AUTOMATIC TUNING OF THE MODIFIED SMITH PREDICTOR CONTROLLERS

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Abstract

A simple relay feedback automatic tuning method is proposed for the modified Smith predictor, to provide a controller for stable, unstable and integrating processes with long dead time. A single asymmetrical relay feedback test is used to obtain a reduced order process model in terms of a second order dynamics plus dead time model. Very simple but straightforward tuning formulae are derived for the controllers which have a simple relationship with the plant model parameters. Thus the plant model blocks in the Smith predictor structure, as well as all the controllers are designed from a single asymmetrical relay test. Excellent performance of the auto-tuned Smith predictor has been substantiated by simulations.

1 Introduction

Luyben [1] proposed an identification method using a relay feedback for the autotuning of controllers. Thereafter several studies have been undertaken for improving relay feedback identification procedures mainly for stable processes. Based on the concept of a dual input describing function DIDF, input-biased relay feedback experiments were proposed for system identification by Shen et al. [2]. Their method can identify a process static gain and its critical point in a single relay test. However, the two estimated points provide insufficient information to obtain a transfer function model possessing two unequal lags plus dead time. Many of the above identification methods are based on analysis using the describing function approximation to a relay which may yield erroneous results on some occasions. Recently, Wang et al. [3] have developed a method that identifies multiple frequency response points for stable processes from a single standard relay test.

In process control, when the time delay is small, a conventional parallel PID controller is commonly used; when the time delay is large, the Smith predictor configuration is common. However, neither the conventional PID controller nor the Smith predictor strategy can be used directly for an integrator or unstable long dead time process. Watanabe and Ito [4], Mataušek et al. [5], and Zhang and Sun [6] presented a modification of the Smith predictor to control the process with an integrator and a long time delay. De Paor and Egan [7] proposed an integrated design procedure for a modified Smith predictor and controller for a class of unstable processes with time delay. However, their control strategy is applicable for a limited range of the values of the normalised dead time for which closed loop asymptotic stability can be achieved besides giving a poor setpoint response in terms of overshoot and settling time. Recently, Hang et al. [8] have presented methods to autotune and self-tune a modified Smith predictor by relay feedback using an analysis based on the describing function approximation to a relay, thus producing errors in the estimation of the plant model parameters. Further, their results are limited to stable processes only. The autotuning approach is very important in industrial practice as no mathematical model of the process is required and also the Smith predictor strategy is commonly used for processes with large time delays.

The paper is organized as follows. Section 2 briefly presents the identification procedure. A brief review of the modified Smith predictor is given in section 3. In section 4, the auto-tuning technique is described followed by simulation studies in section 5. Conclusions are given in Section 6.

2 Identification

The relay feedback method has become a widely accepted approach to system identification in process control. However, this method can lead to erroneous results if the system parameters are estimated from equations derived using the approximate describing function approach [9]. Exact analytical expressions have been derived by the authors in a recent paper [10] for the desired parameters in terms of simple limit cycle measurements. On the basis of these expressions, an identification procedure has been suggested which is applicable for the asymmetrical relay feedback test method for estimation of the parameters of a second order process transfer function model. The procedure is repeated below.

Typical transfer function models in process control are often assumed to be the stable SOPDT models, the unstable SOPDT models and the integrating SOPDT models. Therefore an SOPDT plant model of the following form which possesses certain generality will be considered for this identification problem. Let the second order plant has the transfer function model

$$G_m e^{-\theta_m s}(s) = \frac{k e^{-\theta_m s}}{(T_1 s \pm 1)(T_2 s + 1)} \qquad (1)$$

It is to be noted that when $T_1 \to \infty$ such that k/T_1 obtains a finite constant value, (1) represents an integrating SOPDT model. Let an asymmetrical relay feedback test gives the process input and output signals shown in Figure 1. Then quantities like the period of the oscillation, T_p, the time period for the positive output of the plant, τ, the positive peak output of the oscillation, A_p and the negative peak output of the oscillation, A_v may be measured as well as the area of the plant output signal a_y and the area of the plant input signal a_u over the last stable period of the oscillation. The limit cycle conditions give

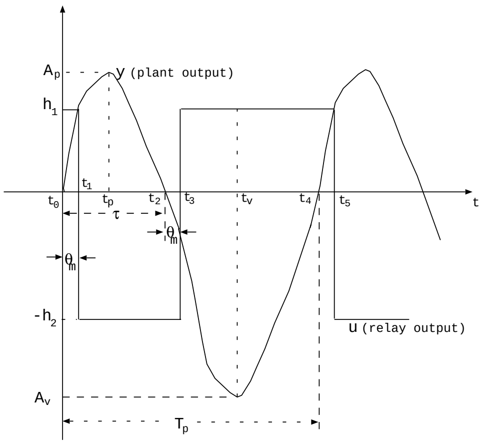

Figure 1: Process input and output signals of period T_p with a relay control.

