Delay-Dependent Robust Stability and H_∞ Control of Jump Linear Systems with Time-Delay [1]

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Abstract

This paper considers the class of continuous-time jump linear system with time-delay and polytopic uncertain parameters. When the time-delay is a known constant, by using the linear matrix inequality (LMI) technique, we first establish delay dependent sufficient condition for robust stability and robust H_∞ control of the class of systems under study. The problem of determining the maximum time-delay under which the system will remain stable is cast into a generalized eigenvalue problem and thus solved by LMI techniques. When the control input contains time-delay, an algorithm to design state feedback controller with constant gain matrix is developed.

Keywords: Jump linear system, polytopic uncertainty, time-delay, LMI, generalized eigenvalue problem.

1 Introduction

Time-delay is a great source of instability and poor performance. During the past decades, stability of linear system with time-delay has received considerable attention. For the progress on the research of deterministic system with time-delay, the reader is referred to Jeung et al. [5], Cao et al. [3], Park [7] and references therein. For jump linear systems with time-delay, Benjelloun and Boukas [1] established a sufficient condition for the stochastic stability in the mean square sense. The robust stabilization problem was addressed by Benjelloun, Boukas and Yang [2]. To the best of our knowledge, the stability of jump linear time-delay system with polytopic uncertainty has never been addressed.

The goal of this paper is to study the delay-dependent stability, stabilizability and H_∞ control problem using LMI technique The rest of this paper is organized as follows. In section 2, the problem is formulated. Section 3 establishes delay dependent conditions for the robust stability and robust H_∞ control problems of JLS. Section 4 is devoted to develop LMI-based robust stabilizability and H_∞ control. Section 5 addresses the stabilization problem when the control input also contains time-delay. Numerical examples are provided in Section 6 to support our theoretical results.

2 Problem Statement

Consider a continuous-time hybrid uncertain linear system with N modes. The mode switching is governed by a Markov process $\{r_t, t \geq 0\}$ which is assumed to take values in the finite state space $\mathcal{S} = \{1, 2, \cdots, N\}$ and have generator $\Pi = (q_{ij}), i, j \in \mathcal{S}, q_{ij} \geq 0, j \neq i, q_{ii} = -\sum_{j\neq i} q_{ij}$. The system dynamics is

$$\Sigma_1 : \begin{cases} \dot{x}_t = A(r_t)x_t + A_1(r_t)x_{t-\tau} + B(r_t)u_t \\ \qquad + B_w(r_t)w_t \\ x_t = \phi(t), r_t = r_0, t \in [-\tau, 0], \\ z_t = C(r_t)x_t + D(r_t)u_t + E(r_t)w_t \end{cases} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state vector, $u_t \in \mathbb{R}^m$ is the control vector, z_t is the output of system, τ is the time delay in the system state, w_t is the system input noise disturbance, which is assumed to be in

$L^2[0,\infty)$. $\phi(\cdot) \in L^2[-\tau, 0]$ is a given initial function which is continuous on $[-\tau, 0]$. $A(r_t), A_1(r_t)$, $B(r_t), B_w(r_t)$, $C(r_t)$, $D(r_t)$, $E(r_t)$ are matrices with appropriate dimensions but unknown. For fixed system mode i, the system matrices are assumed to belong to a convex compact set of polytopic type $\mathcal{M}(i)$, namely,

$$(A(i), A_1(i), B(i), B_w(i), C(i), D(i), E(i)) \in \mathcal{M}(i)$$

where

$$\mathcal{M}(i) = \Big\{(A(i), A_1(i), B(i), B_w(i), C(i), D(i), E(i)) = \sum_{j=1}^{\nu} t_j (A_{ij}, A_{1ij}, B_{ij}, B_{wij}, C_{ij}, D_{ij}, E_{ij}), t_j \geq 0, \sum_{j=1}^{\nu} t_j = 1\Big\} \quad (2)$$

with $A_{ij}, A_{1ij}, B_{ij}, B_{wij}, C_{ij}, D_{ij}, E_{ij}$ being given matrices with appropriate dimensions.

