A Time-frequency Domain Fault Detection Approach Based on Parity Relation and Wavelet Transform

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Abstract: In this paper, problems related to the design of robust residual generators have been studied. The main objective of our study is to make use of the simple time domain design form of the parity relation based approach and the frequency domain analysis known by the H_2-optimization approach, in order to improve the system performance without an essential increase in computation. We establish a relationship between the parity relation based and the H_2 optimal residual generators and show that the optimal parity vector v_s converges to the H_2-optimal post-filter with $s \to \infty$. Making use of the fact that the H_2-optimal post-filter is a narrow band filter and the well known time-frequency domain properties of Wavelet Transform, a time-frequency domain approach is developed, which allows us to design a residual generator based on Wavelet Transform. The significant property of such kind of residual generators is its simple form, low order and high performance. The main results are illustrated by examples.

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1 Introduction

Robustness problem is one of most important topics in the field of model based fault diagnosis. Among a number of existing methods, the so-called parity relation based optimization approach and the H_2-optimization proposed in the late 80'er by Frank and his co-workers [10], [5] are widely accepted as one of the standard solutions to the optimal design of fault detection systems [1], [8], [7].

The main objective of this paper is an integrated use of the parity relation based and the H_2-optimization approaches. As known, the parity relation based approach is a time domain design whose realization involves only solutions of algebraic equations and thus requires no involved computation in design and implementation. In comparison, the H_2-optimization is a frequency domain approach which may provide us with a better system performance due to the use of frequency domain analysis, and requires, on the other side, a strong mathematical and control engineering background in order to achieve the desired performance. The essential idea of this work is to make use of the simple time domain design form of the parity relation based approach and the frequency domain analysis in order to improve the system performance without an essential increase in computation.

The main work to realize the above-mentioned idea consists of the following studies: (i)establishing a relationship between the parity relation based and the H_2 optimization approaches; (ii)using Wavelet Transform technique, which is well known for its power in solving time-frequency domain problems, to integrate the both approaches.

2 Characteristics of parity vector v_s

In this section, we first briefly review the parity space approach and H_2-optimization of observer-based residual generator [6], [8], [7], [9], [10], followed by establishing a relationship between the optimal solutions of these two approaches, which builds the theoretical basis for the further study.

2.1 A brief review

Consider linear time-invariant discrete systems described by

$$x(k+1) = Ax(k) + Bu(k) + E_d d(k) + E_f f(k) \quad (1)$$
$$y(k) = Cx(k) + Du(k) + F_d d(k) + F_f f(k) \quad (2)$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^{k_u}$ the vector of control signals, $y \in \mathbf{R}^m$ the output vector, $d \in \mathbf{R}^{k_d}$ the unknown disturbance vector and $f \in \mathbf{R}^{k_f}$ the vector of faults to be detected. $A, B, C, D, E_f, E_d, F_f$ and F_d are known and of appropriate dimensions.

2.1.1 parity space approach:

A parity relation based residual generator is expressed by

$$r_s(k) = v_s(y_s(k) - H_{u,s}u_s(k)) \quad (3)$$

whose dynamics is governed by

$$r_s(k) = v_s(H_{0,s}x(k-s) + H_{d,s}d_s(k) + H_{f,s}f_s(k)) \quad (4)$$

where vector $v_s = [v_{s,0}\ v_{s,1}\ \cdots\ v_{s,s}] \in \mathbf{R}^{m(s+1)}$ is called parity vector which should be selected from the parity space P_s defined by $P_s = \{v_s |\ v_s H_{0,s} = 0\}$, s is the order of the parity relation, and

$$y_s(k) = \left[y^T(k-s)\ \ y^T(k-s+1) \cdots y^T(k)\right]^T$$
$$u_s(k) = \left[u^T(k-s)\ \ u^T(k-s+1) \cdots u^T(k)\right]^T$$
$$d_s(k) = \left[d^T(k-s)\ \ d^T(k-s+1) \cdots d^T(k)\right]^T$$
$$f_s(k) = \left[f^T(k-s)\ \ f^T(k-s+1) \cdots f^T(k)\right]^T$$
$$H_{0,s} = \left[C^T\ \ A^T C^T \cdots \left(A^T\right)^s C^T\right]^T$$
$$H_{u,s} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}B & \cdots & CB & D \end{bmatrix}$$
$$H_{d,s} = \begin{bmatrix} F_d & 0 & \cdots & 0 \\ CE_d & F_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_d & \cdots & CE_d & F_d \end{bmatrix}$$
$$H_{f,s} = \begin{bmatrix} F_f & 0 & \cdots & 0 \\ CE_f & F_f & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_f & \cdots & CE_f & F_f \end{bmatrix}$$

