Input Design for optimal discrete time Point-to-Point motion of an industrial XY-positioning table.

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Abstract

In this paper a new technique is presented to design input signals for point-to-point control problems with the property of minimal excitation of parasitic system oscillations. This technique is compared to impulse based input design techniques in experiments performed on an industrial XY-positioning table. Impulse Input Shaping was formulated as a solution to an optimisation problem, which has been transformed for the case of point-to-point problems, resulting in a more time-optimal solution. The results from this new optimisation are very impressive considering the simplicity.

1 Introduction

The ever increasing demands on accuracy confront control engineers with the mechanical limitations of modern industrial servo systems. The parasitic flexibilities in the structure of a servo-system have to be taken into account when controlling a high accuracy servo system. In recent years methods have been developed to prevent these unwanted oscillations caused by poorly damped poles of a system, by specific design of the reference trajectory of the system. Singer and Seering [1] presented a simple, effective method that is now known as Impulse Input Shaping, which introduces a small time delay in a shaped reference trajectory. More complex variations have been developed in the form of negative impulse shapers in [2], which reduces this delay significantly. With a priori knowledge about variations of the location of the system poles, more robust input shaping techniques can be used as presented in [3] and [4]. In [5] the concept of multi mode input shapers has been introduced to handle more that one flexible vibration in a system. It was shown that this introduces a lot of time delay unless an optimisation is performed to obtain an optimal impulse sequence for all the poles under consideration. For general trajectories this is however far from practical.

In this paper a new technique is presented that generates an input with optimal vibration reducing properties, very similar to the optimisation techniques presented in [5]. It is shown that for the class of point-to-point trajectories this optimisation can be solved quite easily, and applied to a large class of point to point trajectories. It is also shown that this technique can handle a large number of poles without significantly increasing the time delay as generally happens with impulse input shaping. The technique has been applied to an industrial XY-positioning table with great success, and the results from these experiment are compared to results for conventional Impulse Input Shaping.

2 Point to Point control objective

In short, the principle of input design, or input shaping, is to prevent the excitation of badly damped poles of a system by eliminating energy in the input signal at the specific frequencies corresponding to those poles. Impulse Input shaping produces zero residual vibration (as defined in [3],[1] and [5]) by creating a set of impulses with appropriate timing and amplitude, and convolving these impulses with the original reference trajectory [1]. This effectively leads to the filtering of the specific frequencies from the input signal. The convolution of an impulse sequence with a reference trajectory in general shapes the entire trajectory for minimal residual vibration, which is not necessary for point to point motion.

For point-to-point control, more freedom is available during movement to achieve zero vibration after the movement is complete. The trajectory that is followed to reach the next operating point is not as relevant as the time needed to reach this point within certain bounds; the *cycle time*. At the end of the movement the machinery must usually wait until all oscillations have disappeared before a specific task can be

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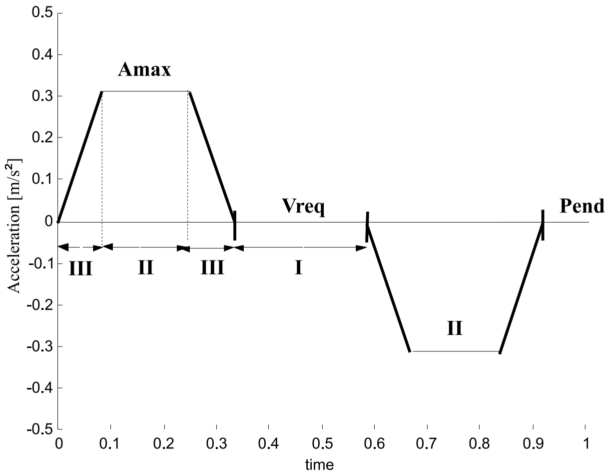

Figure 1: Acceleration profile for Point-To-Point movement

performed. So the main goal for input design methods is this case is the reduction of this time, which is in essence a reduction of the cycle time rather than a reduction of the residual vibrations along an entire trajectory.