For system (1), the concept of its stability is defined as follows:

Definition 2.1 *System (1) with $u_t \equiv 0$ is called to be robustly stochastically stable (SS), if for any $(A(r_t), A_1(r_t), B_w(r_t)) \in \mathcal{M}(r_t)$ there exist a constant $M_0(r_0, \phi(\cdot), \|w\|_2)$, which is dependent on the initial condition $(r_0, \phi(\cdot))$ and satisfies $M_0(r_0, 0, 0) = 0$, such that the state trajectory x_t of (1) satisfies*

$$\int_0^{\infty} \mathbb{E}[\|x_t\|^2 | r_0, \phi(\cdot)] dt \leq M_0(r_0, \phi(\cdot), \|w\|_2), \quad (3)$$

for all $w_t \in L^2[0, \infty)$.

Definition 2.2 *Let $\gamma > 0$ be a positive constant. System (1) is said to be robustly stable with noise attenuation level γ, if there exists a constant $M(r_0, \phi(\cdot))$ with $M(r_0, 0) = 0$, such that*

$$\|z\|_2 \triangleq \mathbb{E}\left[\int_0^{\infty} z_t^\top z_t dt\right]^{1/2} \leq \gamma \left[\|w\|_2^2 + M(r_0, \phi(\cdot))\right]^{1/2}. \quad (4)$$

System (1) is said to be robustly stabilizable with noise attenuation level γ if there exist matrices $K(i), i \in \mathcal{S}$, such that the closed-loop system under control $u_t = K(r_t)x_t$ satisfies (4).

The goal of this paper is to address the following problems:

- How to judge whether a given system is stable or not.
- When the time delay is unknown, for a given system to be stable how long can the time delay be?
- How to design a state feedback controller of the form $u_t = K(r_t)x_t$ that robustly stabilizes the system and at the same time assures a given noise attenuation level.

3 Robust Stability

The first part of this section will address the stability of the following system

$$\Sigma_2 : \dot{x}_t = A(r_t)x_t + A_1(r_t)x_{t-\tau} \quad (5)$$

To this end, let us give the following assumption:

Assumption 3.1 *Let $\{x_t, t \geq 0\}$ be the state trajectory of system (1). Suppose that there exists a constant $\varrho > 0$, such that*

$$\|x_{t+\theta}\| \leq \varrho \|x_t\|, \forall \theta \in [-2\tau, 0].$$

Remark 3.1 *The above assumption doesn't represent a restriction since ϱ can be chosen arbitrarily.*

Definition 3.1 *System Σ_2 is said to be mean square quadratic stable (MSQS) if there exist $X_i > 0, i \in \mathcal{S}, Q > 0, Q_1 > 0$ such that*

$$\begin{pmatrix} J_1(i) & X_i & X_i A^\top(i) & A_1(i)Q_1 & S_i(X) \\ X_i & -\frac{1}{\tau}Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A(i)X_i & \mathbf{0} & -\frac{1}{\tau}Q_1 & \mathbf{0} & \mathbf{0} \\ Q_1 A_1^\top(i) & \mathbf{0} & \mathbf{0} & -\frac{1}{2\tau}Q_1 & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{pmatrix} < 0, \quad (6)$$

$$\begin{pmatrix} Q & Q A_1^\top(i) \\ A_1(i)Q & Q_1 \end{pmatrix} > 0 \quad (7)$$

hold for any $(A(i), A_1(i)) \in \mathcal{M}(i)$, where

$$J_1(i) = X_i[A(i) + A_1(i)]^\top + [A(i) + A_1(i)]X_i + q_{ii}X_i,$$
$$S_i(X) = \mathrm{diag}\{\sqrt{q_{i1}}X_i, \cdots, \sqrt{q_{ii-1}}X_i, \sqrt{q_{ii+1}}X_i, \cdots, \sqrt{q_{iN}}X_i\},$$
$$\mathcal{X}_i = \mathrm{diag}\{X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_N\}.$$

The stability in the MSQS sense is defined on all $(A(i), A_1(i)) \in \mathcal{M}(i), i \in \mathcal{S}$, which are unknown. Thus, conditions (6) and (7) do not permit us to conclude on the stability of system Σ_2. However, noting that (6) and (7) are affine in $A(i), A_1(i)$, one can get the following equivalent definition.