In the case that a perfect decoupling from $d(k)$ is impossible, a robust residual generator is designed under a certain optimization sense becomes necessary. Frank and Wünnenberg [10], [11] proposed the following performance index

$$\min_{v_s \in P_s} J = \min_{v_s \in P_s} \frac{v_s H_{d,s} H_{d,s}^T v_s^T}{v_s H_{f,s} H_{f,s}^T v_s^T} \quad (5)$$

Recently, Ding et al. [4] proved that

$$\min_{v_{s+1} \in P_{s+1}} \frac{v_{s+1} H_{d,s+1} H_{d,s+1}^T v_{s+1}^T}{v_{s+1} H_{f,s+1} H_{f,s+1}^T v_{s+1}^T} < \min_{v_s \in P_s} \frac{v_s H_{d,s} H_{d,s}^T v_s^T}{v_s H_{f,s} H_{f,s}^T v_s^T}$$

Thus, we can claim that

$$\lim_{s \to \infty} \min_{v_s \in P_s} J = \min_{s} \min_{v_s \in P_s} J \quad (6)$$

2.1.2 H_2-optimization of residual generators:

It is now well known that all linear residual generator can be expressed by [7]

$$r(k) = q(z)(y(k) - G_u(z)u(k)) \quad (7)$$

whose dynamics is governed by

$$r(k) = q(z)(G_d(z)d(k) + G_f(z)f(k)) \quad (8)$$

where $q(z) \in \mathbf{RH}_\infty$, the so-called post-filter, is the parameterization vector which also ensures the stability of the residual generator, $G_d(z) = C(zI - A)^{-1}E_d + F_d, G_f(z) = C(zI - A)^{-1}E_f + F_f$. H_2-optimal design of observer-based residual generators is an optimization problem related to the following performance index

$$\min_{q(z)} J = \min_{q(z)} \frac{\int_0^{2\pi} q(e^{j\omega})G_d(e^{j\omega})G_d^*(e^{j\omega})q^*(e^{j\omega})d\omega}{\int_0^{2\pi} q(e^{j\omega})G_f(e^{j\omega})G_f^*(e^{j\omega})q^*(e^{j\omega})d\omega} \quad (9)$$

where $*$ denotes the conjugate transpose of matrix, which is carried out when a perfect decoupling from $d(k)$ is not realizable.

According to the work by Ding and Frank [5], the optimal solution of $q(z)$ to problem (9) is a narrow band limited filter.

2.2 Relationship between two approaches

Let $g_d(i)$, $i = 0 \cdots s$, be the impulse response of the system (1)-(2) to the unknown inputs d. It can be easily proved that

$$g_d(0) = F_d,\ g_d(1) = CE_d,\ \cdots,\ \ g_d(s) = CA^{s-1}E_d$$

Let $\tilde{v}_s$ be the flip of v_s, i.e. $\tilde{v}_s = [v_{s,s}\ v_{s,s-1} \cdots v_{s,0}] = [\rho_0\ \rho_1\ \cdots\ \ \rho_s]$, with $\rho_i = v_{s,s-i}$, $i = 0 \cdots s$. It then turns out

$$
\begin{aligned}
& v_s H_{d,s} \\
& = [\rho_s \ \rho_{s-1} \cdots \ \rho_0] \begin{bmatrix} g_d(0) & 0 & \cdots & 0 \\ g_d(1) & g_d(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ g_d(s) & \cdots & g_d(1) & g_d(0) \end{bmatrix} \\
& = [\psi(s) \ \psi(s-1) \cdots \psi(0)]
\end{aligned}
$$

with

$$\psi(i) = \sum_{l=0}^{i} \rho_{i-l} g_d(l) = \rho_i * g_d(i) = \mathcal{Z}^{-1}[\widetilde{V}_s(z) G_d(z)]$$

where $*$ denotes convolution, $\widetilde{V}_s(z)$ denotes the $\mathcal{Z}$-Transform of $\widetilde{v}_s = [\rho_0 \cdots \rho_i \cdots \rho_s]$ with i as the time independent variable. According to the Parseval Theorem, we have

$$
\begin{aligned}
& \lim_{s \to \infty} v_s H_{d,s} H_{d,s}^T v_s^T = \sum_{n=0}^{\infty} \psi(n) \psi^T(n) \\
& = \frac{1}{2\pi} \int_0^{2\pi} \widetilde{V}_s(e^{j\omega}) G_d(e^{j\omega}) G_d^*(e^{j\omega}) \widetilde{V}_s^*(e^{j\omega}) d\omega \qquad (10)
\end{aligned}
$$

and thus the following theorem.