This means that a trajectory should designed to be as short as possible based on the physical actuator and system constraints, with a constraint of minimal vibrations at the end of the trajectory. In the case of the XY-positioning table described later, a typical trajectory is from one point at constant speed, to another point at constant speed. The physical limitations in the form of a maximal jerk (J_{max}, derivative of acceleration), acceleration (A_{max}) and a predetermined speed that is to be obtained (V_{req}). Such a optimal trajectory can be obtained by optimising the entire trajectory to avoid excitation of specific system poles. This would result in a very short signal, but each time the trajectory is changed, the complex optimisation has to be performed again, which is in general much too time consuming or complicated.

Figure 1 shows the acceleration profile of a typical point-to-point trajectory as described above. Section I denotes the required constant speed v_{req}, section II is the maximal acceleration A_{max} and the grade in section III is caused by the maximal jerk J_{max}. By varying the time of section I we can reach different end positions p_{end} and variation of the length of section II leads to different values of V_{req}. The part of constant acceleration and constant velocity does not excite the system so only part III, denoted as the *jerk phase*, in figure 1 needs to be shaped to avoid excitation of the poles of the system. Our goal therefore is to obtain an input design/shape method that is able to eliminate vibrations for certain badly damped poles, by optimising only the jerk phase of the trajectory. From symmetry we can then use this to obtain an entire profile that has minimal vibration properties. As stated above, without changing this optimal jerk phase a large variety of point-to-point trajectories for a specific system can be generated.

3 Input design through optimisation

This input design problem can be formulated as a minimal time problem, the system must move from one position to another and be free of oscillations in as small a time as possible. Since we cannot change the physical limitations of the system we will only search for a time optimal jerk phase. With $a(t)$ the acceleration profile during the jerk phase, the problem is defined as:

- Objective of time optimality:

$$\min_{a(t)} \int_{t=t_0}^{t=t_N} 1 dt \qquad (1)$$

- cancellation of residual vibration (V) of a set of poles with frequency ω_m and damping ratio ζ_m

$$V(\omega_m, \zeta_m, a(t), t) = 0 \qquad (2)$$

- under the constraints:

$$\begin{aligned} a(t_0) &= 0 && (3)\\ a(t_N) &= A_{max} && (4)\\ \frac{da}{dt}(t_n) &= 0 && (5)\\ \left|\frac{da}{dt}(t_n)\right| &\leq J_{max} && (6) \end{aligned}$$

Which means that for the jerk phase an acceleration profile has to be found of minimum time (t_N as small as possible), while satisfying 5 constrains: The residual vibration at time t_N must be eliminated for a set of poles (ω_m, ζ_m) (2). The jerk phase must begin with zero acceleration (3) and end with maximal acceleration (4). For constant acceleration, the derivative of the acceleration (jerk) must be zero at the end of the jerk phase (5). And finally the magnitude of the jerk may at no time exceed the maximal jerk (6).

This problem clearly does not have a simple solution, but can be transformed into a linear convex optimisation problem in discrete time. If we take the trajectory to be a discrete time signal, $a(t)$ will in fact be a vector of N discrete values defining the jerk phase in discrete time. The time-optimal solution we are looking for will thus be a time vector a_N that satisfies the constraints with the smallest number of samples N. The formulation simplifies with respect to the constraints on the derivative of the acceleration if we work with the derivative of the acceleration. The problem is now simplified to a form where we will look for a simple impulse sequence to be called x that satisfies all constraints.