Proposition 3.1 *Suppose the time-delay τ in system Σ_2 is a given constant. Then, system (1) is MSQS if and only if there exist symmetric and positive-definite matrices $X_i, i \in \mathcal{S}, Q, Q_1$ such that the following LMIs*

$$\begin{pmatrix} J_1(i,j) & X_i & X_i A_{ij}^\top & A_{1ij}Q_1 & S_i(X) \\ X_i & -\frac{1}{\tau}Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A_{ij}X_i & \mathbf{0} & -\frac{1}{\tau}Q_1 & \mathbf{0} & \mathbf{0} \\ Q_1 A_{1ij}^\top & \mathbf{0} & \mathbf{0} & -\frac{1}{2\tau}Q_1 & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{pmatrix} \triangleq \Gamma(i,j) < 0, \quad (8)$$

$$\begin{pmatrix} Q & QA_{1ij}^\top \\ A_{1ij}Q & Q_1 \end{pmatrix} > 0 \quad (9)$$

are feasible for all $j = 1, \cdots, \nu$ at the same time, where $J_1(i,j) = X_i[A_{ij} + A_{1ij}]^\top + [A_{ij} + A_{1ij}]X_i + q_{ii}X_i$.

Proposition 3.1 shows that it is easy to check whether system Σ_2 is MSQS by LMI technique. However, the MSQS doesn't provide any information on the system behavior of the system trajectory. To establish the relationship between MSQS and SS, let us introduce a lemma.

Theorem 3.1 *If system Σ_2 is MSQS, then it is robustly SS.*

Based on the assumption that the time delay τ is constant, Theorem 3.1 establishes an LMI-based stability condition for system Σ_2. Now, we come to consider the problem of determining the upper bound for the time-delay τ. Obviously, when τ is constant but unknown, (8) is nonlinear in τ. Let $v = \frac{1}{\tau}$. Then, (8) can be written as follows:

$$\begin{pmatrix} J_1(i,j) & X_i & X_i A_{ij}^\top & A_{1ij}Q_1 & S_i(X) \\ X_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A_{ij}X_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ Q_1 A_{1ij}^\top & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{pmatrix} < v \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{1}{2}Q_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}. \quad (10)$$

Note that when using gevp of LMI toolbox to solve the generalized eigenvalue problem (GEVP)

$$\begin{aligned} &\min_x \quad v \\ &\mathcal{A}(x) < v\mathcal{B}(x), \end{aligned}$$

the matrix $\mathcal{B}(x)$ must be positive-definite (see [4]). Thus, to cast the problem of maximizing the time-delay τ, i.e., minimizing v, into the framework of GEVP, let us introduce some auxiliary matrices $\Gamma > 0, \Gamma_1 > 0$ and rewrite (10) as follows:

$$\begin{pmatrix} J_1(i,j) & X_i & X_i A_{ij}^\top & A_{1ij}Q_1 & S_i(X) \\ X_i & -\Gamma & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A_{ij}X_i & & -\Gamma_1 & \mathbf{0} & \mathbf{0} \\ Q_1 A_{1ij}^\top & \mathbf{0} & \mathbf{0} & -\frac{1}{2}\Gamma_1 & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{pmatrix} < 0, \quad (11)$$

$$\begin{pmatrix} \Gamma & \mathbf{0} \\ \mathbf{0} & \Gamma_1 \end{pmatrix} < v \begin{pmatrix} Q & \mathbf{0} \\ \mathbf{0} & Q_1 \end{pmatrix}. \quad (12)$$

The optimization problem that determines the upper bound of the delay can be formulated as

$$\mathcal{P}_0 : \quad \begin{aligned} &\min_{P(i), Q, Q_1, \Gamma, \Gamma_1} \quad v \\ &\text{s.t. (9), (11), (12)} \end{aligned}$$

which is a standard GEVP problem and thus can be solved by LMI toolbox efficiently.