Theorem 1 *Given optimization problems*

$$\min_{v_s \in P_s} J_1 = \min_{v_s \in P_s} \frac{v_s H_{d,s} (v_s H_{d,s})^T}{v_s H_{f,s} (v_s H_{f,s})^T} \qquad (11)$$

$$\min_{q(z)} J_2 = \min_{q(z)} \frac{\int_0^{2\pi} q(e^{j\omega}) G_d(e^{j\omega}) G_d^*(e^{j\omega}) q^*(e^{j\omega}) d\omega}{\int_0^{2\pi} q(e^{j\omega}) G_f(e^{j\omega}) G_f^*(e^{j\omega}) q^*(e^{j\omega}) d\omega} \qquad (12)$$

and let $v_{s,opt} = [v_{s,opt,0} \, v_{s,opt,1} \cdots v_{s,opt,s}]$ *and* $q_{opt}(z)$ *are the optimal solutions of (11) and (12) respectively,* $J_{1,opt}$ *and* $J_{2,opt}$ *are the corresponding optimal value respectively, then*

$$\lim_{s \to \infty} J_{1,opt} = J_{2,opt} \qquad (13)$$

$$q_{opt}(z) = \widetilde{V}_{s \to \infty, opt}(z) \qquad (14)$$

where $\widetilde{V}_{s,opt}(z)$ *is the* $\mathcal{Z}$*-Transform of* $\widetilde{v}_{s,opt}$*, which is the flip of* $v_{s,opt}$*, i.e.*

$$\widetilde{v}_{s,opt} = [v_{s,opt,s} \ \ v_{s,opt,s-1} \cdots v_{s,opt,0}] \qquad (15)$$

Proof: If $v_{s \to \infty, opt}$ is the optimal solution to problem (11) when $s \to \infty$, then from (10) and (6), we know that for $q_{opt}(z)$ in the form of (14) and (15) it holds

$$
\begin{aligned}
& J_2 \mid_{q(z) = q_{opt}(z)} = J_1 \mid_{v_s = v_{s \to \infty, opt}} \\
& = \lim_{s \to \infty} \min_{v_s \in P_s} J_1 = \min_s \min_{v_s \in P_s} J_1
\end{aligned}
$$

from which we can further draw the conclusion that $q_{opt}(z)$ makes the performance index J_2 achieving its minimum. Otherwise suppose that $q_c(z)$ instead of $q_{opt}(z)$ is the optimal solution to problem (12), i.e.

$$J_2 \mid_{q(z) = q_c(z)} = \min_{q(z)} J_2 < J_2 \mid_{q(z) = q_{opt}(z)} = \min_s \min_{v_s \in P_s} J_1$$

According to (10), for this $q_c(z)$ we can find a corresponding v_{sc} whose components are just the flip of $\mathcal{Z}^{-1}[q_c(z)]$ and $J_1 \mid_{v_s = v_{sc}} = J_2 \mid_{q(z) = q_c(z)}$. Following this, $J_1 \mid_{v_s = v_{sc}} < \min_s \min_{v_s \in P_s} J_1$, which is apparently contradictory. Thus $q_{opt}(z)$ corresponding to $v_{s \to \infty, opt}$ in the form of (14) and (15) is the optimal solution of optimization problem (12), and *vice versa.*Therefore (13), (14) and (15) hold.

Remark 1 *The above theorem gives a deeper insight into the relationship of parity space method and observer-based approach: (i)The optimal performance index* $J_{1,opt}$ *converges to a limit which is just the optimal performance index* $J_{2,opt}$*; (ii)The flip of* $v_{s \to \infty, opt}$ *are just the inverse* $\mathcal{Z}$*-Transform of* $q_{opt}(z)$*; (iii) Since* $q_{opt}(z)$ *is a narrow band filter, the frequency response of* $v_{s \to \infty, opt}$ *is also narrow band-limited.*

In order to illustrate the above results, we observe the following example.

