Model Estimation and Controller Reduction: Dual Closed Loop Identification Problems

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Abstract

Algorithms for direct controller reduction by identification in closed loop has been recently proposed [9, 6]. In this paper it is shown that the plant model identification in closed loop using closed loop output error identification algorithms and the direct estimation in closed loop of a reduced order controller feature a duality character. Basic schemes, algorithms and properties of the algorithms can be directly obtained by interchanging the plant model and the controller. Experimental results concerning the use of these algorithms for direct controller reduction can be found in a companion paper [7]. In the last part of the paper the interaction between plant model identification in closed loop and direct controller reduction is emphasized.

Keywords: Identification in closed loop, Controller reduction, Recursive identification algorithms, Asymptotic properties.

1 Introduction

Plant model identification in closed loop has received a growing attention in the nineties [4, 2, 8, 3]. One of the reasons is the possibility to estimate approximated plant models (design models) whose precision is enhanced in the frequency regions which are critical for control. The term "control oriented identification" was also used as a generic name for the identification methods which achieve the above objectives.

Closed loop output error (CLOE) identification algorithms [8, 10] allow to identify an approximate design model which features a high accuracy in the critical regions for control design. Effectively the frequency distribution of the asymptotic error (bias) between the estimated and the true model is heavily weighted by the magnitude of the sensitivity function of the true system which explains why this desired property is obtained.

In fact the algorithms search for a plant model which minimises a 2-norm error between the true closed loop transfer function and the simulated closed loop transfer function containing the final estimated model.

Asymptotic frequency bias error distribution can be obtained indicating clearly that the bias is small where some sensitivity functions are high and in addition this bias error is not affected by the measurement noise [8, 3]. Two basic configurations for closed loop output error identification will be considered [8, 3]:

- external excitation superposed to the output of the controller (see Fig.2);
- external excitation superposed to the reference signal (see Fig.3).

While the parameter adaptation algorithm (P.A.A.) is the same for the two configurations, the asymptotic properties of the estimated model will be different.

Using the basic concepts for controller reduction [1], it has been shown in the papers [9, 6] that direct controller reduction can be done by identification in closed loop either using simulated data or real data (which is a unique feature of this approach with respect to other approaches to direct controller reduction). To proceed one need to know the nominal controller and to have an estimated plant model (either the model used for design or a model identified on site in open or closed loop). It has been shown that the algorithms developed for the estimation of reduced order controllers lead to reduced order controllers which minimize a 2-norm with the property that the error between the reduced order controller and the nominal controller is small at the frequencies where some sensitivity functions are high. Two basic configurations have been considered:

- **Input matching:** The reduced order controller tries to minimize the error between the control generated by the two controllers in closed loop operation;

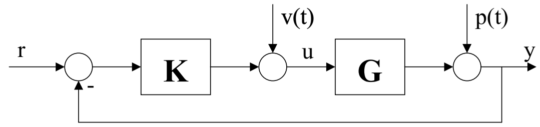

Figure 1: The closed-loop system

- **Output matching:** The reduced order controller tries to minimize the error between the plant output generated by the two controllers in closed loop operation.

The objective of the paper is to show that these two problems (model estimation and controller reduction) are dual and that the effective algorithms, the stability analysis and the asymptotic properties can be obtained by a direct substitution i.e.:
estimated plant model $\longrightarrow$ estimated controller;
nominal controller $\longrightarrow$ plant model.

The only differences will occur at the effective implementation of the algorithms since the controller has a direct transfer term while the plant model has at least one step delay.

The paper is organized as follows: Section 2 will specify the notations. Section 3 will review the basic schemes for closed loop output error identification and their properties .

In Section 4 the dual arguments indicated above will be used and it will be shown that the results obtained correspond to those previously obtained for direct controller reduction by identification in closed loop. Section 5 will mention some experimental results. Section 6 will briefly discuss the interaction between plant model identification in closed loop and direct identification in closed loop of reduced order controllers.

