

Internet Pricing: Comparison and Examples

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Abstract

The central issue of Internet economics is pricing. In [1], we studied the Internet pricing based on the leader-follower game, the cooperative game, and the two-person game theory. In this paper, we continue our study by comparing different pricing schemes with the above approaches. These schemes include Paris Metro Pricing (PMP) and pricing with priority. We show that PMP does not provide better social welfare thus does not provide better cooperative solutions. Numerical examples indicate that the leader-follower game leads to an optimal solution with the same price for both “classes” of users in PMP. This contradicts to the intention of the original design of the scheme.

Keywords: Leader-follower game, cooperative game, social welfare, Paris Metro Pricing.

1 Introduction

The central issue of Internet economics is pricing, which can be used as an effective means to recover cost, to increase competition among different service providers, and to reduce congestion or to control the traffic intensity [6] [7]. There are many approaches in determining a pricing strategy which include the cost-based approach, the optimization-based approach and the recently proposed *edge pricing* [5].

Most of the existing works applying game theory to Internet pricing (optimization-based) adopt a leader-follower game framework: the ISPs set up prices as leaders and the users respond with demands. The ISPs must set up the right prices to induce desirable demands from the users to achieve profits as large as possible. This belongs to the domain of non-cooperative game. In [1], it was proposed to study the Internet pricing problem by using the cooperative game approach. With this approach, all the possible outcomes in a utility space are studied, and the players (ISPs and users) determine, through negotiation or arbitration, a particular outcome as their *fair* solution. This solution depends on the concept of fairness, which can be spec-

ified clearly by a set of axioms. It was shown that the solution of the leader-follower game approach may not be on the Pareto boundary and by cooperation a “fair” solution can be obtained which make both the ISPs and the users better off.

This paper develops further results based on the previous work. We introduce the concept of *social welfare* to explain the linear property of the Pareto boundary. This approach provides a better framework for analyzing the leader-follower and cooperative game. And we extend the model to different pricing schemes, including recently proposed *Paris Metro Pricing* (PMP) [2] [3] and *pricing with priority*. The PMP proposal was inspired by the Paris Metro system and is one of the representative schemes in Internet pricing. Its basic idea is to divide the total bandwidth into several parts which charge different prices. Intuitively, the part with a higher price is expected to attract less traffic and thereby may provide a better service. Detailed study is provided in this paper.

The paper is organized in the following way. In section 2, we briefly review the fundamental concepts of game theory that will be used in this paper. In section 3, the QoS model is described and some analytical formulas are developed based on it. We demonstrate that the utility set has a linear boundary from which the cooperative game solution is picked up and is better than the leader-follower game solution. The above statement is verified by a numerical example. In section 4, the QoS model and analysis are extended to PMP; in the end, we find no improvement of the linear boundary. We illustrate the idea by applying PMP to the same example as in section 3 and compare the result. A numerical example for the leader-follower PMP is also given. In section 5, pricing with priority is discussed and we argue that it may provide a better solution. The paper concludes with some discussions in section 6.

2 Game Theoretical Framework

In this section, basic concepts of game theory are reviewed for the two-party optimization problem. First, the ISP and the user (can be thought as a group of users) are the two players in a game. They can choose

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many *strategies*, which are prices, determined by the ISP, and the traffic intensity submitted to the Internet, determined by the user. The outcomes are the *utilities* of each player, which are the ISP's profit and the user's gain.

From the point of view of the non-cooperative game, this problem is in fact a leader-follower one with the ISP being the leader and the user being the follower. The ISP announces a price c . Then according to this c , the user determines her demand r . With this c and r , we can denote the utilities for the user and the ISP as $U(c, r)$ and $V(c, r)$ respectively. For the non-cooperative game, both players want to maximize their own utilities. Thus given any price c , the user chooses her demand

$$r(c) = \arg[\max_{r \in R} U(c, r)], \quad (1)$$

where R is the set of all r 's. Knowing this reaction of the user, the ISP chooses the price

$$c^* = \arg[\max_{c \in C} V(c, r(c))], \quad (2)$$

where C is the set of all c 's and $r(c)$ is the function specified in (1). Therefore, the solution point in the utility space can be decided as $(U(c^*, r(c^*)), V(c^*, r(c^*)))$ which is a *Nash equilibrium*. Nash equilibrium means no single party can leave this point without the cooperation of the other to improve its own utility.