We now start with $N = 1$ and check if there is a feasible solution satisfying all constraints, if there is not, we increase N by one, and try again. We can repeat this over and over again until at least one feasible solution exists for a given number of samples. We now know that any feasible solution found this way is time-optimal for the discrete signal. If more than one solution exists, which is very likely, we need to make a selection. In this paper a rather arbitrary selection based on minimal energy is used, but more complex and meaningful selections can be used. With x being the vector describing the discrete signal $a(t)$, the problem is now stated as:

$$\text{sub-objective}: \quad \min_x (x^T x) \tag{7}$$

$$\text{constraints}:$$

$$V(\omega_m, \zeta_m, x[t], t) = 0 \tag{8}$$

$$x[1] = 0 \tag{9}$$

$$\sum_{i=1}^{N} \sum_{i=1}^{N} x[i] = A_{max} \tag{10}$$

$$\sum_{i=1}^{N} x[i] = 0 \tag{11}$$

$$\left\| \begin{array}{c} x[1] \\ x[1]+x[2] \\ \vdots \\ x[1]+\cdots+x[N] \end{array} \right\| \leq J_{max} \tag{12}$$

which can be written in the form:

$$\text{sub-objective}: \quad \min_x (x^T x) \tag{13}$$

$$\text{constraints}: \quad Ax = b$$

$$Cx \leq d \tag{14}$$

Which is no more than a feasibility test under some constraints, and a selection from possible feasible solutions once the solution space contains more than a unique solution.

To obtain a reasonable comparison with ZVD (Zero Vibration Derivative) shapers as presented by Singer and Seering [1] an additional constrain on the derivative of the residual vibrations has been introduced that was used by Singer and Seering to obtain more robustness with respect to variations in the frequency of the oscillating poles:

$$\frac{dV(\omega_m, \zeta_m, x(t), t)}{d\omega} = 0 \tag{15}$$

This is only added to obtain a reasonable comparison between the methods, however it is possible to take into account a more direct constraint for demands on the robustness in the optimisation routine.

The iterative procedure described above had been implemented using the MATLAB^{TM}-Optimisation function QUADPROG(QP) and has been applied on the XY-positioning table described below.

4 Experimental results for an industrial setup

The method described above has been used to eliminate oscillations in one direction of a high precision XY-positioning table as shown in figure 2. This machine consists of a large granite base on top of which is a moving mass (the chuck) that is moved by linear electro-motors. The granite base is fitted with dampers to the ground to isolate it from outside disturbances. The measurement system consists of laser-interferometers which measure the position of a reflective surface on the sides of the table. This industrial experimental set-up is especially relevant for industrial applications because the dynamic behaviour is very much similar to that encountered in industrial applications like modern computer manufacturing, pick and place machinery, etc... Two important oscillations of the system have been identified at 804 Hz (internal parasitic flexibility) and 55 Hz (frame resonance). An ZVD Impulse Input shaper has been designed for both of these oscillations, and the new point-to-point optimisation described above has been used for these frequencies. Both methods include the extra constraint on the derivative of the residual vibrations. Also experiments have been performed to eliminate the oscillations of a set of 4 poles at once. This has been done with the optimisation routine to show it can handle more complex systems quite easily. The physical limitations of the system are given by $J_{max} = 1313\ m/s^3$, $A_{max} = 10.5\ m/s^2$, $V_{req} = 0.5\ m/s$ and $P_{end} = 0.08\ m$.

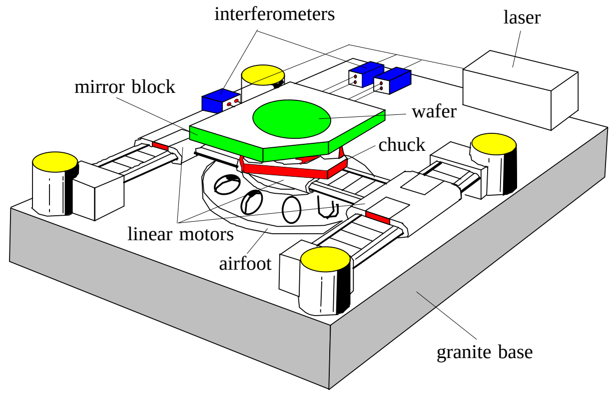

Figure 2: XY-positioning table
picture appearing courtesy of D.de Roover [6]

