Repetitive Learning Controller for CVCF PWM DC/AC Converter

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Abstract

In this paper, a repetitive learning control (RC) scheme is proposed for const-voltage const-frequency (CVCF) pulsewidth modulated (PWM) AC/DC converter. The repetitive controller is designed to force periodic tracking error approach zero asymptotically. The design theory of repetitive learning controller is described systematically and the stability analysis of overall system is discussed. The proposed DC/AC converter offers minimized voltage total harmonics distortion (THD) under parameter uncertainties and load disturbances. Simulation results are provided to illustrate the validity of the proposed scheme.

1 Introduction

CVCF DC/AC PWM converters are widely employed in various of AC power conditioning systems, such as automatic voltage regulator (AVR), uninterruptible power supply (UPS) systems. THD in the output voltage is one important index to evaluate the performance of DC/AC converters, which leads to communication interference, excessive heating in capacitors and transformers, solid-state device malfunctions *etc.* Nonlinear loads, causing periodic distortion, are major sources of THD in AC power systems.

To minimize THD, several high-precision control scheme for DC/AC converters are proposed. In [1] [2] [3], a deadbeat (or OSAP) controller is proposed. Sliding mode controller (SMC) [4] [5] and hysteresis controller (HC) [6] are proposed to overcome parameter uncertainties and load disturbance. However, the deadbeat control is highly dependent on the accuracy of the parameters; random switching pattern of SMC or HC will impose excessive stress on power device and cause the difficulty of low-pass filtering.

The repetitive learning control (RC) method [7], based on the internal model principle [8], is proposed in [9] to achieve high accuracy in the presence of uncertainties for servomechanism. Applications of RC [10] include robots [11], disc drives [12], steel casting process [13], satellite [14]. Without complete design method and stability analysis of RC system, [15] applies it to DC/AC converter with preliminary results .

In this paper, the design of discrete time RC controller is systematically presented. According to corresponding method, a plug-in RC controller is developed for the OSAP controlled CVCF PWM DC/AC converter. The stability of overall system is discussed. To show the validity of proposed method, computer simulation results are illustrated.

2 Design of Discrete Time Repetitive Controller

In the discrete time domain, a periodic signal with a period N, can be generated by a delay chain with a positive feedback loop (as shown in Fig.1). Note that the z-transform of a periodic signal, $w(k)$, can be expressed as:

$$W(z) = \frac{z^{-N}}{1-z^{-N}} W_0(z) = \frac{1}{z^N - 1} W_0(z) \qquad (1)$$

where $W_0(z) = w(0) + w(1)z^{-1} + ... + w(N-1)z^{-(N-1)}$, represents the z-transform of the first periodic sequence of $w(k)$; $N = f/f_c$ with f being the reference signal frequency and f_c being the sampling frequency.

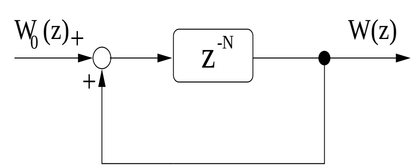


Figure 1: Periodic signal generator.

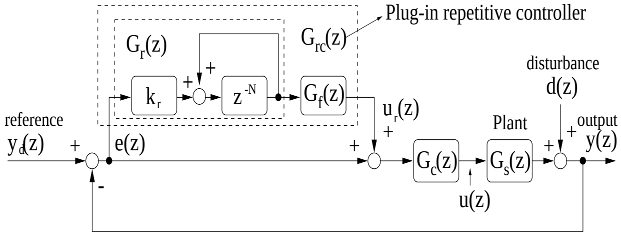


Figure 2: Plug-in repetitive control system.

According to internal model principle, the zero error tracking of any reference input in the steady state can be achieved if a generator of the reference input is included in the stable closed-loop system. Therefore, for a periodic reference input signal, the RC controller should include a repetitive signal generator as follows [10]:

$$G_r(z) = \frac{k_r z^{-N}}{1 - z^{-N}} \tag{2}$$

where k_r is *repetitive control gain*.