2 Notations

Consider the system shown in Fig. 1 where the plant model is given by:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{1}$$

where:

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \\ &= 1 + z^{-1}A^*(z^{-1}) \qquad (2) \\ B(z^{-1}) &= b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} \\ &= z^{-1}B^*(z^{-1}) \qquad (3) \end{aligned}$$

and the nominal controller by:

$$K(z^{-1}) = \frac{R(z^{-1})}{S(z^{-1})} \tag{4}$$

One defines the following sensitivity functions:

- $S_{yp}(z^{-1}) = \dfrac{1}{1+KG} = \dfrac{A(z^{-1})S(z^{-1})}{P(z^{-1})}$;
- $S_{up}(z^{-1}) = \dfrac{-K}{1+KG} = \dfrac{-A(z^{-1})R(z^{-1})}{P(z^{-1})}$;
- $S_{yv}(z^{-1}) = \dfrac{G}{1+KG} = \dfrac{z^{-d}B(z^{-1})S(z^{-1})}{P(z^{-1})}$;
- $S_{yr}(z^{-1}) = \dfrac{KG}{1+KG} = \dfrac{z^{-d}B(z^{-1})R(z^{-1})}{P(z^{-1})}$.

where

$$P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \tag{5}$$

The system of Fig.1 will be denoted the "true closed loop system". Throughout the paper we will consider feedback systems which will use either an estimation of G (denoted $\hat{G}$) or a reduced order estimation of K (denoted $\hat{K}$). The corresponding sensitivity functions will be denoted as follows:

- S_{xy} - Sensitivity function of the true closed loop system (K, G).
- $\hat{S}_{xy}$ - Sensitivity function of the nominal simulated closed loop system (nominal controller K + estimated plant model $\hat{G}$).
- $\hat{\hat{S}}_{xy}$ - Sensitivity function of the simulated closed loop system using a reduced order controller (reduced controller $\hat{K}$ + estimated plant model $\hat{G}$).

Similar notations are used for $P(z^{-1})$: $\hat{P}(z^{-1})$ when using K and $\hat{G}$, $\hat{\hat{P}}(z^{-1})$ when using $\hat{K}$ and $\hat{G}$.

3 Closed loop output error identification algorithms (CLOE)

The two basic schemes are shown in Fig. 2 and Fig. 3.

The output of the plant is given by:

$$\begin{aligned} y(t+1) &= -A^*y(t) + B^*u(t-d) = \theta^T\psi(t) \qquad (6) \\ \psi(t) &= [-y(t), \ldots, -y(t-n_A+1), u(t-d), \\ &\qquad \ldots, u(t-d-n_B+1)] \qquad (7) \\ \theta &= [a_1, \ldots, a_{n_A}, b_1, \ldots, b_{n_B}] \qquad (8) \\ u(t) &= -\frac{R}{S}y(t) + r_u(t) \qquad (9) \end{aligned}$$

where $r_u(t) = r(t)$ in the scheme of Fig. 2 and $r_u(t) = \dfrac{R}{S}r(t)$ in the scheme of Fig. 3.

The output of the closed loop adjustable predictor is given by:

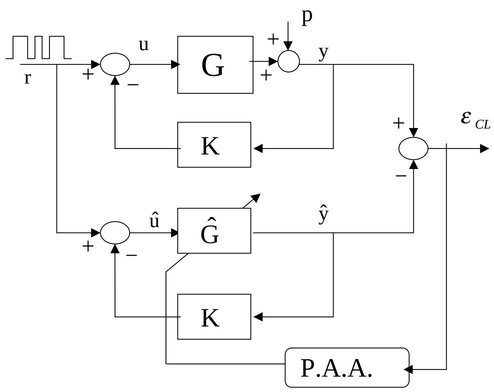

Figure 2: Closed loop output error identification (external excitation added to the controller output).

a priori:

$$\hat{y}^0(t+1) = -\hat{A}^*(t,q^{-1})\hat{y}(t) + \hat{B}^*(t,q^{-1})\hat{u}(t-d) = \hat{\theta}^T(t)\phi(t) \quad (10)$$

$$\hat{u}(t) = -\frac{R(q^{-1})}{S(q^{-1})}\hat{y}(t) + r_u(t) \quad (11)$$

a posteriori:

$$\hat{y}(t+1) = \hat{\theta}^T(t+1)\phi(t) \quad (12)$$

where

$$\hat{\theta}^T(t) = [\hat{a}_1, \ldots, \hat{a}_{n_{\hat{A}}}, \hat{b}_1, \ldots, \hat{b}_{n_{\hat{B}}}] \quad (13)$$

$$\phi(t) = [-\hat{y}(t), \ldots, -\hat{y}(t-n_{\hat{A}}+1), \hat{u}(t), \ldots, \hat{u}(t-d-n_{\hat{B}}+1)] \quad (14)$$

