Sydney, Australia, Dec., 2000

Observability Conditions of Induction Motors at Low Frequencies

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Abstract

The paper studies the observability conditions for the induction motor. We first review the sufficient condition derived by from the classical geometrical analysis, then we present some particular case where these conditions turn out to be also necessary. The purpose of the paper is to clarify observability problems found in particular when the machine is operating at low frequencies without angular velocity sensor. The observability analysis also clarifies the fact that the observability problems are physically due to low frequency operation conditions, rather than to low rotational speed operation (even if these two cases coincides in absence of torque load).

Keywords: Induction motors, observability.

1 Introduction

Industry concerned by induction motors, are continuously seeking for cost reductions in their products. These reductions often impose the minimization of the number of sensor used for control purposes because they substantially contribute to increase the complexity and cost of the full installation (additional cables, maintenance, etc.), and the default probability.

The scalar control (U/f), which is basically an open-loop control, does not need (in theory) any sensor (in practice, voltage and current sensors are used to increase the motor performance, and the level of security). Vector controllers (i.e. Field Oriented Control), and nonlinear feedbacks perform over the scalar controller, but they need to measure the motor speed in order to compute the coordinates transformation often required to estimate the orientation frame.

The so called *sensorless* feedback for induction motors, implies that the speed control of the machine should be performed only on the basis of output current injection. This imposes the design of observers for flux, and angular velocity.

As noted by several authors, the observer design is particularly difficult when it comes to operate the machine at low frequencies (when the input voltage rotates at very low frequency), which is an operation regime that often coincides with the operation at low velocities. This operation regime is included in many applications where the motor should be keep in stand still mode for a period of time.

Previous works on sensorless control do not really account for this possible lack of observability that may arise at low frequency operation. For instance, [9] have proposed a two-time scale model decomposition leading to a combined observer/estimator strategy. An observer is separately designed for the current and flux vectors, while the mechanical angular velocity is obtained through an estimator (open-loop observer). Other possibility is to resort to iterative sliding mode design as proposed in [3]. In these two approaches the observability problems do not explicitly come out. The former uses an estimator and consequently is robust to the lack of observability. The latter does not take into account the lack of observability property, neither.

An commonly strategy intended to recover the observability property lost at low frequency, is to inject a high frequency singal to the control input (see for instance, [8] and [6]). This idea relates to the notion of universal inputs. Nevertheless, there is no previous study formally demonstrating such a possibility.

Observability problems at low frequency has been detected by many authors and praticiens in the field [8]. Nevertheless, few works have been addressing this problem from a formal observability point of view. The purpose of this paper is thus to provide a formal analysis, yielding to a characterization of the observability properties of the 6-dimension state (current, flux, rotational speed, and torque load) of the induction motor. We also compare this results to the standard 5-dimension model (rotational speed is assumed constant).

We first review some of the sufficient condition derived from classical geometrical analysis, then we present some particular cases where these condition turn out to be also necessary. It is thus formally shown that the system observability is lost when the motor operates at constant flux* and zero rotational velocity. This operation mode corresponds to constant input voltage (zero frequency). The analysis clarifies thus the fact that observability problems are physically due to low frequency operation conditions, rather than at low velocities, even if in some case, these two conditions happen to occur simultaneously (absence of torque load).

In this study we consider the induction motor model in the (α, β)-coordinates:

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} &= \begin{bmatrix} -\gamma & 0 & \frac{K}{T_r} & p\omega_r K \\ 0 & -\gamma & -p\omega_r K & \frac{K}{T_r} \\ \frac{L_m}{T_r} & 0 & -\frac{1}{T_r} & -p\omega_r \\ 0 & \frac{L_m}{T_r} & p\omega_r & -\frac{1}{T_r} \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} \\ \frac{d\omega_r}{dt} &= \frac{p}{J}\frac{L_m}{L_r}(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}) + \frac{\tau_l}{J} \\ \dot{\tau}_l &= 0 \end{aligned} \tag{1}$$

where $I_s = [i_{s\alpha}, i_{s\beta}]^T$ is the stator current vector, $\Psi_s = [\psi_{r\alpha}, \psi_{r\beta}]^T$ is the rotor flux vector, and τ_l is the load torque. ω_r is the rotor angular velocity, u_s is the stator voltage, and the other constants are electrical parameters of the machine.