On the other hand, in the cooperative game, the two players are called bargainers. They work on the set of all the points in the utility space corresponding to all feasible strategies with a starting point $S_0 = (U_0, V_0)$ which will be the outcome of the game if no agreement can be reached by the two bargainers. The solution point is chosen based on certain *fairness* criteria that both parties agree upon. They can be clearly expressed by a set of four axioms. The first three are universally acceptable which include *Symmetry*, *Pareto optimality* and *Invariance with respect to utility transformations* whereas the last one varies according to different arbitration schemes such as Nash, Raiffa-Kailai-Smorodinsky and modified Thomson solutions. Details are discussed in [4], which also provides a general framework that unifies the concept of fairness with the above three solutions as special cases. In this paper, we will use the Nash solution with the fourth axiom to be *Independence of irrelevant alternatives* although other solutions can be chosen. The Nash solution was shown to be the point on the Pareto boundary that maximize the product of $(U - U_0) \times (V - V_0)$, or equally, the tangent point of the hyperbolae $(U - U_0) \times (V - V_0) = \text{constant}$ with the Pareto boundary.

3 Model, Analysis and Example

Assume the user generates requests which form a Poisson process with arrival rate λ . Each request requires a

unit bandwidth to serve for a time length exponentially distributed with a unit mean without loss of generality. Thus the service rate is just the bandwidth μ . Each request sets up a required mean response time denoted as s which is a random variable with density distribution function $f(s)$. The response time here counts from the moment the user submits the request till the end of the service to this request by the server. Based on this s and also other considerations such as the price, QoS and the importance of the request, the user may choose to submit or discard it. If she chooses to submit it, then after service her gain from this request can be determined by $g(s, s')$ with s' being the realized mean response time. On the other hand, if she chooses to discard the request, then no gain and no payment. It is reasonable to require the gain function $g(s, s')$ to be non-increasing for s' since smaller s' reflects better service.

Queueing theory proves to be an efficient tool to analyze the Internet pricing issue [9] [13]. With the above assumptions, the link can be modeled simply as an M/M/1 queue [8]. Suppose when a request arrives with a required response time s , the user submits it with a probability of $\alpha(s)$ or discards it with $1 - \alpha(s)$. Overall, the arrival rate to the Internet is $\lambda\alpha$ where

$$\alpha = \int_0^\infty \alpha(s) f(s) ds. \quad (3)$$

It is easy to verify that the service time s' has a density distribution function

$$p(s') = (\mu - \lambda\alpha) e^{-(\mu - \lambda\alpha)s'}. \quad (4)$$

The user's gain for this request if submitted is

$$G(s) = \int_0^\infty g(s, s') p(s') ds'. \quad (5)$$

As a whole, the user's utility and the ISP's revenue are, respectively,

$$U = \lambda \int_0^\infty G(s) \alpha(s) f(s) ds - \lambda \alpha c, \quad (6)$$

$$V = \lambda \alpha c. \quad (7)$$

Now the ISP chooses her strategy c while the user chooses her strategy α and $\alpha(s)$ for all s . However, the values of $\alpha(s)$ are totally dependent on α . In fact, given any value of α , the user must choose $\alpha(s)$ in the way so as to maximize her utility U as well as satisfy (3). Define

$$I_\alpha = \{s : G(s) \geq G(s'), \forall s \in I_\alpha, s' \notin I_\alpha; \alpha = \int_{I_\alpha} f(s) d(s); I_\alpha \subset [0, \infty)\}, \quad (8)$$

then it's easy to verify that

$$\alpha(s) = \begin{cases} 1 & s \in I_\alpha \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

In the end, the utilities of the two parties can be denoted as $U(c, \alpha)$ and $V(c, \alpha)$.