The closed loop output error is defined as:

$$a\,priori: \quad \varepsilon^0_{CL}(t+1) = y(t+1) - \hat{y}^0(t+1) \quad (15)$$

$$a\,posteriori: \quad \varepsilon_{CL}(t+1) = y(t+1) - \hat{y}(t+1) \quad (16)$$

and the parameter adaptation algorithm (P.A.A.) will be given by:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon_{CL}(t+1) \quad (17)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t) \quad (18)$$

$$0 < \lambda_1(t) \le 1 \quad ; \quad 0 \le \lambda_2(t) < 2; \quad F(0) > 0$$

$$\varepsilon_{CL}(t+1) = y(t+1) - \hat{y}(t+1) = \frac{y(t+1) - \hat{y}^0(t+1)}{1 + \Phi^T(t)F(t)\Phi(t)} \quad (19)$$

Specific algorithms are obtained by an appropriate choice of the observation vector $\Phi(t)$ as follows:

- CLOE

$$\Phi(t) = \phi(t) \quad (20)$$

- F-CLOE

$$\Phi(t) = \frac{S(q^{-1})}{\hat{P}(q^{-1})}\phi(t) \quad (21)$$

where:

$$\hat{P}(q^{-1}) = \hat{A}(q^{-1})S(q^{-1}) + q^{-d}\hat{B}(q^{-1})R(q^{-1}) \quad (22)$$

and $\hat{A}(q^{-1})$ and $\hat{B}(q^{-1})$ correspond to an *a priori* estimation of $\hat{G}$.

- AF-CLOE

$$\Phi(t) = \frac{S(q^{-1})}{\hat{P}(t,q^{-1})} \quad (23)$$

where:

$$\hat{P}(t,q^{-1}) = \hat{A}(t,q^{-1})S(q^{-1}) + q^{-d}\hat{B}(t,q^{-1})R(q^{-1}) \quad (24)$$

and $\hat{A}(t,q^{-1})$ and $\hat{B}(t,q^{-1})$ correspond to the current estimates of $\hat{G}$.

When $n_{\hat{A}} = n_A$, $n_{\hat{B}} = n_B$, then the closed loop output error goes to zero (in a deterministic environment) and unbiased estimates are obtained in a stochastic environment (when $p(t)$ is independent with respect to $r_u(t)$ and of finite power) if:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2}; \max_t \lambda_2(t) \le \lambda < 2 \quad (25)$$

is a strictly positive real transfer function, where:

$$H(z^{-1}) = \begin{cases} S/P & \text{for} \quad CLOE \\ \hat{P}/P & \text{for} \quad F-CLOE \\ 1 & \text{for} \quad AF-CLOE \end{cases} \quad (26)$$

(in the last case this is a local result) [8, 10].

When $n_{\hat{A}} < n_A$, $n_{\hat{B}} < n_B$, it can be shown that all the signals are bounded provided that [8]:

– $r_u(t)$ is norm bounded;

– It exists a reduced order model such that:

$$y(t+1) = -\hat{A}^*(q^{-1})\,y(t) + \hat{B}(q^{-1})\,u(t-d) + \eta(t+1) \quad (27)$$

where $\eta(t+1)$ is norm bounded for a norm bounded $r(t)$;

– The passivity condition (25) is satisfied.