An input signal is designed for poles with an oscillation of 804 Hz and damping $\zeta = 0.00133$. Figure 3 shows the acceleration profile resulting from the 2 methods and the physical optimal profile. Both methods result in a slight elongation with respect to the fastest profile, with only a small difference:

Standard trajectory	:	$38.4\ ms$
Impulse Input shaper	:	$39.0\ ms = +0.6\ ms$
Optimised	:	$38.8\ ms = +0.4\ ms$

This is because the oscillatory frequency is so high that the half-period delay that is typical for Impulse shaping only represents a very short time. The time response (figure 4) shows the same small difference and the power spectral density plot (PSD)(figure 5) illustrates that the vibration reduc-

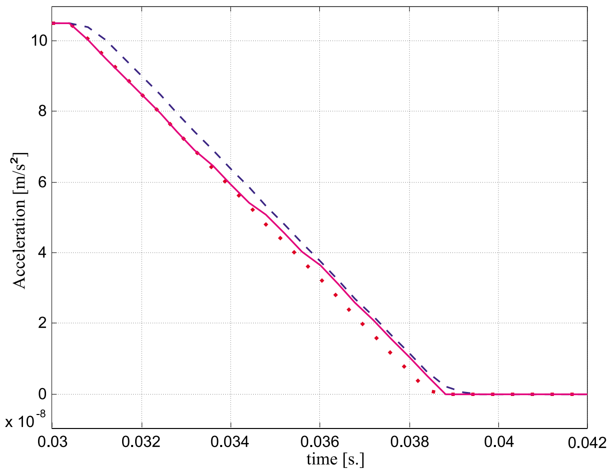

Figure 3: Acceleration profile, shaped 804 Hz vibration
original=dotted, impulse shaper=dashed, optimisation=full line

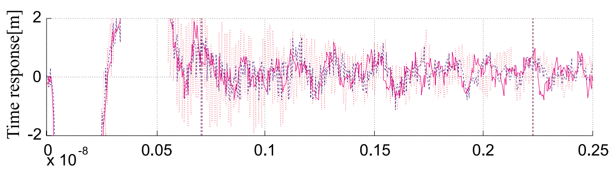

Figure 4: Time response for shaped 804 Hz inputs
original=dotted, impulse shaper=dashed, optimisation=full line

ing properties are very similar for the 2 methods. The cycle time is reduced by a spectacular 20% for both methods!:

$$\begin{aligned} \text{Standard} &: 71.2\,ms \\ \text{Impulse} &: 57.2\,ms = -14\,ms = -19.7\,\% \\ \text{Optimised} &: 56.8\,ms = -20.2\,ms = -20.2\,\% \end{aligned}$$

The same design is followed for poles with an oscillation of 55 Hz and damping $\zeta = 0.0150$. Figure 6 shows the acceleration profiles, now clearly showing distinctly different results in set-up time as well as the form of the profile. The optimisation routine clearly provides a shorter setup time:

$$\begin{aligned} \text{Standard} &: 38.4\,ms \\ \text{Impulse} &: 44.4\,ms = +6.0\,ms = +15.6\,\% \\ \text{Optimised} &: 43.2\,ms = +4.8\,ms = +12.5\,\% \end{aligned}$$

The demands on accuracy are not met for either of the shapers at this frequency because another oscillation of 85 Hz that was not visible beforehand, but which does cause unwanted oscillations. We can now look at the cumulative power spectral density plot defined as:

$$cumPSD(p) = \int_{-\infty}^{p} |S_{xx}(\omega)|^2 d\omega \tag{16}$$

with $S_{xx}(\omega)$ the power spectral density (PSD) of x at frequency ω. This cumPSD in figure 7 shows that especially the optimisation method excites this frequency quite