Theorem 3.1 establishes the stability of system Σ_2, based on which the robust stability of system Σ_1 can be developed. To this end, let us introduce a definition.

Definition 3.2 *System (1) with $u_t \equiv 0$ is said to be internally MSQS, if there exist symmetric and positive matrices $X_i, i \in \mathcal{S}, Q, Q_1$ such that its parameters satisfy (8) and (9).*

By definition, internally MSQS of system (1) means that it is MSQS in the case of zero noise disturbance. In the general case, the following proposition shows that internally MSQS implies SS.

Proposition 3.2 *Suppose that system (1) with $u_t \equiv 0$ is internally MSQS and $w(\cdot) \in L^2[0, \infty)$, then system (1) is SS.*

With Theorem 3.2, we get the robust stability of system Σ_1 as follows:

Theorem 3.2 *Suppose that there exist symmetric and positive-definite matrices $X_i, i \in \mathcal{S}, Q, Q_1$ such that*

$$\left(\begin{array}{ccccc} J_1(i,j) & B_{wij} & X_i C_{ij}^\top & J_{14}(i,j) & S_i(X) \\ B_{wij}^\top & -\gamma^2 I & E_{ij}^\top & \left[\begin{array}{cc} B_{wij}^\top & \mathbf{0} \end{array}\right] & \mathbf{0} \\ C_{ij}X_i & E_{ij} & -I & \mathbf{0} & \mathbf{0} \\ J_{14}^\top(i,j) & \left[\begin{array}{c} B_{wij} \\ \mathbf{0} \end{array}\right] & \mathbf{0} & -\frac{1}{\tau}\mathcal{Q} & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{array}\right) < 0, \quad (13)$$

$$\left(\begin{array}{cc} Q & QA_{1ij}^\top \\ A_{1i}Q & Q_1 \end{array}\right) > 0 \quad (14)$$

hold for all $j = 1, \cdots, \nu$ at the same time, where

$$J_{14}(i,j) = \left(\begin{array}{cccc} \mathbf{0} & X_i A_{ij}^\top & X_i & A_{1ij}Q_1 \end{array}\right),$$
$$\mathcal{Q} = \mathrm{diag}\{Q_1, Q_1, Q, \frac{1}{3}Q_1\}.$$

Then system (1) is robustly SS and

$$\|z\|_2 \le \gamma\left[\|w\|_2^2 + \lambda\left(\int_{-\tau}^0\int_\theta^0 \phi^\top(s)\phi(s)dsd\theta + \int_{-\tau}^0\int_{\theta-\tau}^0 \phi^\top(s)\phi(s)dsd\theta\right)\right]^{1/2}, \quad (15)$$

where
$\lambda = \max\{\max_{i,j}[\lambda_{max}(A_{ij}^\top Q_1^{-1}A_{ij})], \lambda_{max}(Q^{-1})\}$.

Proof: The proof will be provided in a complete version.

4 Robust Stabilizability and H_∞ Control

This section is devoted to designing a state feedback controller of the following form

$$u_t = K(r_t)x_t \quad (16)$$

that stabilizes system (1) in the SS sense and the closed-loop system attains a given noise attenuation level. Substituting (16) into (1) yields

$$\left\{\begin{array}{l} \dot{x}_t = [A(r_t) + B(r_t)K(r_t)]x_t + A_1(r_t)x_{t-\tau} \\ \qquad + B_w(r_t)w_t, \\ x_t = \phi(t), t \in [-\tau, 0), \\ z_t = [C(r_t) + D(r_t)K(r_t)]x_t + E(r_t)w_t. \end{array}\right. \quad (17)$$

Replacing A_{ij} and C_{ij} with $A_{ij} + B_{ij}K(i)$ and $C_{ij} + D_{ij}K(i)$ in (13) and letting $Y_i = K(i)X_i$, we get the following theorem from Theorem 3.2.

