process model is reliable and low robustness degree design is accepted, the cancellation controller is the proper one.

As it is well known [12], the procedure for obtaining a finite settling time behavior with assuming steady state requirements leads to planning two diophantine equations:

$$\begin{aligned} 1 - M(z^{-N}) &= (1 - z^{-N})^{\lambda+1} \Lambda(z^{-N}) \\ M(z^{-1}) &= [1 + z^{-1} + \dots + z^{-(N-1)}]^{R+1} \Omega(z^{-1}) \end{aligned} \quad (26)$$

being λ the order of the polynomial reference (for instance, $\lambda=0$ for step, $\lambda=1$ for ramp,...), where Λ and Ω are polynomials to be determined. The solution of the proposed equation system could require a sophisticated procedure, because there are discrete transfer functions at different periods.

The second equation, using the $W(\cdot)$ polynomial notation will be:

$$M(z^{-1}) = W_R(z^{-1}) \Omega(z^{-1})$$

Moreover, to avoid the intersampling ripple, this condition should be expressed as:

$$M(z^{-1}) = W_R(z^{-1}) \bar{B}_p(z^{-1}) \Omega(z^{-1})$$

or, alternatively,

$$\begin{aligned} 1 - M(z^{-N}) &= (1 - z^{-N})^{\lambda+1} \Lambda(z^{-N}) \\ M(z^{-1}) &= W_R(z^{-1}) W_A(z^{-1}) B_p(z^{-1}) \Omega(z^{-1}) \end{aligned} \quad (27)$$

easier to solve.

An experimental example was developed on a portic robotics environment. Assume a CT process model such as

$$G(s) = \frac{1}{(1+10s)(1+20s)}$$

It has been shown, [13], that a single rate cancellation control, with $T=3$ sec introduces hidden oscillations. A dual-rate controller to track a ramp, as usually required for a robotic environment, is designed, according to (26).

The resulting controller with $N=3$, is given by:

1) Minimum-Time

$$\begin{aligned} G_1^{NT} &= \frac{1}{1 - 2z^{-1} + z^{-2}} \\ G_2^T &= \frac{1.33 - 3.47z^{-1} - 3z^{-2} - 0.81z^{-3}}{0.0024 + 0.002z^{-1}} \end{aligned}$$

As it can be seen in the output response, fig. 5, some hidden oscillations occur.

If the process numerator is included in the closed loop dual rate model, following (27), a new controller is designed,

2) Finite-Time

$$\begin{aligned} G_1^{NT} &= \frac{1}{1 - 2z^{-1} + z^{-2}} \\ G_2^T &= 58.9 - 49.7z^{-1} - 94.43z^{-3} + 79.6z^{-4} + 37.6z^{-6} - 31.7z^{-7} \end{aligned}$$

the response being ripple-free, as shown in fig. 6.

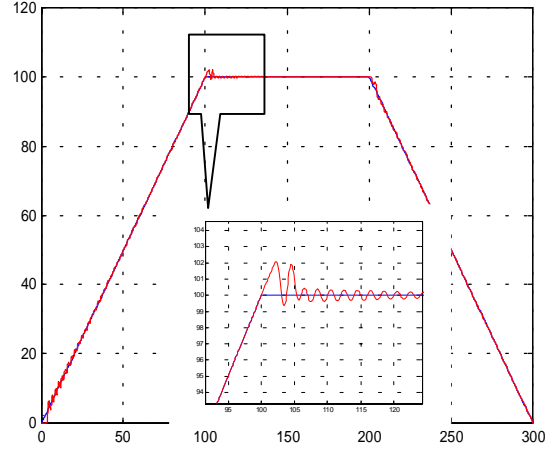


Fig. 5. Cancellation controller

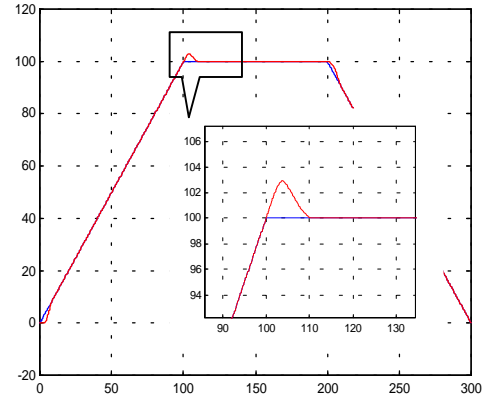


Fig. 6. Ripple free MR control

4.3 Pole Placement Design.

Now, a feedforward/feedback controller, fig. 7, is considered in the sense of [14]. The goal is to obtain a more robust design.