Example 1: Consider system model

$$A = \begin{bmatrix} 1 & -1.3 \\ 0.25 & -0.25 \end{bmatrix}, B = [2 \ \ 1]^T, \ C = [0 \ \ 1]$$

$$E_d = [0.4 \ \ 0.5]^T, \ E_f = [0.6 \ \ 0.1]^T, \ \ D = F_d = F_f = 0.$$

It is easily to prove that $J_{2,opt} = 0.4444$ and $q_{opt}(z)$ is a narrow band filter at $\omega = 0$. Fig. 1 demonstrates that the optimal performance index $J_{1,opt}$ converges to $J_{2,opt}$ with $s \to \infty$. Fig. 2 shows that the frequency band of v_s becomes narrower with the increase of s.

3 Wavelet Transform of vector and matrix

Wavelet Transform is a powerful time-frequency analysis tool for signal. It can be regarded as a bank of filters in different frequency bands, which are well localized both in time and frequency domain. That means on one hand, these filters can provide good frequency selectors, but on the other hand, the length of the filters can keep short simultaneously. In this paper, Wavelet Transform is used as a bridge between time and frequency domain approach.

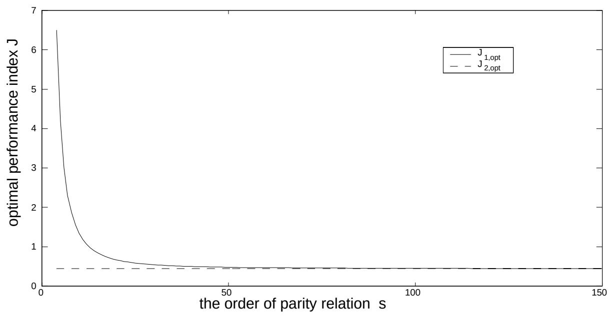

Figure 1: the change of $J_{1,opt}$ with s

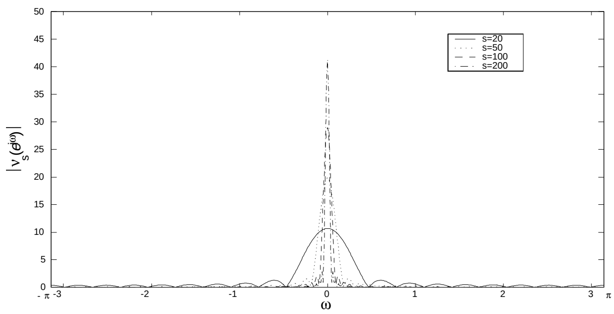

Figure 2: the frequency response of parity vector v_s

3.1 Wavelet Transform of signal

The orthonormal Wavelet Transform (WT) of a signal $x(k), k = 1 \cdots l$, is calculated using Mallat algorithm and includes the detail coefficients $WT_x^d(j,k), j = 1 \cdots j_m, k = 1 \cdots n_j$, and the approximation coefficients $WT_x^a(j_m,k), k = 1 \cdots n_{j_m}$[3][2]. WT has the following useful characteristics: (i)For a given scale j, $WT_x^d(j,k)$ is equivalent to the output of $x(k)$ through a bandpass filter FP_j which is well localized both in time and frequency domain, and with the increase of j, the central frequency of FP_j moves from high frequency band to low. $WT_x^a(j_m,k)$ is equivalent to the output of $x(k)$ through a lowpass filter FL_{j_m} which is also well localized both in time and frequency domain; (ii)The efficiency of Mallat algorithm is very high; (iii) For each scale j, the length of WT of $x(k)$ is $n_j \approx \frac{l}{2^j}$; (iv) The inner product of two signals $x(k)$ and $y(k)$ in time domain is equal to the inner product of their WT in time-frequency domain, namely

$$\sum_{k=1}^{l} x(k)y(k) = \sum_{j=1}^{j_m}\sum_{k=1}^{n_j} WT_x^d(j,k)WT_y^d(j,k) + \sum_{k=1}^{n_{j_m}} WT_x^a(j_m,k)WT_y^a(j_m,k) \tag{16}$$

3.2 Wavelet Transform of vector and matrix

When i is regarded as the time independent variable, the parity vector $v_s = [v_{s,0} \cdots v_{s,i} \cdots v_{s,s}]$ can be regarded as a signal and its WT can be calculated. In this paper, we use the detail WT vector $WT_{v_s,j}^d$ and approximation WT vector WT_{v_s,j_m}^a to denote all of the detail WT coefficients of v_s at scale j and all of the approximation WT coefficients of v_s at j_m respectively, i.e.