$$\frac{1}{\lambda_1}\left[\frac{(e^{\lambda_1(T_p-\tau-\theta_m)} - e^{\lambda_1(T_p-\theta_m)})(h_1+h_2)}{1-e^{\lambda_1 T_p}} - h_1\right] - \frac{1}{\lambda_2}\left[\frac{(e^{\lambda_2(T_p-\tau-\theta_m)} - e^{\lambda_2(T_p-\theta_m)})(h_1+h_2)}{1-e^{\lambda_2 T_p}} - h_1\right] = 0 \qquad (2)$$

and

$$\frac{1}{\lambda_1}\left[\frac{(e^{\lambda_1(T_p-\theta_m)} - e^{\lambda_1(\tau-\theta_m)})(h_1+h_2)}{1-e^{\lambda_1 T_p}} + h_2\right] - \frac{1}{\lambda_2}\left[\frac{(e^{\lambda_2(T_p-\theta_m)} - e^{\lambda_2(\tau-\theta_m)})(h_1+h_2)}{1-e^{\lambda_2 T_p}} + h2\right] = 0 \qquad (3)$$

where $\lambda_1 = \mp 1/T_1$ and $\lambda_2 = -1/T_2$. If t_p is the time instant at which the positive peak output of the plant occurs, then the expression for A_p

$$A_p = \mp k[h_2 - (h_1+h_2)(R_1^{\frac{-\lambda_2}{\lambda_1-\lambda_2}} R_2^{\frac{\lambda_1}{\lambda_1-\lambda_2}})] \qquad (4)$$

where

$$R_1 = [\frac{1-e^{\lambda_1(T_p-\tau)}}{1-e^{\lambda_1 T_p}}] \qquad (5)$$

$$R_2 = [\frac{1-e^{\lambda_2(T_p-\tau)}}{1-e^{\lambda_2 T_p}}] \qquad (6)$$

Similarly, if t_v is the time instant at which the negative peak output of the plant occurs, then the expression for A_v is given by

$$A_v = \mp k[(h_1+h_2)(R_3^{\frac{-\lambda_2}{\lambda_1-\lambda_2}} R_4^{\frac{\lambda_1}{\lambda_1-\lambda_2}}) - h_1] \qquad (7)$$

where

$$R_3 = [\frac{1-e^{\lambda_1 \tau}}{1-e^{\lambda_1 T_p}}] \qquad (8)$$

$$R_4 = [\frac{1-e^{\lambda_2 \tau}}{1-e^{\lambda_2 T_p}}] \qquad (9)$$

It is to be noted that expressions (2-9) require $\lambda_1 \neq \lambda_2$. The areas of the plant output and input signals become

$$a_y = \int_0^{T_p} y(t)dt = \mp k((h_1+h_2)\tau - h_1 T_p) \qquad (10)$$

$$a_u = \int_0^{T_p} u(t)dt = (h_1+h_2)\tau - h_1 T_p \qquad (11)$$

The steady state gain from (10) and (11) is

$$\frac{a_y}{a_u} = k \qquad (12)$$

In principle, (2), (3), (4), (7) with the R values substituted and (12) can be solved for all the unknown parameters T_1, T_2, θ_m and k of the transfer function model given in (1) by the asymmetrical relay feedback test involving five measurements of A_p, A_v, T_p, τ and a_y. In practice, the measurements required to obtain (12) may be more difficult to implement and therefore the first four mentioned equations may be used to obtain the four unknowns. On the other hand, if a_y is measured so that k can be found from (12) the solution of the equations is much simpler. λ_1 and λ_2 can be found from (4) and (7) and then θ_m from (2) or (3). When equations (2), (3), (4) and (7) are used because a_y is not measured, a non- linear algebraic equation solver has to be used to solve the four simultaneous equations and problems may arise in that convergence may take place to a false solution if poor initial estimates are used.

If there is noise then it affects the accuracy of the measurements. Depending on the time available it may be possible to improve the accuracy of a measurement by using averages over several periods. In principle if the noise has zero mean then average over many cycles should give the correct answer. The other problem is simply one of investigating the effect of putting errors into the tuning formulas. This is more difficult if several parameters are measured and explicit expressions for the tuning parameters are unavailable.