Clear knowledge about the utility set can be attained if we look at the social welfare S which is defined as the

sum of the user's utility and the ISP's revenue. By (6) and (7),

$$S = U + V = \lambda \int_0^\infty G(s) \alpha(s) f(s) ds. \quad (10)$$

Thus S has nothing to do with the price c but is totally determined by α . In fact, by observing (6), when the user chooses those $\alpha(s)$ to maximize her own utility U for the given α , she maximizes the social welfare $S(\alpha)$ at the same time. In other words, given the value of α , the point (U, V) is on the straight line $U + V = S = \text{constant}$. As the price c increases from 0 to $\frac{S}{\lambda\alpha}$, the point moves along this line from $(S, 0)$ to $(0, S)$ with a fraction of $\lambda\alpha$. The intuition is the social welfare is allotted to the user and the ISP as their utilities. The higher the price, the more the ISP's revenue whereas the less the user's utility. Also, from the fact that there must exist certain value of α , say α^* , at which $S(\alpha)$ reaches the maximum $S(\alpha^*)$, we deduce that the Pareto boundary of the utility set has a linear property.

Now by the game theoretical framework clarified in section 2, the solution for the leader-follower game is the Nash equilibrium $(U(c^*, \alpha(c^*)), V(c^*, \alpha(c^*)))$, where

$$\alpha(c) = \arg[\max_{\alpha \in R} U(c, \alpha)], \quad (11)$$

$$c^* = \arg[\max_{c \in C} V(c, \alpha(c))]. \quad (12)$$

As we know, the solution for cooperative game is on the Pareto boundary. Thus it should be $(U(c_{opt}, \alpha^*), V(c_{opt}, \alpha^*))$ where c_{opt} needs to be decided based on certain fairness criteria that both bargainers agree upon. Normally, $\alpha^* \neq \alpha(c^*)$, thus $S(\alpha^*) > S(\alpha(c^*))$. It is reasonable to pick up the solution of the leader-follower game as the starting point of the bargaining problem. Then the Nash solution can be easily calculated to be $(U(c^*, \alpha(c^*)) + \frac{S(\alpha^*) - S(\alpha(c^*))}{2}, V(c^*, \alpha(c^*)) + \frac{S(\alpha^*) - S(\alpha(c^*))}{2})$. Comparing the Nash solution with the Nash equilibrium, we can see both the user and the ISP are better off with an increase of $\frac{S(\alpha^*) - S(\alpha(c^*))}{2}$. That is to say, a better result is obtained for both parties by cooperation.

To illustrate the idea, we provide a numerical example.

Example 1. Assume the bandwidth is $\mu = 1000$ packets per second; for simplicity, the arrival rate of user requests is also $\lambda = 1000$ packets per second. Then α is just the traffic intensity and it may vary from 0 to 1. $f(s) = e^{-s}$; $g(s, s') = 10e^{-s}[u(s') - u(s' - s)]$ where $u(s')$ is the *unit step* function. Then by (4) and (5), we have $G(s) = 10e^{-s}(1 - e^{-(\mu - \lambda\alpha)s})$ and $\alpha(s)$ can be calculated by (8) and (9).

Now for the leader-follower game, given different price c , the user can respond with different α , leading to different curves in the utility space. Seven such curves are plotted in Figure 1, with c increasing from 1 to 7. For any value of c , the user chooses an α to maximize her utility, which are indicated by the dashed

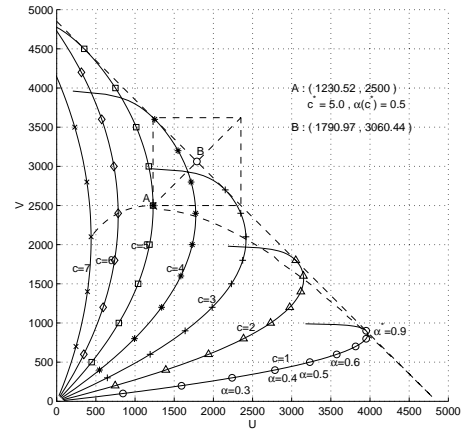


Figure 1: Optimal policies for the leader-follower and cooperative game - Nash equilibrium and Nash solution.