The asymptotic frequency distribution of the bias when the estimated plant model does not belong to the model set is given by [8, 10]:

$$\hat{\theta}^* = \arg\min_\theta \int_{-\pi}^{\pi} |S_{yp}|^2 \left[|G-\hat{G}|^2|\hat{S}_{yp}|^2\phi_r(\omega) + \phi_p(\omega) \right] d\omega = \arg\min_\theta \int_{-\pi}^{\pi} \left[|S_{yv} - \hat{S}_{yv}|^2\phi_r(\omega) + |S_{yp}|^2\phi_p(\omega) \right] d\omega \quad (28)$$

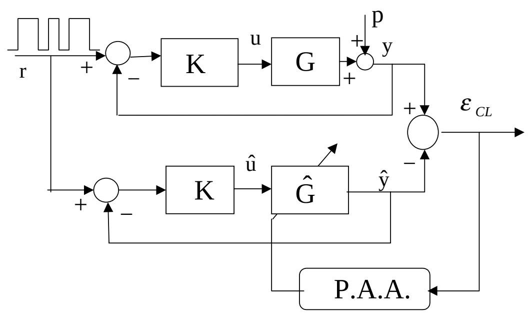

Figure 3: Closed loop output error identification (external excitation added to the reference signal).

for the scheme of Fig. 2 and by:

$$\begin{aligned}\hat{\theta}^* &= \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 \Big[|G-\hat{G}|^2 |\hat{S}_{up}|^2 \phi_r(\omega) \\ &\qquad + \phi_p(\omega) \Big] d\omega \\ &= \arg\min_{\theta} \int_{-\pi}^{\pi} \Big[|S_{yr}-\hat{S}_{yr}|^2 \phi_r(\omega) \\ &\qquad + |S_{yp}|^2 \phi_p(\omega) \Big] d\omega \qquad (29)\end{aligned}$$

for the scheme in Fig. 3.

In short, if $r(t)$ is a discrete time white noise (for exemple a PRBS) the algorithm will search for the best $\hat{G}$ which will minimize either the 2-norm between the sensitivity functions with respect to an input disturbance of the true closed loop system and of the estimated closed loop system for the scheme of Fig.2 or the 2-norm between the complementary sensitivity functions of the true closed loop system and of the estimated closed loop system for the scheme of Fig.3 (the noise does not affect the results).

4 Algorithms for direct estimation in closed loop of reduced order controllers

We will consider that a model of the plant is available (denoted by $\hat{G}$). This model can be the plant model used for design or an identified model (in open loop or in closed loop) which passed the validation tests.

We will assume of course that the nominal controller of orders n_R and n_S is known (denoted by K).

We will make the dual type modifications in the scheme of figures 2 and 3 summarized in Table 1. The resulting schemes are shown in Fig.4 and Fig.5.

The signal $x(t)$ is defined as:

$$x(t) = r(t) - u(t) \qquad (30)$$

in Fig.4 and

$$x(t) = \hat{G}[r(t) - u(t)] \qquad (31)$$

in Fig.5.

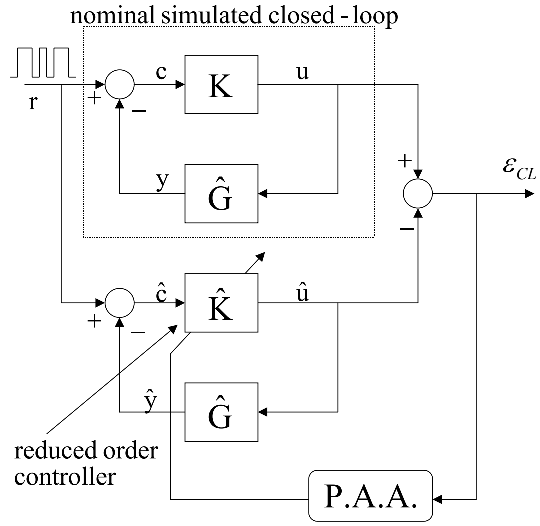

Figure 4: Closed loop input matching (CLIM).

As it will be shown, the scheme of Fig.4 corresponds to closed loop input matching objective (CLIM algorithm) and the scheme of Fig.5 corresponds to closed loop output matching objective (CLOM algorithm) for controller reduction.