Let's consider a RC system as shown in Fig.2, where $y_d(z)$ is reference input signal, $y(z)$ is output signal, $d(z)$ is disturbance signal, $e(z)$ is the tracking error signal, $G_s(z)$ is the transfer function of the plant, plug-in RC controller $G_{rc}(z)$ is the feedforward compensator, and $G_c(z)$ is the original conventional feedback controller. Before the plug-in of RC controller $G_{rc}(z)$, $G_c(z)$ is chosen so that the following closed-loop transfer function is asymptotically stable.

$$\begin{aligned} H(z) &= \frac{G_c(z)G_s(z)}{1+G_c(z)G_s(z)} \\ &= \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \\ &= \frac{z^{-d}B^+(z^{-1})B^-(z^{-1})}{A(z^{-1})} \end{aligned} \tag{3}$$

where d is the known number of pure time step delays; $B^-(z^{-1})$ is the uncancelable portion of $B(z^{-1})$; $B^+(z^{-1})$ is the cancelable portion of $B(z^{-1})$. In order to provide a margin of stability to the filter, any zeros of $B(z^{-1})$ that is close to 1 can be included in $B^-(z^{-1})$ [11].

And the filter $G_f(z)$ in the RC controller $G_{rc}(z)$ is chosen in the following form

$$G_f(z) = \frac{z^{-n_u}A(z^{-1})B^-(z)}{B^+(z^{-1})b} \tag{4}$$

where $B^-(z)$ is obtained from $B^-(z^{-1})$ with z^{-1} replaced by z; b is a scalar chosen so that $b \geq [B^-(1)]^2$; n_u is the order of $B^-(z^{-1})$, and z^{-n_u} makes the filter realizable. Equation (4) is an implementation of *Zero Phase Error Tracking Controller* (ZPETC) as a filter design for $G_f(z^{-1})$ [16].

From Fig.2, the transfer functions from $y_d(z)$ and $d(z)$ to $y(z)$ in the overall closed loop system are respectively derived as

$$\begin{aligned} \frac{y(z)}{y_d(z)} &= \frac{(1+G_r(z)G_f(z))G_c(z)G_s(z)}{1+(1+G_r(z)G_f(z))G_c(z)G_s(z)} \\ &= \frac{(1-z^{-N}+k_r z^{-N}G_f(z))H(z)}{1-z^{-N}(1-k_r G_f(z)H(z))} \end{aligned} \tag{5}$$

$$\frac{y(z)}{d(z)} = \frac{1-z^{-N}}{1+G_c(z)G_s(z)} \frac{1}{1-z^{-N}(1-k_r G_f(z)H(z))} \tag{6}$$

And the error transfer function for the overall system is

$$\begin{aligned} G_e(z) &= \frac{e(z)}{y_d(z)-d(z)} \\ &= \frac{1-z^{-N}}{1+G_c(z)G_s(z)} \frac{1}{1-z^{-N}(1-k_r G_f(z)H(z))} \end{aligned} \tag{7}$$

From (3) (5) (6) and (7), it can be concluded that the overall closed-loop system is stable if the following two conditions hold: 1) The roots of $1 + G_c(z)G_s(z) = 0$ are inside the unit circle; and 2)

$$\|1 - k_r G_f(z)H(z)\| < 1 \tag{8}$$

Obviously, if the angular frequency ω of the reference input $y_d(t)$ and the disturbance $d(t)$ approaches $\omega_m = 2\pi m f$ $(m = 0, 1, 2, \cdots, N/2)$, then $z^{-N} \to 1$, $\lim_{\omega\to\omega_m} \|G_e(j\omega)\| = 0$, and thus

$$\lim_{\omega\to\omega_m} \|e(j\omega)\| = 0 \tag{9}$$

Equation (9) means that zero steady-state error is obtained with RC controller for any periodic reference input whose frequency is less than half of the sampling frequency.

Because the open loop poles of the RC controller are on the stability boundary, the stability of the overall system is sensitive to unmodeled dynamics [11]. In order to enhance the robustness of the system, a low-pass filter $Q(z, z^{-1})$ is used in RC controller as follows [11]:

$$G_r(z) = \frac{k_r Q(z, z^{-1})z^{-N}}{1 - Q(z, z^{-1})z^{-N}} \tag{10}$$

where

$$Q(z, z^{-1}) = \frac{\sum_{i=0}^{m} \alpha_i z^i + \sum_{i=1}^{m} \alpha_i z^{-i}}{2\sum_{i=1}^{m} \alpha_i + \alpha_0} \tag{11}$$

where α_i ($i = 0, 1, \cdots, m$; $m = 0, 1, 2, \cdots, N/2$) are coefficients to be designed.