This system is characterized by the input u_s, the output I_s, and our problem is to characterize the condition under which the state $x = [I_s^T, \Psi_s^T, \omega_r, \tau_l]^T$, can be observed, from its output I_s . This model is named the 6-dimensional model, whereas if ω_r is considered constant (the last two equation of the above model are substituted by $\dot{\omega}_r = 0$), then we will refer to the 5-dimensional model, which is pertinent for many practical considerations.

*We mean by constant flux, a flux vector with zero rotational velocity. It should not be confused with flux vector of only constant norm

2 Observability rank conditions

In this section we present and comment some particular case leading to sufficient conditions such that model (1), is locally observable (in the weak sense), see [7], [10]. For that, we consider systems of the form:

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)u = F(x(t), u(t)), \\ y &= h(x(t)) \end{aligned} \tag{2}$$

with $x(t_0) = x_0$, and the following observability definition:

Definition 2.1 (see, [4]) *Rank condition.* A locally weakly observable condition for the system (2) is that the rank of the observability matrix O of dimension $+\infty \times n$ defined as:

$$O = \begin{pmatrix} dh \\ dL_F h \\ dL_F^2 h \\ \vdots \end{pmatrix}$$

be equal to the state vector dimension n; $L_f^i h(x)$ being the Lie derivative of h of order i, along $F(x, u)$.

2.1. Sufficient observability conditions: special cases

5-dimensional model As appointed in [7] [10], in the particular case of constant velocity ($\dot{\omega}_r = 0$), a sufficient condition for local observability as defined by Definition 2.1, is that:

$$\dot{\psi}_{r\alpha} \neq 0 \quad \text{or} \quad \dot{\psi}_{r\beta} \neq 0 \tag{3}$$

To see this, we can look at the two sub-spaces O_1 et O_2:

$$O_1 = \begin{bmatrix} h_1 \\ L_{f+gu}h_1 \\ L_{f+gu}^2 h_1 \\ h_2 \\ L_{f+gu}h_2 \end{bmatrix} \text{ and } O_2 = \begin{bmatrix} h_1 \\ L_{f+gu}h_1 \\ h_2 \\ L_{f+gu}h_2 \\ L_{f+gu}^2 h_2 \end{bmatrix} \tag{4}$$

the associated observability matrix are:

$$J_1 = \frac{\partial}{\partial X}(O_1) \text{ and } J_2 = \frac{\partial}{\partial X}(O_2) \qquad (5)$$

computing the corresponding determinants give:

$$\begin{aligned} |J_1| &= -(pK)^3 [\dot{x}_3] \left(\frac{1}{T_r^2} + x_5^2 \right) \\ |J_2| &= -(pK)^3 [\dot{x}_4] \left(\frac{1}{T_r^2} + x_5^2 \right) \end{aligned}$$

Remark 2.1 Each of the rank of J_1 and J_2 is input independent ($\dot{x}_3$ and $\dot{x}_4$ depending only on I_s, and Ψ_s). Thus, any u results in a locally observable system over the subspace $\Omega = \{x : |\dot{x}_3| + |\dot{x}_4| \neq 0, \dot{x}_5 = 0\}$.

Remark 2.2 A part for the trivial and uninteresting case of zero norm flux operation, the previous condition indicated (under the hypothesis of constant velocity) that a sufficient observability conditions can be fulfilled as long as the motor operates away from constant flux conditions.