3 The modified Smith Predictor structure

The modified controller structure ([11]) based on a simple and straightforward modification of the conventional Smith predictor is shown in Figure 2. Providing an internal feedback loop across the delay free part of the process model, unstable and integrating (pseudo-unstable) processes are stabilised first. Of the three controllers, G_{c1} in the inner loop is provided to stabilise an unstable process or integrating process. The other two controllers, G_c and G_{c2} are then used to take care of servo-tracking and disturbance rejection respectively by considering the inner loop as an open- loop stable process. When $G_{c1} = 0$ and $G_{c2} = 0$, the standard Smith predictor is obtained. The closed loop response to setpoint and disturbance inputs is given by

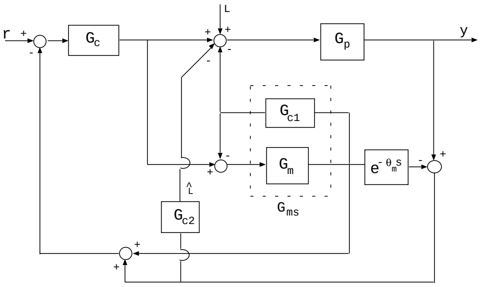

Figure 2: Structure of the modified Smith predictor.

$$Y(s) = Y_r(s)R(s) + Y_L(s)L(s) \qquad (13)$$

where

$$Y_r(s) = \frac{GG_c e^{-\theta s}(1 + G_{c2}G_m e^{-\theta_m s})}{D_1} \qquad (14)$$

$$Y_L(s) = \frac{Ge^{-\theta s}(1 + G_m[G_c + G_{c1} - G_c e^{-\theta_m s}])}{D_1} \qquad (15)$$

and $D_1 = (1 + G_m[G_c + G_{c1}])(1 + G_{c2}Ge^{-\theta s}) + G_c(Ge^{-\theta s} - G_m e^{-\theta_m s})$. $G_m e^{-\theta_m s}$ and $G_p(s) = Ge^{-\theta s}$ are the transfer functions of the plant model and the plant, respectively. Based on the assumption that the model used perfectly matches the plant dynamics that is $G_p = G_m e^{-\theta_m s}$, (14) and (15) reduce to

$$Y_r(s) = \frac{G_m G_c e^{-\theta_m s}}{1 + G_m(G_c + G_{c1})} = \bar{Y_r}(s)e^{-\theta_m s} \qquad (16)$$

$$Y_L(s) = \frac{G_m e^{-\theta_m s}(1 + G_m(G_c + G_{c1}) - G_m G_c e^{-\theta_m s})}{(1 + G_m(G_c + G_{c1}))(1 + G_m G_{c2} e^{-\theta_m s})} \qquad (17)$$

4 Controller Design

The form of the primary PI controller is

$$G_c = \frac{K_p(T_i s + 1)}{T_i s} \qquad (18)$$

and that of the inner feedback controller is

$$G_{c1} = \frac{K_f(T_f s + 1)}{(T_f/N)s + 1} \qquad (19)$$

Then, by ignoring the influence of the low-pass filter time constant T_f (since $T_f/N << 1$) and using G_{c1} the transfer function of the stabilised process model becomes

$$G_{ms} = \bar{k}/((\bar{T_1}s + 1)(T_2 s + 1)) \qquad (20)$$

where $T_f = T_2$. The suitable values of the constants $\bar{k}$ and $\bar{T_1}$ for stable, unstable and integrating SOPDT process models are shown in the Table 1. The most common design goal in process control is to obtain a critically damped closed loop system, which is as fast as possible. (18) and (20) enable one to obtain the delay free part of (16) as

Table 1: modified process transfer functions

Type	$G_m(s)$	$G_{ms}(s)$	*Constants*
m_1	$\frac{k}{(T_1 s+1)(T_2 s+1)}$	$\frac{k}{(\bar{T_1} s+1)(T_2 s+1)}$	$\bar{k} = \frac{k}{kK_f+1}$ $\bar{T_1} = \frac{T_1}{kK_f+1}$
m_2	$\frac{k}{(T_1 s-1)(T_2 s+1)}$	$\frac{\bar{k}}{(\bar{T_1} s+1)(T_2 s+1)}$	$\bar{k} = \frac{k}{kK_f-1}$ $\bar{T_1} = \frac{T_1}{kK_f-1}$
m_3	$\frac{k}{s(T_2 s+1)}$	$\frac{\bar{k}}{(\bar{T_1} s+1)(T_2 s+1)}$	$\bar{k} = \frac{1}{K_f}$ $\bar{T_1} = \frac{1}{kK_f}$