With the above changes one has

a priori:

$$\begin{aligned}\hat{u}^0(t+1) &= -\hat{S}^*(t,q^{-1})\hat{u}(t) + \hat{R}(t,q^{-1})\hat{x}(t) \\ &= \hat{\theta}_c^T(t)\phi_c(t) \qquad (32)\end{aligned}$$

a posteriori : $\quad \hat{u}(t+1) = \hat{\theta}_c^T(t+1)\phi_c(t) \qquad (33)$

where

$$\begin{aligned}\hat{x}(t+1) &= -\hat{A}^*(q^{-1})\hat{x}(t) - \hat{B}^*(q^{-1})\hat{u}(t-d) \\ &\quad + A(q^{-1})r(t) \qquad (34)\end{aligned}$$

for the scheme of Fig.4,

$$\begin{aligned}\hat{x}(t+1) &= -\hat{A}^*(q^{-1})\hat{x}(t) - \hat{B}^*(q^{-1})\hat{u}(t-d) \\ &\quad + \hat{B}^* r(t-d) \qquad (35)\end{aligned}$$

for the scheme of Fig.5, and

$$\hat{\theta}_c^T(t) = [\hat{s}_1(t), \ldots, \hat{s}_{n_{\hat{S}}}(t), \hat{r}_0(t), \ldots, \hat{r}_{n_{\hat{R}}}(t)] \qquad (36)$$

$$\begin{aligned}\phi_c(t) &= [-\hat{u}(t), \ldots, -\hat{u}(t-n_{\hat{S}}+1), \hat{x}(t+1), \\ &\quad \ldots, \hat{x}(t-n_{\hat{R}}+1)] \qquad (37)\end{aligned}$$

The closed loop input error will be given by:

a priori:

$$\varepsilon_{CL}^0(t+1) = u(t+1) - \hat{u}^0(t+1) \qquad (38)$$

a posteriori:

$$\varepsilon_{CL}(t+1) = u(t+1) - \hat{u}(t+1) \qquad (39)$$

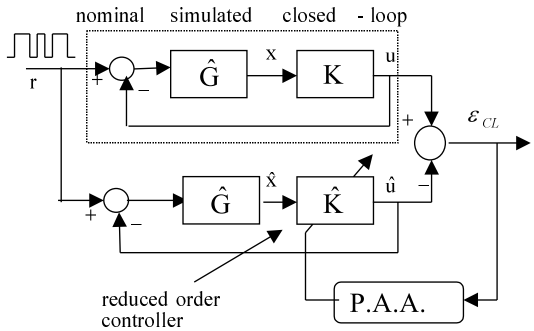

Figure 5: Closed loop output matching (CLOM).

and from Eqs. (17) through (18) one gets the P.A.A.

$$\hat{\theta}_c(t+1) = \hat{\theta}_c(t) + F(t)\Phi(t)\varepsilon_{CL}(t+1) \quad (40)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t) \quad (41)$$

$$0 < \lambda_1(t) \leq 1 \quad ; \quad 0 \leq \lambda_2(t) < 2; \quad F(0) > 0$$

$$\varepsilon_{CL}(t+1) = \frac{u(t+1) - u^0(t+1)}{1 + \Phi^T(t)F(t)\Phi(t)} \quad (42)$$

The corresponding specific algorithms will be

- CLIM / CLOM

$$\Phi(t) = \phi_c(t) \quad (43)$$

- F-CLIM / F-CLOM

$$\Phi(t) = \frac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})}\,\phi_c(t) \quad (44)$$

where:

$$\hat{P}(q^{-1}) = \hat{A}(q^{-1})S(q^{-1}) + q^{-d-1}\hat{B}(q^{-1})R(q^{-1}) \quad (45)$$

is a known quantity and therefore there is no need to estimate this polynomial in line.

The corresponding transfer functions involved in the passivity conditions for stability becomes:

$$H(z^{-1}) = \begin{cases} \dfrac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})} & \text{for} \quad \mathrm{CLIM/CLOM} \\ 1 \quad \text{for} & \mathrm{F-CLIM/F-CLOM} \end{cases} \quad (46)$$

(since the exact polynomial of the nominal simulated closed loop is known).