6-dimensional model The more general case of the 6-dimension model, leads (under the hypothesis of $\dot{\psi}_{r\alpha} = \dot{\psi}_{r\beta} = 0$) to the following sufficient rank condition:

$$\dot{\omega}_r \neq 0 \qquad (6)$$

this condition derives from the subspace O_3 generated from:

$$O_3 = \begin{bmatrix} h_1 \\ h_2 \\ L_{f+gu}h_1 \\ L_{f+gu}h_2 \\ L^2_{f+gu}h_1 \\ L^2_{f+gu}h_2 \end{bmatrix} \qquad (7)$$

The determinant of this matrix, under the particular hypothesis of constant flux ($\dot{x}_3 = \dot{x}_4 = 0$), and letting the flux to be defined in the d direction of the $(d-q)$-frame (i.e. $x_3 = \Psi_d$, $x_4 = 0$), is given as

$$|J_3| = -\frac{K^4 p^3}{J T_r} x_3^2 \dot{x}_5 \qquad (8)$$

This expression cancel when the flux norm is zero, i.e. $x_3 = 0$ or when $\dot{x}_5 = \dot{\omega}_r = 0$. The first condition being irrelevant (zero flux), the second one implies constant velocity under constant flux operation. Thus, the observability properties of the 6-dimension model cannot be established for all operation modes, and in particular under stationary operation condition of constant velocity, and zero excitation frequency. As a conclusion, we have that only in case of constant flux operation, this analysis indicate that a sufficient condition for state observability is that the mechanical angular velocity is not constant.

Remark 2.3 In both of the previous observability analysis (5-dimension and 6 dimension model), if the angular velocity is assumed to be constant, the rank condition is not satisfied if the flux is also constant. It seems thus that operation conditions at zero velocity (stand-still), the only possibility of recovering observability is by keeping the flux varying. Nevertheless, it should keep in mind that, it is not possible from the present analysis (that only provide sufficient conditions) to conclude on the lack of observability since the studied observability spaces have not been proved to be involutive.

Further studies on the necessary and sufficient conditions for local observability are studied next, by looking at all the possible directions of the observability space.

3 Necessary and sufficient observability conditions

Model (1) can be rewritten in a more compact form, by introducing the following notation: $u = \frac{1}{K\sigma L_s} u_s$, $w = pT_r\omega_r$, $\gamma_M = \frac{KL_m}{T_r} = \frac{1-\sigma}{\sigma T_r}$, $r_1 = KT_r \frac{p^2}{J} \frac{L_m}{L_r}$, $\tau = T_r \frac{p}{J} \tau_l$ and

$$\begin{aligned} I = &\ K^{-1} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \\ \Psi = &\ \begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} \\ \mathcal{I} = &\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \mathcal{J} = &\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

where $\tau_e = \frac{p(1-\sigma)}{\sigma L_r} I^T \mathcal{J} \Psi$ is the electro-mechanical torque, and τ_l, the load torque, is assumed to be constant. with this notation, the motor model is:

$$\begin{aligned} \dot{I} &= -\gamma I + T_r^{-1}(\mathcal{I} - w\mathcal{J})\Psi + u \\ \dot{\Psi} &= \gamma_M I - T_r^{-1}(\mathcal{I} - w\mathcal{J})\Psi \\ \dot{w} &= r_1 I^T \mathcal{J} \Psi - \tau \\ \dot{\tau} &= 0 \end{aligned}$$

In what follows, we define $x = [I^T, \Psi^T, w, \tau]^T$ being the state vector, and $y = I$, the stator currents, are the system outputs. Let $I^{(m)}$ be the m-th time-derivative (equivalently $L^m_{f+gu}h$), and $\mathcal{L}_m = \frac{\partial}{\partial x} I^{(m)}$ the m-th component (double lines vector) of the observability matrix. We can, easily compute[†]:

$$\begin{array}{llcccc} \mathcal{L}_0 & = [& \mathcal{I}, & 0, & 0, & 0 \ ] \\ \mathcal{L}_1 & = [& -\gamma\mathcal{I}, & T_r^{-1}(\mathcal{I} - w\mathcal{J}), & -T_r^{-1}\mathcal{J}\Psi, & 0 \ ] \end{array}$$

remark that it is possible to exchange $\mathcal{L}_1$, by $\bar{\mathcal{L}}_1$, because together with $\mathcal{L}_0$ they span the same subspace, i.e.