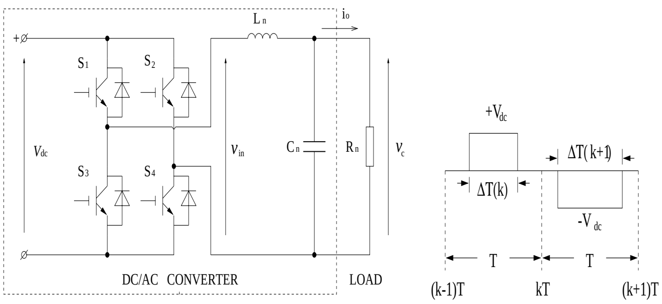
Notice that $Q(z, z^{-1})$ is a moving average filter that has zero phase shift and bring all open loop poles inside the unit circle except the one at +1. A first order filter

$Q(z,z^{-1}) = (z+2+z^{-1})/4$ is generally sufficient. On the other hand, high frequency periodic disturbance are not perfectly canceled by this controller. In this case, a trade-off is made between tracking precision and system robustness [17]. And correspondingly, equation (8) is modified as follows [12]:

$$\|1 - k_r G_f(z)H(z)\| < \|\frac{1}{Q(z,z^{-1})}\| \qquad (12)$$

3 Repetitive Controller for CVCF PWM DC/AC Converter

3.1 Modeling CVCF PWM DC/AC Converter



(a) CVCF PWM DC/AC converter.

(b) PWM waveform for v_{in}.

Figure 3: PWM converter and PWM waveform.

The dynamics of the CVCF PWM DC/AC converter (as shown in Fig.3 (a)) can be described as follows [1]:

$$\begin{bmatrix} \dot{v}_c \\ \ddot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L_nC_n} & -\frac{1}{C_nR_n} \end{bmatrix} \begin{bmatrix} v_c \\ \dot{v}_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_nC_n} \end{bmatrix} v_{in} \qquad (13)$$

where v_c is the output voltage; i_o is the output current; v_{dc} is the dc bus voltage; L_n, C_n, and R_n are the nominal values of the inductor, capacitor and load, respectively; as shown in Fig.3 (b), the control input v_{in} is a PWM voltage pulse of magnitude v_{dc} (or -v_{dc}) with width ΔT centered in the sampling interval T.

For a linear system $\dot{x} = Ax + Bu$, its sampled-data equation can be expressed as

$$x(k+1) = e^{AT}x(k) + \int_0^T e^{A(T-\tau)}Bu(\tau)d\tau \qquad (14)$$

Therefore, sampled-data form for (13) can be derived as follows:

$$\begin{bmatrix} v_c(k+1) \\ \dot{v}_c(k+1) \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \begin{bmatrix} v_c(k) \\ \dot{v}_c(k) \end{bmatrix} \pm \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \Delta T(k) \qquad (15)$$

where coefficients $\varphi_{11} = 1 - \frac{T^2}{2L_nC_n}$, $\varphi_{21} = -\frac{T}{L_nC_n} + \frac{T^2}{2L_nC_n^2R_n}$, $\varphi_{12} = T - \frac{T^2}{2C_nR_n}$, $\varphi_{22} = 1 - \frac{T}{C_nR_n} - \frac{T^2}{2L_nC_n} + \frac{T^2}{2C_n^2R_n^2}$, $g_1 = \frac{ET}{2L_nC_n}$, $g_2 = \frac{E}{L_nC_n}(1 - \frac{T}{2C_nR_n})$.

3.2 Problem Formulation

Consider the CVCF PWM DC/AC converter described by equation (15) and its output equation

$$y(k) = v_c(k) \qquad (16)$$

The objective of the controller is to force the tracking error between $y(k)$ and its sinusoidal reference $y_d(k)$ with the period of $N*T$ to approach zero asymptotically.

3.3 Controller Design for Converter

According the theory in section 2, the controller for CVCF PWM DC/AC comprises of conventional feedback controller and plug-in RC controller.