$$WT_{v_s,j}^d = [WT_{v_s}^d(j,1) \cdots WT_{v_s}^d(j,n_j)],\ j = 1 \cdots j_m$$
$$WT_{v_s,j_m}^a = [WT_{v_s}^a(j_m,1) \cdots WT_{v_s}^a(j_m,n_{j_m})]$$

Let h_q denote the q_{th} column of $H_{d,s}$, $q = 1 \cdots (s+1)k_d$, then the WT vectors $WT_{h_q,j}^d$, $j = 1 \cdots j_m$, and WT_{h_q,j_m}^a of h_q can be defined in the same way as those of v_s, the only difference is they are column vectors. Furthermore, we can define the WT matrices $WT_{H_{d,s},j}^d$, $j = 1 \cdots j_m$ and $WT_{H_{d,s},j_m}^a$ for $H_{d,s}$ as

$$WT_{H_{d,s},j}^d = \left[WT_{h_1,j}^d\ WT_{h_2,j}^d \cdots WT_{h_{(s+1)k_d},j}^d\right]$$
$$WT_{H_{d,s},j_m}^a = \left[WT_{h_1,j_m}^a\ WT_{h_2,j_m}^a \cdots WT_{h_{(s+1)k_d},j_m}^a\right]$$

WT vectors of $y_s(k)$ and WT matrices for $H_{f,s}$, $H_{u,s}$ and $H_{0,s}$ can be defined in the same way.

The WT vectors and matrices can be calculated with Mallat Algorithm, but for the convenience of theory analysis, it can be expressed by matrix operation, i.e.

$$WT_{v_s,j}^d = v_s\left(M_j^d\right)^T,\ WT_{v_s,j_m}^a = v_s\left(M_{j_m}^a\right)^T$$
$$WT_{H_{d,s},j}^d = M_j^d H_{d,s},\ WT_{H_{d,s},j_m}^a = M_{j_m}^a H_{d,s}$$

where

$$M_j^d = \begin{bmatrix} \overline{M}_{j,0,0}^d \times I_m & \cdots & \overline{M}_{j,0,s}^d \times I_m \\ \vdots & & \vdots \\ \overline{M}_{j,n_j,0}^d \times I_m & \cdots & \overline{M}_{j,n_j,s}^d \times I_m \end{bmatrix}$$
$$M_{j_m}^a = \begin{bmatrix} \overline{M}_{j_m,0,0}^a \times I_m & \cdots & \overline{M}_{j_m,0,s}^a \times I_m \\ \vdots & & \vdots \\ \overline{M}_{j_m,n_{j_m},0}^a \times I_m & \cdots & \overline{M}_{j_m,n_{j_m},s}^a \times I_m \end{bmatrix}$$
$$\overline{M}_j^d = WT_{I_{s+1}}^j,\ \overline{M}_{j_m}^a = WT_{I_{s+1},j_m}^a$$

in which $\overline{M}_{j,p,q}^d$, $p = 0 \cdots n_j$, $q = 0 \cdots s$, and $\overline{M}_{j_m,p,q}^a$, $p = 0 \cdots n_{j_m}$, $q = 0 \cdots s$, are the element of $\overline{M}_j^d$ and $\overline{M}_{j_m}^a$ at the p_{th} row and q_{th} column respectively, I_m and I_{s+1} are unit matrices with dimension m and $s+1$ respectively.

4 Fault detection approach based on parity space and Wavelet Transform

4.1 Basic idea and problem formulation

Recall that in Section 2 we have shown that the H_2-optimal post-filter can be approximated by a parity vector with an arbitrary accuracy. Although we can make use of this fact to simply the optimization and implementation of residual generators, since using parity space approach only solving algebraic equations is needed, a large s implies heavy on-line computation.

On the other hand, the results achieved in Section 2 show that the optimal parity vector v_s converges to the H_2-optimal post-filter with $s \to \infty$ and moreover matrices $H_{d,s}$, $H_{f,s}$ contain information of $G_d(z), G_f(z)$. Motivated by this fact, we use Wavelet Transform as a tool to get and use frequency domain information provided by v_s, $H_{d,s}$ and $H_{f,s}$.