$$\begin{aligned} \bar{\mathcal{L}}_1 &= T_r(\mathcal{I} - w\mathcal{J})^{-1}(\mathcal{L}_1 + \gamma\mathcal{L}_0) \\ &= [\ 0\ ,\ \mathcal{I}\ ,\ -(\mathcal{I} - w\mathcal{J})^{-1}\mathcal{J}\Psi\ ,\ 0\ ] \end{aligned}$$

Noticing that $\dot{\Psi} + \dot{I} = (\gamma_M - \gamma)I + u$ and $\frac{\partial}{\partial x}u = 0$, and taking the m-th time-derivative, we can write:

$$\begin{aligned} \frac{\partial}{\partial x}\Psi^{(m+1)} &= -\frac{\partial}{\partial x}I^{(m+1)} + (\gamma_M - \gamma)\frac{\partial}{\partial x}I^{(m)} \\ &= \mathcal{LC}(\mathcal{L}_m, \mathcal{L}_{m+1}) \end{aligned}$$

where $\mathcal{LC}(.,.)$ stands for a linear combination between line vectors.

Taking the $(m+1)$ derivative of the first state equation ($\dot{I} = ...$), and substituting $\frac{\partial}{\partial x}\Psi^{(m+1)}$ using the above expression, we get:

$$\mathcal{L}_{m+2} = \mathcal{LC}(\mathcal{L}_m, \mathcal{L}_{m+1}) - \mathcal{J}\frac{\partial}{\partial x}((w\Psi)^{(m+1)})$$

for $m = 0$, this expression becomes:

$$\mathcal{L}_2 = \mathcal{LC}(\mathcal{L}_0, \mathcal{L}_1) - \mathcal{J}\frac{\partial}{\partial x}(\frac{d}{dt}(w\Psi))$$

using the model equations, $\mathcal{L}_2$ can be calculated:

$$\begin{aligned} \mathcal{L}_2 &= \mathcal{LC}(\mathcal{L}_0, \mathcal{L}_1) \\ &\quad + \mathcal{J}[\ 0\ ,\ \dot{w}\mathcal{I} + r_1\Psi I^T\mathcal{J}\ ,\ \dot{\Psi}\ ,\ -\Psi\ ] \end{aligned}$$

For the same reasons as before, it is possible to exchange $\mathcal{L}_2$ by $\bar{\mathcal{L}}_2$, as:

$$\begin{aligned} \bar{\mathcal{L}}_2 &= -[\ 0\ ,\ \dot{w}\mathcal{I} + r_1\Psi I^T\mathcal{J}\ ,\ \dot{\Psi}\ ,\ -\Psi\ ] \\ &\quad + (\dot{w}\mathcal{I} + r_1\Psi I^T\mathcal{J})\bar{\mathcal{L}}_1 \\ &= [\ 0\ ,\ 0\ ,\ (\delta\mathcal{I} - \kappa\mathcal{J})\Psi - \dot{\Psi}\ ,\ \Psi\ ] \end{aligned}$$

where $\delta = r_1(\mathcal{I} - w\mathcal{J})^{-1}\Psi + \frac{w\dot{w}}{1+w^2}$ and $\kappa = \frac{\dot{w}}{1+w^2}$.