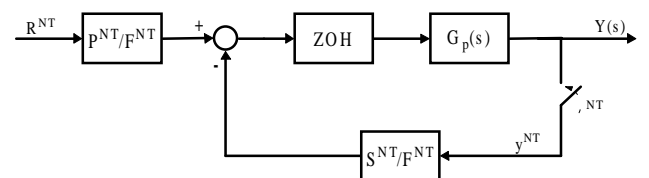


Fig. 7 Three-term control

As it is well known, for a single rate three-term controller, the closed loop transfer function is:

$$M^T = \frac{P^T B_p^T}{A_p^T F^T + B_p^T S^T}; \text{ or } M^{NT} = \frac{P^{NT} B_p^{NT}}{A_p^{NT} F^{NT} + B_p^{NT} S^{NT}} \quad (28)$$

The control strategy is based on holding the T design fix, trying to reduce the hidden oscillations that appears

when some high T is imposed. In this case, the implementation is based on the assumption of an NT controller on a T environment, that is to say, it is like to consider the T expanded NT structures. Analytically:

$$\left(\frac{B_p^{NT} P^{NT}}{A_p^{NT} F^{NT} + B_p^{NT} S^{NT}} \right)^T = \frac{(B_p^{NT})^T (P^{NT})^T}{(A_p^{NT})^T (F^{NT})^T + (B_p^{NT})^T (S^{NT})^T}$$

That leads to assume as interface for different sampling periods design, given by:

$$H^T = \left(\frac{G_p^{NT}}{M^{NT}} \right)^T \frac{M^T}{G_p^T} \quad (29)$$

Again, to avoid the intersampling ripple the numerator of M^T and M^{NT} are taken as those of the process, that is those of G_p^T and G_p^{NT} .

In order to show the pole assignment controller design strategy, the following CT process model is assumed

$$G(s) = \frac{100}{s^2 + 2s + 100}$$

The control goal is to get a closed loop transfer function given by:

$$M(s) = \frac{100}{s^2 + 20s + 100}$$

to follow step reference input.

The process responses for a) single rate control $T=0.25$, b) single rate control for NT , with $N=2$, and c) the “MR T based control” are shown in fig. 8.

As it is evident some hidden oscillations appears for $N=2$, but their reduction is possible by increasing the multi rate N . Additional requirements could be stated in order to get lower oscillations, that it is to say to achieve perfect steady state tracking.

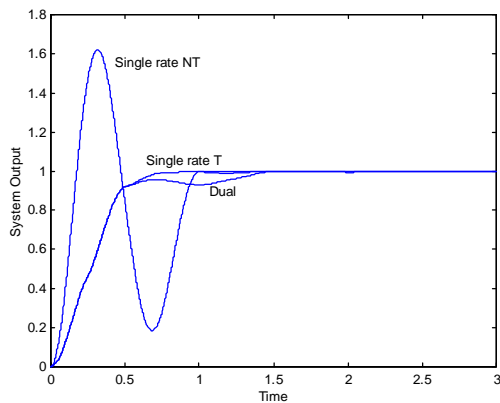


Fig. 8. Pole Placement MR control.

5. CONCLUSIONS

The design of MR controllers in order to achieve with a slow output sampling, the same performances than the fast sampling control system has been discussed. The main result is a simple design approach based on the fast and slow controlled process behavior.

The controller implementation is straightforward, mainly for stepwise references. It allows for a reduction in

the measurement processing. It is also convenient in the case of slower sampling due to technical limitations.

Different kinds of MR controllers with non-conventional structure have been developed. PID controllers have been proposed, with slight modifications to avoid the intersampling ripple. Also, the finite time settling time design was revised and the ripple is also cancelled.

Finally, MR controllers based on the three-term controllers have been also considered. In this case the strategy was to use the low frequency single rate design and introduce an interface refinement. In any case, promising results are obtained when applied to simple processes.

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