The crucial step is to examine the properties of the estimated reduced order controller. To do this we will directly use the expressions (28) and (29) in which we will take $\phi_p(\omega) = 0$ (no noise) and we will make the substitution indicated in Table 1. One gets for the scheme of Fig.4 (closed loop input matching)

Plant model identification in closed loop Fig.2, Fig.3		Identification of reduced order controller in closed loop Fig.4, Fig.5
controller (K)	$\longrightarrow$	available plant model ($\hat{G}$)
true plant model (G)	$\longrightarrow$	nominal controller (K)
estimated plant model ($\hat{G}$)	$\longrightarrow$	estimated (reduced order) controller ($\hat{K}$)
y, $\hat{y}$	$\longrightarrow$	u, $\hat{u}$
u, $\hat{u}$	$\longrightarrow$	x, $\hat{x}$

Table 1: Duality between plant model identification in closed loop and direct estimation of reduced order controller in closed loop

$$\hat{\theta}_c^* = \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yp}|^2 |K - \hat{K}||\hat{\hat{S}}_{yp}|\phi_r(\omega)d\omega = \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{up} - \hat{\hat{S}}_{up}|^2 \phi_r(\omega)d\omega \quad (47)$$

and for the scheme of Fig.5 (closed loop output matching):

$$\hat{\theta}_c^* = \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yp}|^2 |K - \hat{K}||\hat{\hat{S}}_{yv}|\phi_r(\omega)d\omega = \arg\min_{\theta_c} \int_{-\pi}^{\pi} |\hat{S}_{yr} - \hat{\hat{S}}_{yr}|^2 \phi_r(\omega)d\omega \quad (48)$$

When $r(t)$ is a discrete time white noise these expressions correspond exactly to the 2-norm expression which we would like to minimize for closed loop input matching and closed output matching respectively.

A detailed analysis when one uses real data can be found in [9] and [6]. In this case it can be shown that the noise will not affect the minimization procedure.

5 Experimental results

Experimental results using these algorithms for controller order reduction in active suspension control can be found in the companion paper [7].

6 Interaction between plant model identification in closed loop and controller reduction by identification in closed loop

In order to do a direct controller reduction which tries to approach as much as possible the nominal closed loop characteristics we need a model of the plant (independently of the technique used for controller reduction).

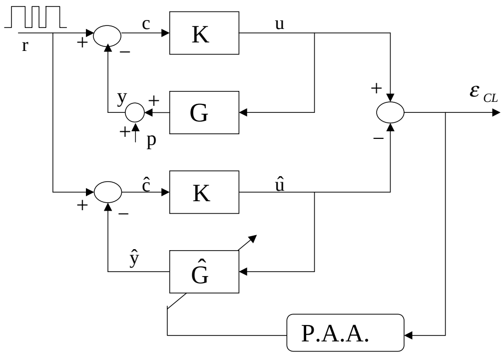

Figure 6: Closed loop input error identification scheme.

If one has access to the real plant and the nominal controller can be implemented one can do an identification of the plant model in closed loop operation. This will allow to find the best model which in feedback with the nominal controller will give the best approximation of the nominal true closed loop system.

However one can ask: what is the most appropriate identification scheme if the use of the identified model is for controller reduction? Or equivalently: what is the good identification criterion?

If one uses the same type of reasoning as the one used in "iterative identification in closed loop and controller redesign" [4], [2] the answer will be that one should use for identification the same criterion as for controller order reduction. Therefore for CLIM the good identification criterion will be

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{up} - \hat{S}_{up}|^2 \phi_r(\omega) d\omega \qquad (49)$$

and for CLOM:

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{yr} - \hat{S}_{yr}|^2 \phi_r(\omega) d\omega \qquad (50)$$

Therefore for CLOM it is reasonable to do a closed loop identification of the plant model using CLOE algorithm in the configuration given in Fig.3 (see Eq.(29)).

For CLIM it is reasonable to do a closed loop identification of the plant model using a new algorithm where one uses a closed loop input error as a measure of the discrepancy between the two closed loops. The corresponding diagram is shown in Fig.6. Details of the algorithms can be found in [5].

7 Conclusions

The objective of this paper was to show that recently developed algorithms for direct identification in closed loop of reduced order controllers can be obtained straightforward from closed loop output error identification algorithms by duality arguments. This duality arguments can also be used to directly obtain the properties of algorithms used for identification of reduced order controllers. In addition of stressing the duality between identification of (reduced order) models in closed loop and identification of reduced order controller in closed loop the paper pointed out the interaction between these two operations.

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