[†]Where 0 is a matrix of appropriated dimension

Let Δ defined as:

$$\begin{aligned} \Delta &:= det\begin{bmatrix} \mathcal{L}_0 \\ \bar{\mathcal{L}}_1 \\ \bar{\mathcal{L}}_2 \end{bmatrix} = det\,[O_\Delta] \\ &= det\begin{bmatrix} \mathcal{I} & 0 & 0 & 0 \\ 0 & \mathcal{I} & -(\mathcal{I} - w\mathcal{J})^{-1}\mathcal{J}\Psi & 0 \\ 0 & 0 & (\delta\mathcal{I} - \kappa\mathcal{J})\Psi - \dot{\Psi} & \Psi \end{bmatrix} \\ &= \Psi^T\mathcal{J}\dot{\Psi} + \frac{\dot{w}}{1+w^2}\Psi^T\Psi \qquad (9) \end{aligned}$$

Then, if $\Delta \neq 0$, the dimension of the space generated by these line vectors, is 6. Thus, the 6-dimension model is observable. Else[‡], the dimension of the space O_Δ shrinks to 5. To the system be observable, a new direction needs to be found by looking at the high-order derivatives of I. This direction, represented by the vector L should then satisfy $L^TV \neq 0$, where V is any vector orthogonal to O_Δ:

$$V = \begin{bmatrix} 0 \\ (w\mathcal{I} + \mathcal{J})^{-1}\mathcal{J}\Psi \\ 1 \\ \frac{\Psi^T\dot{\Psi}}{\Psi^T\Psi} - \delta \end{bmatrix}$$

If $\Delta = 0$ and $\mu_m := \mathcal{L}_{m+3}V = 0 \ \ \forall m \geq 0$, the space generated by the time-derivatives of the output I is at most of dimension 5: the system is non observable. If there at least a $m \geq 0$, such that $\mu_m \neq 0$, then $\mathcal{L}_{m+3}$ provides the missing direction to span the complete 6-dimensional space.

Assuming $\Psi \neq 0$, we have proved the following:

Theorem 3.1 *A necessary and sufficient condition for the system (1)* **not** *being observable is that the following two conditions hold simultaneously:*

(i) $\Delta = 0$,

(ii) $\forall m \geq 0$, $\mu_m = 0$,

with Δ and μ_m as defined before.

Corollary 3.1 *If and only if, any of the following two conditions holds:*

(i) $\Delta \neq 0$

(ii) $\exists m \geq 0$*, such that* $\mu_m \neq 0$

[‡]We exclude the uninteresting case of $\Psi = 0$

system (1) is observable.

It is instructive to analyze some particular cases derived from Theorem 3.1, and Corollary 3.1.

3.1. Particular conditions for observability

From (i) of Theorem (3.1), we have that if $\Delta \neq 0$, then the system is observable. Note that (9), can also be rewritten as:

$$\frac{\Delta}{\Psi^T\Psi} = \omega_\Psi + \frac{\dot{w}}{1+w^2} \tag{10}$$

where $\omega_\Psi := \frac{d}{dt}\{\angle\Psi\} = \frac{\Psi^T \mathcal{J}^T \dot{\Psi}}{\Psi^T\Psi}$. Thus, there are two clear cases when $\Delta \neq 0$:

- if the flux rotor angle is constant ($\omega_\Psi = 0$), it is sufficient that the rotational velocity ω is not constant
- if the velocity is constant ($\dot{w} = 0$), then it is sufficient that the flux is not constant.

Remark 3.1 these two conditions have been obtained separately in the previous sections for the fifth and sixth dimension model. Here they are derived from condition (i) of Corollary 3.1.

3.2. Particular conditions for lost of observability

Here we derive some particular cases leading to conditions under which the system observability is lost.

Introducing, $Z = (\mathcal{I} - w\mathcal{J})^{-1}\Psi$, which allows to write the expression of Δ, and μ_m in the following compacting form:

$$\begin{aligned}
\Delta &= \frac{Z^T \mathcal{J}^T \dot{Z}}{Z^T Z} =: \omega_Z \\
\mu_m &= \Psi^{(m+2)} + w^{(m+1)}\mathcal{J} Z \\
&\quad - r_1 \sum_{j=0}^{m} C^j_{m+2} \Psi^{(j)} I^{(m+1-j)T} Z \\
&\quad - (m+2)\frac{Z^T \dot{Z}}{Z^T Z}\Psi^{(m+1)}
\end{aligned}$$

with $C^j_{m+1} = \frac{(m+1)!}{j!(m+1-j)!}$ being constant.