Since the optimal parity vector v_s with high order is a narrow band filter in the frequency domain, when v_s is expressed in time-frequency domain using Wavelet Transform, only a part of the coefficients are none zero and all the other coefficients should be equal to zero or very small. If we only use these non-zero WT coefficients of v_s as a new parity vector, since they carry almost all the information of v_s , the performance index $J_{1,opt}$ in this case will be approximately equal to $J_{1,opt}$ in the infinite order case. So the H_2 optimal performance index $J_{2,opt}$ can be approximately obtained by using parity space method with a very low order parity vector.

4.2 WT-based residual generator

Let $WT_{v_s} = [WT^d_{v_s,1} \cdots WT^d_{v_s,j_m} \;\; WT^a_{v_s,j_m}]$, then according to (3),(4) and (16), the parity relation based residual generator can be written as

$$r_s(k) = WT_{v_s} \times \left(\begin{bmatrix} WT^d_{y_s(k),1} \\ \vdots \\ WT^d_{y_s(k),j_m} \\ WT^a_{y_s(k),j_m} \end{bmatrix} - \begin{bmatrix} WT^d_{H_{u,s},1} \\ \vdots \\ WT^d_{H_{u,s},j_m} \\ WT^a_{H_{u,s},j_m} \end{bmatrix} u_s(k) \right) \quad (17)$$

and the dynamic of the residual generator becomes

$$r_s(k) = WT_{v_s} \begin{bmatrix} WT^d_{H_{0,s},1} \\ \vdots \\ WT^d_{H_{0,s},j_m} \\ WT^a_{H_{0,s},j_m} \end{bmatrix} x(k-s) + WT_{v_s} \left(\begin{bmatrix} WT^d_{H_{d,s},1} \\ \vdots \\ WT^d_{H_{d,s},j_m} \\ WT^a_{H_{d,s},j_m} \end{bmatrix} d_s(k) + \begin{bmatrix} WT^d_{H_{f,s},1} \\ \vdots \\ WT^d_{H_{f,s},j_m} \\ WT^a_{H_{f,s},j_m} \end{bmatrix} f_s(k) \right) \quad (18)$$

When order s is high, the optimal v_s must be a narrow band filter. If v_s is bandpass in frequency domain, then there must be a scale $j_{opt} \in [1, 2, \cdots j_m]$ which can ensure that the frequency band of $WT^d_{v_s,j_{opt}}$ covers the frequency band of the optimal v_s and $WT^d_{v_s,j_{opt}}$ has relatively large modulus, while all the other WT vectors are nearly equal to zero. Similarly, if the optimal v_s is lowpass in frequency domain, then $WT^a_{v_s,j_m}$ must have relatively large modulus and all the other WT vectors are nearly equal to zero. So among the $j_m + 1$ WT vectors of optimal v_s, only one has large modulus, and all the others are almost zeros. Suppose this special WT vector is $WT^{ad_{opt}}_{v_s,j_{opt}}$, where $j_{opt} \in [1, 2, \cdots j_m]$, and ad_{opt} can be a or d to include the two different cases. In this section, suppose ad_{opt} and j_{opt} are already known , but the coefficients of $WT^{ad_{opt}}_{v_s,j_{opt}}$ are unknown.

Since the main information is included in the frequency band of $WT^{ad_{opt}}_{v_s,j_{opt}}$, removing of the other WT vectors from the residual generator will only slightly influence the dynamic of residual generator. Then the WT-based residual generator can be given by:

$$r_{wt}(k) = v_{wt} \left(WT^{ad_{opt}}_{y_s(k),j_{opt}} - WT^{ad_{opt}}_{H_{u,s},j_{opt}} u_s(k) \right) \quad (19)$$

in which $v_{wt} = WT^{ad_{opt}}_{v_s,j_{opt}}$, and the dynamic of the residual generator becomes

$$r_{wt}(k) = v_{wt} WT^{ad_{opt}}_{H_{0,s},j_{opt}} x(k-s) + v_{wt} \left(WT^{ad_{opt}}_{H_{d,s},j_{opt}} d_s(k) + WT^{ad_{opt}}_{H_{f,s},j_{opt}} f_s(k) \right) \quad (20)$$

Since the length of v_{wt} is about $n_{j_{opt}} \approx \frac{s+1}{2^{j_{opt}}}$, the order of the parity vector is decreased much more. But because the dynamic of r_s in (18) and r_{wt} in (20) are almost equal to each other, the optimal performance index will be similar.