3.3.1 Conventional Feedback Controller:

Another form of the dynamics (15) (16) can be obtained as follows:

$$\begin{aligned} y(k+1) = \; & -p_1y(k) - p_2y(k-1) + m_1u(k) \\ & +m_2u(k-1) \end{aligned} \qquad (17)$$

where $u(k) = \pm\Delta T(k)$; $p_1 = -(\varphi_{11} + \varphi_{22})$, $p_2 = \varphi_{11}\varphi_{22} - \varphi_{21}\varphi_{12}$, $m_1 = g_1$, $m_2 = g_2\varphi_{12} - g_1\varphi_{22}$.

If the control law for the plant (17) is chosen as follows

$$\begin{aligned} u(k) = \; & \frac{1}{m_1}[y_d(k) - m_2u(k-1) + p_1y(k) \\ & +p_2y(k-1)] \end{aligned} \qquad (18)$$

then $y(k+1) = y_d(k)$. It yields deadbeat response $H(z) = z^{-1}$. (18) is called *One Sampling Ahead Preview* (OSAP) controller [1].

3.3.2 Plug-in Repetitive Controller:

In addition to a sampling time tracking delay, the OSAP controller depend on the accurate model with L_n, C_n and R_n. In practice, inevitable parameter uncertainties ΔL, ΔC and load disturbance ΔR will bring more tracking error. Therefore, a RC controller is used to overcome the periodic disturbance and parameters variation.

According to design theory mentioned in section 2, $G_f(z) = 1/H(z) = z$, and the RC controller $G_{rc}(z)$

is proposed as follows

$$G_{rc}(z) = G_r(z)G_f(z) = \frac{k_r z^{-N+1} Q(z, z^{-1})}{1 - Q(z, z^{-1}) z^{-N}} \quad (19)$$

Because the model of the plant is linear system and the high frequency noises will be filtered by low-pass LC filter of the converter, no low-pass filter $Q(z, z^{-1})$ is needed. We set $Q(z, z^{-1}) = 1$ in this case.

In sampled-data form, the RC controller can be expressed as follows

$$u_r(k) = u_r(k - N) + k_r e(k - N + 1) \quad (20)$$

In fact, (20) is an anticipatory learning control law [18].

3.4 Robustness Analysis

In practice, converter parameters are $L = L_n + \Delta L$, $C = C_n + \Delta C$, $R = R_n + \Delta R \in (R_{min}, \infty)$. Therefore, the difference equation for the actual plant becomes

$$\begin{aligned} y(k+1) = \quad & -a_1 y(k) - a_2 y(k-1) + b_1 u(k) \\ & + b_2 u(k-1) \end{aligned} \quad (21)$$

where $a_1 = p_1 + \Delta p_1$, $a_2 = p_2 + \Delta p_2$, $b_1 = m_1 + \Delta m_1$ and $b_2 = m_2 + \Delta m_2$ are calculated at the basis of the practical parameters L, C and R.

When an OSAP controller (18) is applied to the plant (21), the closed-loop transfer function $H(z)$ becomes

$$H(z) = \frac{(b_1 + b_2 z^{-1})}{M(s) - P(s)} \quad (22)$$

where $M(s) = (z + a_1 + a_2 z^{-1})(m_1 + m_2 z^{-1})$, $P(s) = (p_1 + p_2 z^{-1})(b_1 + b_2 z^{-1})$.

When $L = L_n$, $C = C_n$, $R = R_n$, a deadbeat response $H(z) = z^{-1}$ is achieved.

According to the stability analysis in section 2, the overall system is stablized if 1) all the poles of (22) are inside the unit circle; 2) $\|1 - k_r z H(z)\| < 1$.

3.5 Simulation

3.5.1 System Parameters: System parameters are given as follows: $C_n = 700\mu F$; $L_n = 450\mu H$; $R_n = 2\Omega$; $C = 800\mu F$, $L = 500\mu H$; $y_d(t)$ is $70V$ (peak); $v_{dc} = 100V$; $f = 50Hz$; $f_c = 1/T = 4KHz$.

Based on above parameters, as shown in Fig.5 (a), when $R > 1.5\Omega$, all the poles of $H(z)$ are located inside the unity circle, the system is stable. Fig.5 (b) shows the maximum gain for $zH(z)$ in frequency domain is approximately equal to 13.5. According to $\|1 - k_r z H(z)\| < 1$, system is stable if $k_r \in (0, 0.15)$. In our case, $k_r = 0.05$.