As previously, we study the cases where the norm of the flux is different from zero. From this expression, we can see that, if:

$$w^{(m+1)} = \Psi^{(m+1)} = 0, \forall m \geq 0 \tag{11}$$

then:

$$\Delta = \mu_m = 0, \forall m \geq 0$$

and the motor states becomes inobservable[§]. Condition (11) is indeed a sufficient condition for the lost of observability.

Equation (11), implies $\omega_\Psi = 0$, and together with the flux equation of the motor model, and the motor torque expression, i.e.

$$\omega_\Psi = \frac{R_r}{p\Psi^T\Psi}\tau_e + \omega_r \tag{12}$$

yields

$$\omega_\Psi = \frac{T_r R_r}{p\Psi^T\Psi}\tau_e + w = 0 \tag{13}$$

that describes a straight line in the $\tau_e - w$ plane. This line describes an inobservability line, around which sensorless controllers may be tested.

Note that motor operation at constant flux and velocity implies that the current and voltage input are also necessarily constant, and thus that motor is under zero excitation frequency.

4 Physics behind the observability conditions

A couple of observations are worth to be enlighten in connection with the observability conditions derived previously.

It is interesting to underline some physical aspects related to the condition $\Delta = 0$. In particular, when $\dot{x}_3 = \dot{x}_4 = 0$, or equivalent when the flux angle is constant $\omega_\Psi = 0$, then we have from (13) $\frac{R_r}{p\Psi^T\Psi}\tau_e + \omega_r = 0$. The electro-magnetic torque, and the mechanical angular velocity have opposed sign. The motor is thus in generation mode with direct current circulating in the stator. It is thus clear that no information can be obtained at the rotor side. These problems clearly appear when the stator voltage vector u is operating at low frequency, and hence the flux vectors are likely to be constant.

[§] If (11)holds, w, Ψ, Z, and I are constant, then all the terms of Δ, and μ_m vanish

Under this operation conditions, we have

$$\begin{aligned} 0 &= -\gamma I + T_r^{-1}(\mathcal{I} - w\mathcal{J})\Psi + u \\ 0 &= \gamma_M I - T_r^{-1}(\mathcal{I} - w\mathcal{J})\Psi \end{aligned}$$

the input torque becomes proportional to the current $I = (\gamma - \gamma_M)^{-1}u$. It is thus impossible to get speed estimation based solely on measured voltage and current.

In "steady state" –when all vectors are rotating at the same speed– the voltage excitation frequency ω_u is equal to the flux frequency ω_Ψ. Assume also constant motor speed operation, thus from (10) the inobservability condition $\Delta = 0$, writes as:

$$\omega_u = \omega_\Psi = -\frac{\dot{w}}{w^2+1} = 0$$

this condition does not hold if ω_u is different from zero regardless the value of the motor speed. This shows that, in reality, problems comes when the system operated under zero stator frequency (all stator vectors are constant), and not necessarily when the rotational velocity of the motor is zero.

However, note from equation (13), that under constant speed ($\tau_e = \tau_l$), this equation becomes:

$$\frac{R_r}{p\Psi^T\Psi}\tau_l + \omega_r = 0 \qquad (14)$$

and thus, if there is no load, zero excitation frequency corresponds to zero motor speed, in which case the system becomes inobservable.

5 Conclusion

Observability issues concerning the induction motor have been clarified and formally stated. We have characterized sufficient conditions leading to observable and inobservable situations. In particular we have shown that a sufficient condition for lost of observability is that the excitation voltage frequency is zero and the motor is operating at constant speed. In absence of load, and under "steady-steady" operation (all vectors rotate at constant frequency, and motor speed is constant), zero motor speed correspond to zero excitation frequency, and thus to an observability lost.

Acknowledgements

This work was initiated during a meeting at Gordes France in May 1999. Authors would like to thanks all other participant to this meeting: D. Lubineau, E. Delmotte, L. Loron. This research was partially funded by the "French Minister of the Research and Technology".

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