$$\bar{Y_r}(s) = \frac{G_{ms}G_c}{1 + G_{ms}G_c} = \frac{\bar{k}K_p}{T_2 s(\bar{T_1}s + 1) + \bar{k}K_p} = \frac{1}{(\tau_{cl}s + 1)^2} \qquad (21)$$

assuming $T_i = T_2$. It is assumed that the open loop speed of response is decided by the faster pole $(T_2 s+1)$. τ_{cl}, the closed loop design parameter can be chosen approximately equal to T_2 since there is no reason for speeding up of the closed loop response than the open loop speed of response. It is apparent from (17) that the controller G_{c2} is essential for load disturbance rejection for the unstable and integrating processes and that stabilization of the characteristic equation is essential. That means one needs to consider stabilization of

$$1 + G_m G_{c2} e^{-\theta_m s} \qquad (22)$$

from which the design values of the controller G_{c2} can be obtained. Let the form of the controller be $G_{c2} =$

$\frac{K_d(T_d s+1)}{(T_d/M)s+1}$ where again $T_d/M << 1$. For the typical integrating SOPDT process

$$1 + G_m G_{c2} e^{-\theta_m s} = 1 + \frac{k K_d e^{-\theta_m s}}{s} = 0 \qquad (23)$$

when $T_d = T_2$. Choosing K_d to give a phase margin, ϕ_m, of 60^o [5] gives

$$K_d = \frac{\pi - 2\phi_m}{2k\theta_m} = \frac{0.5236}{k\theta_m} \qquad (24)$$

Similarly, for the unstable SOPDT process, letting $T_d = T_2$, (22) can be written as

$$1 + G_m G_{c2} e^{-\theta_m s} = 1 + \frac{k K_d e^{-\theta_m s}}{T_1 s - 1} = 0 \qquad (25)$$

De Paor and O'Malley [12] suggested a proportional controller, K_d, for the stabilisation of an unstable FOPDT process based on the optimum phase margin criterion which gives

$$K_d = \sqrt{\frac{T_1}{\theta_m k^2}} \qquad (26)$$

with the constraint $\theta_m/T_1 < 1$. Thus simple expressions for all the controller parameters are found and tabulated in the Table 2.

Table 2: Controller parameters

Type	G_c	G_{c1}	G_{c2}
m_1	$kK_p = \frac{T_1}{T_2}$ $T_i = T_2$	$kK_f = \frac{2T_1}{T_2} - 1$ $T_f = T_2$	- -
m_2	$kK_p = \frac{T_1}{T_2}$ $T_i = T_2$	$kK_f = \frac{2T_1}{T_2} + 1$ $T_f = T_2$	$kK_d = \sqrt{\frac{T_1}{\theta_m}}$ $T_d = T_2$
m_3	$kK_p = \frac{1}{T_2}$ $T_i = T_2$	$kK_f = \frac{2}{T_2}$ $T_f = T_2$	$kK_d = \frac{0.5236}{\theta_m}$ $T_d = T_2$

5 Simulation studies

In this section three examples have been considered to show the worth of the proposed autotuning method.

Example 1
Consider the higher order plant transfer function ([8])

$$G_p(s) = \frac{1}{(1+s)^{20}} \qquad (27)$$

For $h_1 = 1.2$ and $h_2 = 0.8$, measurements on the limit cycle in a simulation gave $T_p = 39.3723$, $\tau = 20.8081$, $A_p = 1.1297$, $A_v = 0.7632$, $a_y = 5.6306$ and $a_u = 5.6306$. The identified SOPDT model by the proposed method is $e^{-13.0633s}/((3.6483s+1)(3.6193s+1))$ while Hang et al proposed a model of $\frac{e^{-11.85s}}{(1+3.6s)^2}$. The controller parameters using the first row of Table 2 are $K_p = 1.0080$, $K_f = 1.0160$, and $T_i = T_f = 3.6193$ while Hang et al suggested a PI controller with $K_p = 0.5$ and $T_i = 3.60$. The responses by both methods are shown in Figure 3 for $T_f/N = 0.01$. The improved performance of the proposed method comes from more accurate modeling and an improved controller design technique.