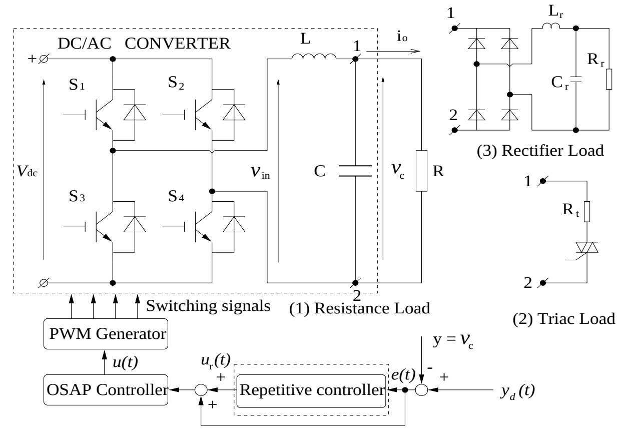
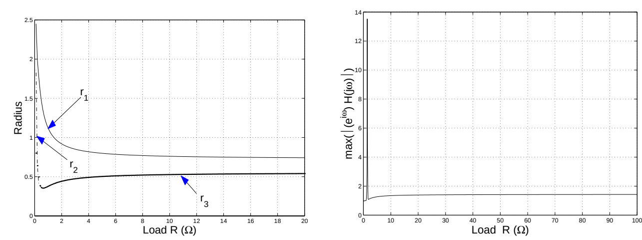


Figure 4: Simulation system.



(a) Radius of poles of $H(z)$.

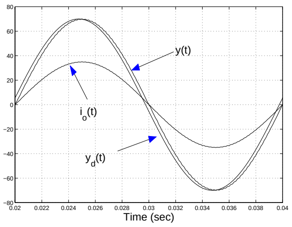
(b) Maximum $\|j\omega H(j\omega)\|$.

Figure 5: Stability Analysis.

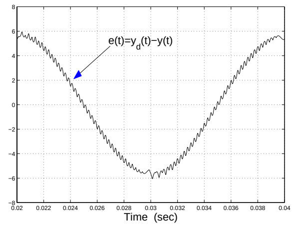
3.5.2 Simulation Results: Fig.6 and Fig.7 shows the simulation results of the only OSAP controlled and RC plus OSAP controlled CVCF PWM DC/AC converter with identical resistance load $R = 2\Omega$, respectively. With RC controller, the peak of tracking error $e(t)$ is reduced from about $6V$ to be less than $0.4V$ after about 0.9 second.

Fig.8 and Fig.9 shows the simulation results of the only OSAP controlled and RC plus OSAP controlled CVCF PWM DC/AC converter with with identical triac load (R_1=2Ω, triggering angle=60°, respectively. With RC controller, the peak of tracking error $e(t)$ is reduced from about $19V$ to be less than $0.4V$ after about 2.1 second.

Fig.10 and Fig.11 shows the simulation results of the only OSAP controlled and RC plus OSAP controlled CVCF PWM DC/AC converter with with identical uncontrolled rectifier load (L_1=5$e^{-5}\Omega$, C_1=5$e^{-2}\Omega$, R_1=3Ω), respectively. With RC controller, the peak of tracking error $e(t)$ is reduced from about $11V$ to be less than $0.4V$ after about 1.6 second.

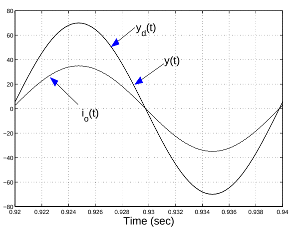


(a) Steady-state reference voltage $y_d(t)$, output voltage $y(t)$, output current $i_o(t)$.

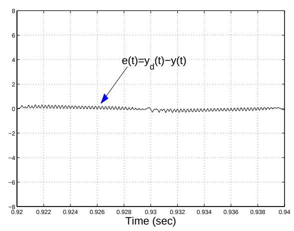


(b) Steady-state tracking error $e(t) = y_d(t) - y(t)$.

Figure 6: OSAP controlled results with load R=2Ω.