Theorem 4.1 *Let $\gamma > 0$ be a given constant. If there exist a set of matrices $X_i > 0, Y_i, i \in \mathcal{S}$ such that*

$$\left(\begin{array}{ccccc} \tilde{J}_{11}(i,j) & B_{wij} & \tilde{J}_{13}(i,j) & \tilde{J}_{14}(i,j) & S_i(X) \\ B_{wij}^\top & -\gamma^2 I & E_{ij}^\top & \left[\begin{array}{cc} B_{wij}^\top & \mathbf{0} \end{array}\right] & \mathbf{0} \\ \tilde{J}_{13}^\top(i,j) & E_{ij} & -I & \mathbf{0} & \mathbf{0} \\ \tilde{J}_{14}^\top(i,j) & \left[\begin{array}{c} B_{wij} \\ \mathbf{0} \end{array}\right] & \mathbf{0} & -\frac{1}{\tau}\mathcal{Q} & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{array}\right) < 0, \quad (18)$$

$$\left(\begin{array}{cc} Q & QA_{1ij}^\top \\ A_{1i}Q & Q_1 \end{array}\right) > 0 \quad (19)$$

hold for all $j = 1, \cdots, \nu$ at the same time, where

$$\tilde{J}_{11}(i,j) = [A_{ij} + A_{1ij}]X_i + B_{ij}Y_i + Y_i^\top B_{ij}^\top + X_i[A_{ij} + A_{1ij}]^\top + q_{ii}X_i,$$

$$\tilde{J}_{13}(i,j) = X_i C_{ij}^\top + Y_i^\top D_{ij}^\top,$$

$$\tilde{J}_{14}(i,j) = \left(\begin{array}{cccc} \mathbf{0} & X_i A_{ij}^\top + Y_i^\top B_{ij}^\top & X_i & A_{1ij}Q_1 \end{array}\right),$$

$$\mathcal{Q} = \mathrm{diag}\{Q_1, Q_1, Q, \frac{1}{3}Q_1\}.$$

Then controller (16) with $K_i = Y_i X_i^{-1}$ robustly stabilizes system (1) and the closed-loop system verifies noise attenuation level γ.

In the above theorem τ is assumed to be known and constant. When τ is an unknown constant, its upper bound can be obtained by solving a GEVP. Let $v = 1/\tau, \gamma^2 = \delta$. Then (18) is equivalent to

$$\left(\begin{array}{ccccc} \tilde{J}_{11}(i,j) & B_{wij} & \tilde{J}_{13}(i,j) & \tilde{J}_{14}(i,j) & S_i(X) \\ B_{wij}^\top & -\delta I & E_{ij}^\top & \left[\begin{array}{cc} B_{wij}^\top & \mathbf{0} \end{array}\right] & \mathbf{0} \\ \tilde{J}_{13}^\top(i,j) & E_{ij} & -I & \mathbf{0} & \mathbf{0} \\ \tilde{J}_{14}^\top(i,j) & \left[\begin{array}{c} B_{wij} \\ \mathbf{0} \end{array}\right] & \mathbf{0} & -\tilde{\Gamma} & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{array}\right) < 0, \quad (20)$$

$$\left(\begin{array}{cc} \Gamma & \mathbf{0} \\ \mathbf{0} & \Gamma_1 \end{array}\right) < v\left(\begin{array}{cc} Q & \mathbf{0} \\ \mathbf{0} & Q_1 \end{array}\right), \quad (21)$$

where $\tilde{\Gamma} = \text{diag}\{\Gamma_1, \Gamma_1, \Gamma, \frac{1}{3}\Gamma_1\}$. The optimization can be formulated as follows:

$$\mathcal{P}_1 : \quad \min_{v, X_i>0, Y_i, Q, Q_1, \Gamma, \Gamma_1} v$$
$$\text{s.t. } (19), (20), (21)$$

From above discussion, we get the following theorem.