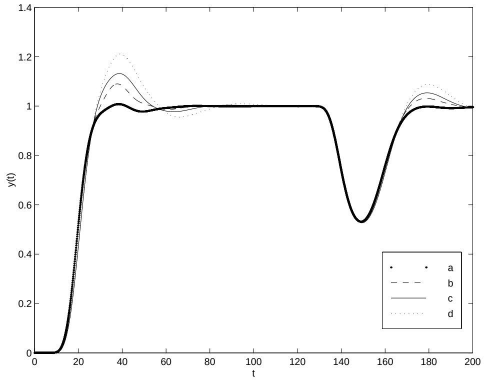

Figure 3: Responses of Example 1 to input step and later on a load disturbance: a)proposed, b) -10% error in proposed θ_m, c)Hang et al, d) -10% error in their θ_m

Example 2
For the unstable plant transfer function ([11])

$$G_p(s) = \frac{2e^{-3.5s}}{(10s-1)(2s+1)} \qquad (28)$$

use of a simulation with an asymmetrical relay test with $h_1 = 1.2$, $h_2 = 1.0$ gave $T_p = 37.7922$, $\tau = 23.6721$, $A_p = 1.5057$, $A_v = 1.1260$, $a_y = 13.4560$ and $a_u = 6.7280$. The unstable SOPDT model was estimated as $2e^{-3.5s}/((10s-1)(2s+1))$ to within 10^{-4}. Using the second row of Table 2, the controller parameters calculated are $K_p = 2.5$, $K_f = 5.5$, $K_d = 0.8452$, and $T_i = T_f = T_d = 2$. The responses of the two controller settings are shown in Figure 4 for $T_f/N = 0.01$ and $T_d/M = 0.01$ with a step load disturbance of magnitude $L = -0.5$ applied at $t = 110$. It is apparent that the results are far superior to those of De Paor and Egan [7]. Further, it has been observed that the proposed method gives robust responses to perturbations in both the unstable time constant and dead time.

Example 3
Consider the integrating process transfer function

$$G_p(s) = \frac{e^{-5s}}{s(s+1)^2(3s+1)} \qquad (29)$$

Following similar identification procedure, an integrating SOPDT model $G_m e^{-\theta_m s} = \frac{e^{-6.5672s}}{s(3.4945s+1)}$ was ob-

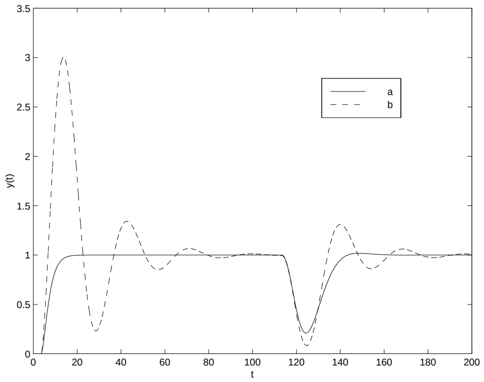

Figure 4: Responses of Example 2:a) proposed, b) De Paor and Egan.

tained from an asymmetrical relay test. Again, using the third row of Table 2, the controller parameters are $K_p = 0.2862$, $K_f = 0.5723$, $K_d = 0.0797$, and $T_i = T_f = T_d = 3.4945$. The unit step input and step load disturbance responses of 0.1 magnitude, for these controller settings are compared with those obtained using the method of Mataušek et al. [5] which does not use autotuning and requires a mathematical model, are shown in Figure 5 for $T_f/N = 0.01$ and $T_d/M = 0.01$. The closed loop performances show the superiority of the proposed control technique. Also, the figure shows the effect of a 10% error in estimating the dead time.

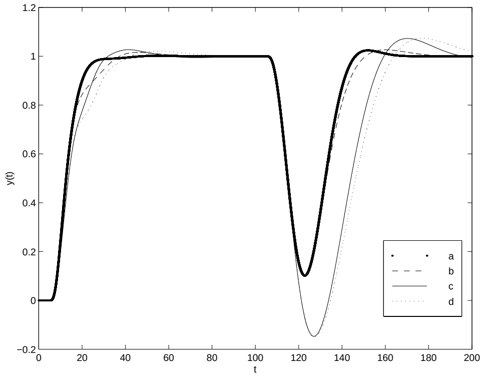

Figure 5: Responses of Example 3:a)proposed, b) 10% error in proposed θ_m, c)Matausek et al, d) 10% error in their θ_m

6 Conclusions

Simple and effective automatic tuning formulae are derived for the modified Smith predictor structure assuming second order model transfer functions with time delay for stable, unstable and integrating processes. The method has been shown to work effectively even when the plant has a higher order transfer function. The robustness of the proposed controller is apparent from results obtained using incorrect time delay values in the plant model. Illustrative examples have been given to show the simplicity and robustness of the controllers for controlling the class of processes considered. The method is particularly valuable in practice since no plant model is required, only the type information for use with Table 1, and the performing of one relay feedback test.

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