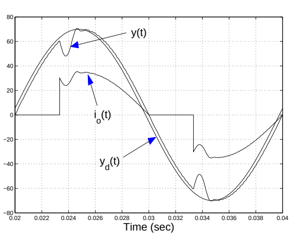


(a) Steady-state reference voltage $y_d(t)$, output voltage $y(t)$, output current $i_o(t)$.

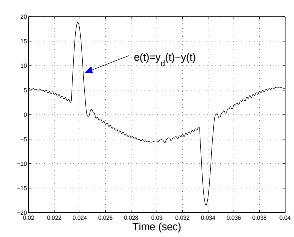


(b) Steady-state tracking error $e(t) = y_d(t) - y(t)$.

Figure 7: RC plus OSAP controlled results with load R=2Ω.

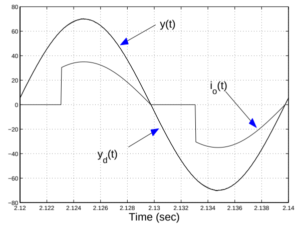


(a) Steady-state reference voltage $y_d(t)$, output voltage $y(t)$, output current $i_o(t)$.

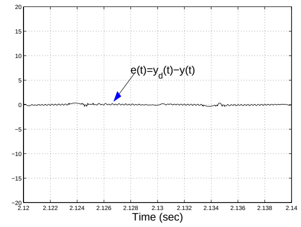


(b) Steady-state tracking error $e(t) = y_d(t) - y(t)$.

Figure 8: OSAP controlled results with triac load (R_1=2Ω, triggering angle=60°).

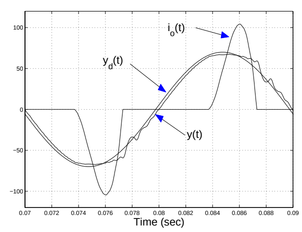


(a) Steady-state reference voltage $y_d(t)$, output voltage $y(t)$, output current $i_o(t)$.

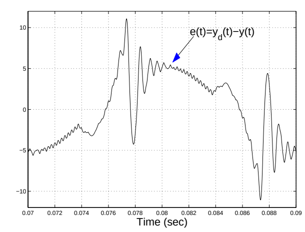


(b) Steady-state tracking error $e(t) = y_d(t) - y(t)$.

Figure 9: RC plus OSAP controlled results with triac load (R_1=2Ω, triggering angle=60°).

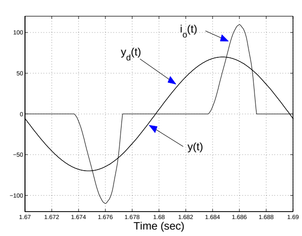


(a) Steady-state reference voltage $y_d(t)$, output voltage $y(t)$, output current $i_o(t)$.

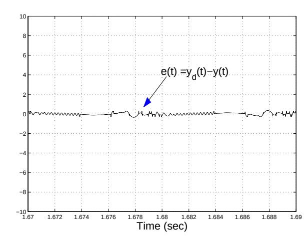


(b) Steady-state tracking error $e(t) = y_d(t) - y(t)$.

Figure 10: OSAP controlled results with uncontrolled rectifier load (L_1=5e^{-5}Ω, C_1=5e^{-2}Ω, R_1=3Ω).



(a) Steady-state reference voltage $y_d(t)$, output voltage $y(t)$, output current $i_o(t)$.



(b) Steady-state tracking error $e(t) = y_d(t) - y(t)$.

Figure 11: RC plus OSAP controlled results with uncontrolled rectifier load (L_1=5e^{-5}Ω, C_1=5e^{-2}Ω, R_1=3Ω).

4 Conclusion

In this paper, a plug-in discrete time repetitive learning controller for CVCF PWM DC/AC converter is proposed. The system offers minimized output voltage THD and fast response under different loads and parameter uncertainties. The tracking error caused by periodic loads disturbances (such as rectifier and triac loads) and parameter uncertainties (ΔL and ΔC) are eliminated by the plug-in repetitive learning controller. It is shown that the proposed RC scheme is zero tracking error control strategy for the CVCF PWM DC/AC converter. Theoretical analysis and simulation results are provided to testify the validity of the proposed control scheme.

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