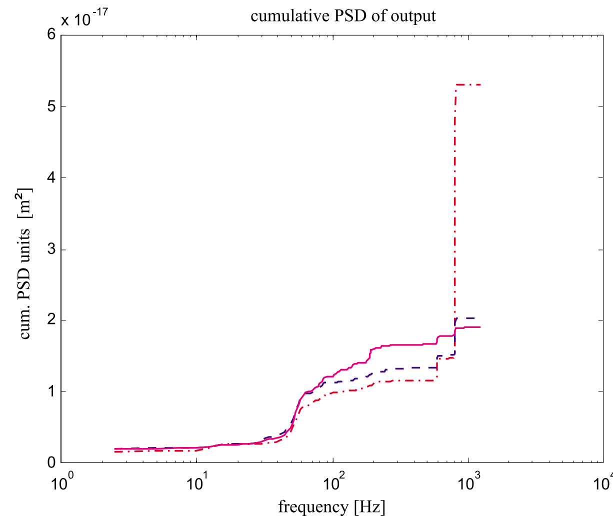

Figure 5: cumPSD of the output for the shaped 804 Hz inputs
original=dotted, impulse shaper=dashed, optimisation=full line

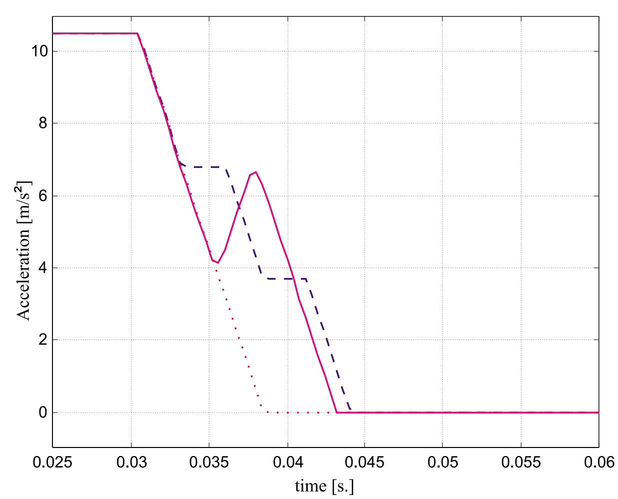

Figure 6: Acceleration profile for shaped 55 Hz vibrations
original=dotted, impulse shaper=dashed, optimisation=full line

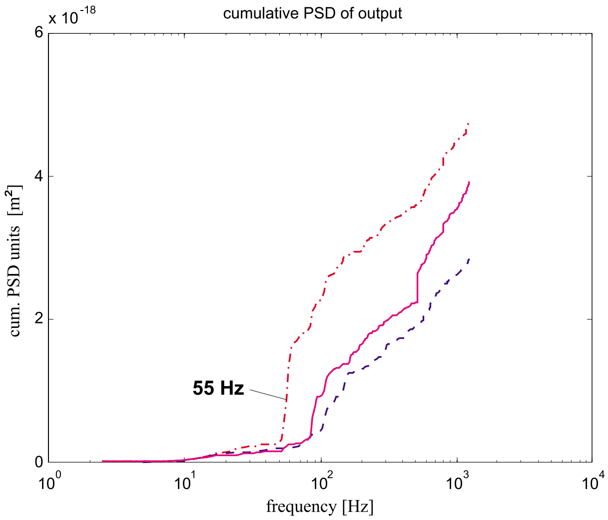

Figure 7: cumPSD of the output for the shaped 55 Hz inputs
original=dotted, impulse shaper=dashed, optimisation=full line