Theorem 4.2 *Let v_0, δ_0 be the solution to $\mathcal{P}_1$, then controller (16) with $K(i) = Y_i X_i^{-1}$ robustly stabilizes system (1) in the SS sense for all $\tau \in [0, \frac{1}{v_0}]$ and the closed-loop system attains the optimal noise attenuation level $\sqrt{\delta_0}$.*

5 Robust Stabilization for General Time-Delay System

The results of previous sections are based on the assumption that the control input doesn't contain delay effect. This section will address the robust stabilization problem of JLS with polytopic uncertain parameters when the control input contains time-delay, i.e.,

$$\Sigma_3 : \begin{cases} \dot{x}_t = A(r_t)x_t + B(r_t)u_t + A_1(r_t)x_{t-\tau} \\ \qquad + B_1(r_t)u_{t-\tau} \\ x_t = \phi(t), t \in [-\tau, 0). \end{cases} \tag{22}$$

To stabilize the jump linear systems, in most cases controller of form (16) are used. However, in some cases, e.g., when the plant state x_t is perfectly available but the mode observation is not present, to design a state feedback controller the mode has to be estimated, which will increase the mode uncertainties. To avoid this shortcoming, state feedback controller with constant gain matrix, i.e.,

$$u_t = K x_t \tag{23}$$

is often used (see [6]). For our model, here we will concentrate on the design of feedback controller of form (23).

By using Theorem 3.1, we get the following proposition.

Proposition 5.1 *Let $v = 1/\tau$ and suppose that there exist matrices $Q > 0, Q_1 > 0$ such that the following inequalities*

$$\begin{pmatrix} J_3(i,j) & X_i & * & [A_{1ij} + B_{1ij}K]Q_1 & S_i(X) \\ X_i & -vQ & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ [A_{ij} + B_{ij}K]X_i & \mathbf{0} & -vQ_1 & \mathbf{0} & \mathbf{0} \\ Q_1[A_{1ij} + B_{1ij}K]^\top & \mathbf{0} & \mathbf{0} & -\frac{v}{2}Q_1 & \mathbf{0} \\ S_i^\top(X) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathcal{X}_i \end{pmatrix} < \delta I, \tag{24}$$

$$\begin{pmatrix} Q & * \\ [A_{1ij} + B_{1ij}K]Q & Q_1 \end{pmatrix} > 0 \tag{25}$$

are feasible for some matrices $X_i > 0, K$ and scalars δ, v, where $J_3(i,j) = X_i[A_{ij} + B_{ij}K + A_{1ij} + B_{1ij}K]^\top + [A_{ij} + B_{ij}K + A_{1ij} + B_{1ij}K]X_i + q_{ii}X_i$. Let v_0, δ_0 be the solution of the following optimization problem:

$$\mathcal{P}_2 : \quad \min_{X_i, K} v + \delta$$
$$\text{s.t. } (24), (25)$$

If $\delta_0 < 0$, then the controller (23) robustly stabilizes system (22) for any $\tau \in [0, 1/v_0]$ in the SS sense.

Obviously, (24) is not linear with respect to K and $X_i, i \in \mathcal{S}$. Thus $\mathcal{P}_2$ can not be directly solved using LMI toolbox. However, the following iterative algorithm can provide a suboptimal solution.

Algorithm 5.1 *(Control design algorithm):*

Step 1. set initial $X_i, i \in \mathcal{S}$ and the error bound $\varepsilon > 0$.

Step 2. For a set of given matrices $X_i > 0, i \in \mathcal{S}$ solve the following linear objective minimization problem

$$\mathcal{P}_3 : \quad \min_{K} v + \delta$$
$$\text{s.t. } (24), (25) \tag{26}$$

and denote the optimal solution by v_1, δ_1.