4.3 Design of WT-based parity vector

$WT^{ad_{opt}}_{v_s,j_{opt}}$ can be obtained from the WT of traditional optimal parity vector v_s. But in order to get an optimal v_{wt}, we directly design v_{wt} using the same method as in parity space approach. Designing of v_{wt} is equal to designing the WT coefficients of v_s under scale j_{opt}.

Using the matrix form of WT, (20) becomes

$$r_{wt}(k) = v_{wt} M^{ad_{opt}}_{j_{opt}} H_{0,s} x(k-s) + v_{wt} \left(M^{ad_{opt}}_{j_{opt}} H_{d,s} d_s(k) + M^{ad_{opt}}_{j_{opt}} H_{f,s} f_s(k) \right)$$

Similarly as in traditional parity space approach, let $N_{wt,basis}$ be the orthonormal basis of the left null space of $M^{ad_{opt}}_{j_{opt}} H_{0,s}$, and let $v_{wt} = p_{wt} N_{wt,basis}$, then the dynamic of $r_{wt}(k)$ is

$$r_{wt}(k) = p_{wt} \left(\overline{H}_{d,wt} d_s(k) + \overline{H}_{f,wt} f_s(k) \right) \quad (21)$$

where

$$\overline{H}_{d,wt} = N_{wt,basis} M^{ad_{opt}}_{j_{opt}} H_{d,s}$$
$$\overline{H}_{f,wt} = N_{wt,basis} M^{ad_{opt}}_{j_{opt}} H_{f,s} \quad (22)$$

Define the optimization objective as

$$J_{wt} = \min_{p_{wt}} \frac{p_{wt}\overline{H}_{d,wt}\left(\overline{H}_{d,wt}\right)^T p_{wt}^T}{p_{wt}\overline{H}_{f,wt}\left(\overline{H}_{f,wt}\right)^T p_{wt}^T} \tag{23}$$

then the optimal parity vector p_{wt} can be found using the same method in traditional parity space approach [11].

4.4 Choose the optimal scale and calculate the WT-based residual signal

There are two kinds of method for choosing j_{opt} and ad_{opt}: (i) Design the optimal v_s using traditional parity space approach, analyze the frequency band of v_s and choose j_{opt} and ad_{opt} to make the frequency band of $WT^{ad_{opt}}_{v_s,j_{opt}}$ cover the frequency band of the optimal v_s; (ii) Design $j_m + 1$ parity vectors under different frequency bands (scales), and use the scale corresponding to the minimal performance index as j_{opt} and ad_{opt}.

According to (19), residual signal $r_{wt}(k)$ can be calculated in the following steps: (i)Calculate $v_{wt}WT^{ad_{opt}}_{H_{u,s},j_{opt}}$ off-line; (ii)Calculate $WT^{ad_{opt}}_{y_s(k),j_{opt}}$ on-line using Mallat algorithm ; (iii)Calculate $r_{wt}(k)$ on-line.

4.5 Example

In order to illustrate the proposed approach and show its efficiency, we consider the following example.

Example 2: Consider the same system as given in Example 1. Daubechies wavelet filter is used in WT-based method.

When traditional parity space method is used, $J_{1,opt}$ =0.44580 for $s = 256$, and $J_{1,opt} = 1.55834$ when $s = 9$, while in WT-based method, an optimal v_{wt} with length 10 can make the optimal performance index $J_{wt,opt}$ reach 0.44585. The simulation results show that, WT based parity space approach results in an optimal parity vector with a very low order which delivers a performance almost equal to the one obtained by using the traditional parity space approach with a much more higher order parity vector.

5 Conclusion

In this paper, problems related to the design of robust residual generators have been studied. The main objective of our study is an integrated use of the parity relation based and the H_2-optimization approaches, in order to make use of the simple time domain design form of the parity relation based approach and the frequency domain analysis for improving the system performance without an essential increase in computation.

We have first established a relationship between the parity relation based and the H_2 optimal residual generators and demonstrated that the optimal parity vector v_s converges to the H_2-optimal post-filter with $s \to \infty$. Making use of the fact that the H_2-optimal post-filter is a bandpass and the well known time-frequency domain properties of Wavelet Transform, we have then developed a time-frequency domain approach which allows us to design a residual generator based on WT, which is presented in the form similar to the parity relation based residual generator. The significant property of such kind of residual generators is its low order and high performance. The main results have been illustrated by examples.

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