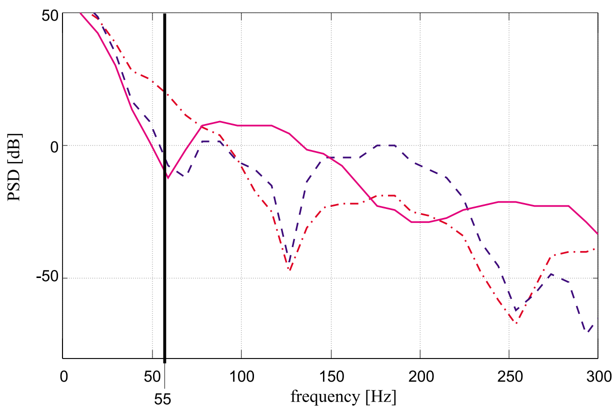

Figure 8: PSD of the shaped 55 Hz inputs
original=dotted, impulse shaper=dashed, optimisation=full line

strongly. This can be explained by looking at the PSD of the input signal (figure 8). The energy reduction for the optimisation is a bit better at 55 Hz, but other frequencies are excited a bit more than by using an Impulse Input shaper. The reduced setup time would suggest that with correct knowledge of all the important poles the optimisation routine will result in a slightly faster cycle time than the Impulse Input shaper would provide for point-to-point control problems.

Finally the routine is used to design an input taking into account 4 sets of poles:

$$
\begin{aligned}
f &= 614\,Hz \quad \zeta = 0.0156 \\
f &= 804\,Hz \quad \zeta = 0.0133 \\
f &= 494\,Hz \quad \zeta = 0.0150 \\
f &= 730\,Hz \quad \zeta = 0.0089
\end{aligned}
$$

The addition of 3 extra sets of poles to the optimisation re-

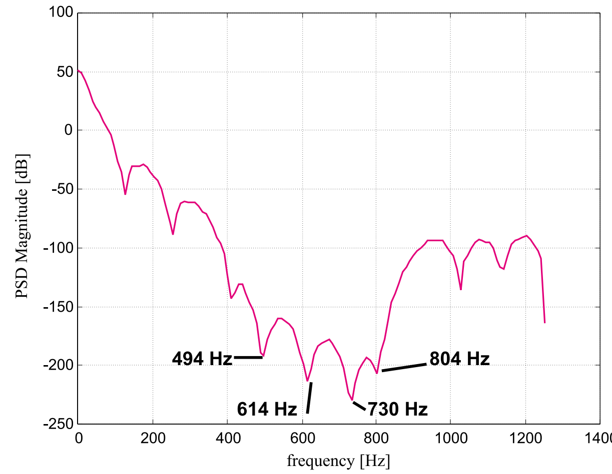

Figure 9: PSD of the shaped multi-mode input

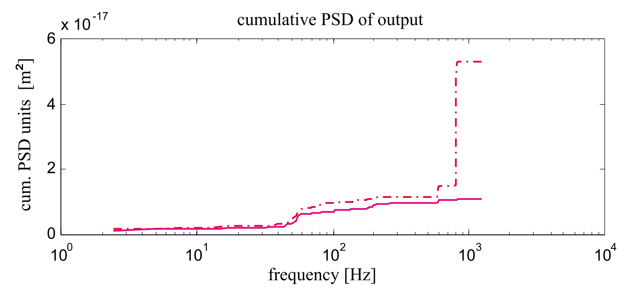

Figure 10: cumPSD of the output for the multi-mode signal
original=dash-dotted, optimisation=full line

sults in a trajectory with a setup time of 40.8 ms, which is only 2 ms longer than the setup time for only the 804 Hz shaper. From the Power spectral density plot in figure 9 can be seen that the input signal does not contain much energy at the given frequencies, which results in the cumPSD given in figure 10. As one can see, the resulting input perfectly eliminates the oscillations from the output.

5 Conclusions

This paper shows a new input design technique for the class of point to point control problems that results in less time delay than conventional techniques without losing much of the flexibility of such techniques. This technique is also able to handle several parasitic vibrations of a system at once without the need of increasingly complex optimisations. It has been shown that these techniques provide a very useful addition to existing input design techniques for industrial applications with very useful results already presented on an industrial setup. It is therefore the authors' opinion that for point to point control problems this technique is preferable to classical input design techniques.

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