Step 3. With K obtained in Step 2, solve the following optimization problem

$$\mathcal{P}_4 : \quad \min_{X_i} v + \delta$$
$$\text{s.t. } (24), (25)$$

and denote the optimal solution by v_2, δ_2.

Step 4 If $|\frac{1}{v_2} - \frac{1}{v_1}| \le \varepsilon$ *and* $\delta_2 < 0$*, stop, otherwise, go to Step 2 with* X_i *obtained in Step 3.*

6 Illustrative Examples

Consider an uncertain system with time-delay and Markov jumps. The system is assumed to have two modes. The Markov process that governs the mode switching has generator Π. The uncertain convex domain has two vertices.

Example 6.1 *Consider a system described by (1). The system data are as follows:* $\Pi = \begin{pmatrix} -3 & 3 \\ 2 & -2 \end{pmatrix}$, $A_{11} = \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix}$, $A_{12} = \begin{pmatrix} -2.1 & -0.1 \\ -0.1 & -1 \end{pmatrix}$, $A_{111} = A_{112} = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$. $A_{21} = \begin{pmatrix} -2 & 0 \\ 0 & -0.91 \end{pmatrix}$, $A_{22} = \begin{pmatrix} -1.9 & 0.1 \\ 0.1 & -0.89 \end{pmatrix}$, $A_{121} = A_{122} = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$. $B_{11} = B_{21} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$, $B_{21} = B_{22} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B_{w11} = B_{w21} = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix}$, $B_{w12} = B_{w22} = \begin{pmatrix} 0.8 & 0 \\ 0 & 1 \end{pmatrix}$ $C_{11} = C_{21} = \begin{pmatrix} 1 & 0.4 \end{pmatrix}$, $C_{12} = C_{22} = \begin{pmatrix} 0.5 & 1 \end{pmatrix}$ $D_{11} = D_{21} = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $D_{21} = D_{22} = \begin{pmatrix} 0 & 1 \end{pmatrix}$ $E_{11} = E_{21} = \begin{pmatrix} 0.9 & 1 \end{pmatrix}$, $E_{21} = E_{22} = \begin{pmatrix} 0.4 & 1 \end{pmatrix}$ *With these data, solving* $\mathcal{P}_1$ *gives a set of gains matrices*

$$K(1) = \begin{pmatrix} -4.3275 & 204.9649 \\ 0.4154 & -36.3945 \end{pmatrix},$$

$$K(2) = \begin{pmatrix} -2.1361 & 122.1252 \\ 0.1238 & 22.1735 \end{pmatrix},$$

the upper bound for the time-delay 0.8283 *and noise attenuation level* $\gamma = 66.9677$.

Example 6.2 *The second example is to illustrate Algorithm 5.1. Consider a system with two modes and for every mode the uncertain domain has two vertices. Parameters* A_{11}, A_{12}, A_{21}, A_{22}, A_{111}, A_{112}, A_{121}, A_{122}, B_{11}, B_{12}, B_{21}, B_{22} *are the same as in example 1. The other parameters are as follows:* $B_{111} = B_{121} = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix}$, $B_{112} = B_{122} = \begin{pmatrix} 0.8 & 0 \\ 0 & 1 \end{pmatrix}$

Using Algorithm 5.1, we get the suboptimal solution as follows:

$$K = \begin{pmatrix} -0.0326 & 0.7645 \\ 0.5891 & -4.2984 \end{pmatrix}$$

and the upper bound for time-delay $\tau_0 = 0.2301$.

7 Conclusion and Discussion

This paper addresses the delay-dependent robust stability problem of jump linear system with time-delay and polytopic uncertain parameters. The time-delay is assumed to be a constant but unknown. The robust delay-dependent stability and stabilization problems are cast into the framework of generalized eigenvalue problem and thus the delay bound and a robust state feedback controller are provided by using the LMI technique. When the control input contains time-delay, an algorithm that allows the design of suboptimal controller with constant gain matrix is developed. Simulation results show that the procedures are efficient.

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