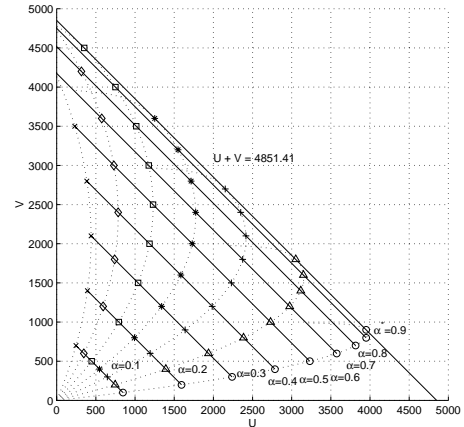


Figure 2: Linear Pareto boundary.

curve. Knowing this, the ISP decides a price to maximize her profit, thus reaching a Nash equilibrium denoted as point A. At this point, the optimal strategies are $c^* = 5.0$ and $\alpha(c^*) = 0.5$ leading to the utilities $(U, V) = (1230.52, 2500)$. The social welfare is $S(\alpha(c^*)) = 3730.52$ but it is far away from the best. As we know, S is totally determined by α which corresponds to a straight line $U + V = S = \text{constant}$ in the utility space, see Figure 2. It reaches the maximum $S(\alpha^*) = 4851.41$ on the Pareto boundary when $\alpha^* = 0.9$. For cooperative game, the solution point is picked up from this boundary. With A to be the starting point, the Nash solution is indicated as point B in Figure 1 with $(U, V) = (1790.97, 3060.44)$. Both parties are better off with 560.45 by cooperation.

4 Comparison with PMP

In this section, we extend the QoS model in section 3 to PMP. Without loss of generality, we study the case where the total bandwidth μ is divided into two parts: μ_1 and $\mu_2 = \mu - \mu_1$. The user's property remains un-

changed, except that when a request arrives with a required response time s , she may choose to submit it only to the first class with a probability of $\alpha(s)$, or only to the second class with $\beta(s)$, or to both classes with $\gamma(s)$, or discard it with $1 - \alpha(s) - \beta(s) - \gamma(s)$. It is reasonable to consider the case where the request is submitted to both classes since that will reduce the average response time which we can see from the below analysis. Thus, two M/M/1 queues are formed, one with arrival rate $\lambda\alpha$ and service rate μ_1 while the other with arrival rate $\lambda\beta$ and service rate μ_2 , where

$$\alpha = \int_0^\infty (\alpha(s) + \gamma(s))f(s)ds, \quad (13)$$

$$\beta = \int_0^\infty (\beta(s) + \gamma(s))f(s)ds. \quad (14)$$

Same as in section 3, the density distribution functions of the service time for the two queues are

$$p_1(s') = (\mu_1 - \lambda\alpha)e^{-(\mu_1 - \lambda\alpha)s'}, \quad (15)$$

$$p_2(s') = (\mu_2 - \lambda\beta)e^{-(\mu_2 - \lambda\beta)s'}. \quad (16)$$

Now, if the request is submitted only to the first queue, the user's gain is

$$G_1(s) = \int_0^\infty g(s, s')p_1(s')ds'. \quad (17)$$

If the request is submitted only to the second queue, the user's gain is

$$G_2(s) = \int_0^\infty g(s, s')p_2(s')ds'. \quad (18)$$

For the third case where the request is submitted to both queues, the response time is the minimum of the two. Since the service times for the two queues are exponentially distributed with rate $\mu_1 - \lambda\alpha$ and $\mu_2 - \lambda\beta$ respectively, it is easy to verify that the response time for this request is also exponentially distributed, but with rate $(\mu_1 - \lambda\alpha) + (\mu_2 - \lambda\beta) = \mu - \lambda\alpha - \lambda\beta$. Therefore, the user's gain is

$$G_{12}(s) = \int_0^\infty g(s, s')p_{12}(s')ds', \quad (19)$$

where

$$p_{12}(s') = (\mu - \lambda\alpha - \lambda\beta)e^{-(\mu - \lambda\alpha - \lambda\beta)s'}. \quad (20)$$

For the last case where the request is discarded, the user's gain is 0. Finally, the user's utility is calculated to be

$$U = \lambda \int_0^\infty (G_1(s)\alpha(s) + G_2(s)\beta(s) + G_{12}(s)\gamma(s))f(s)ds - \lambda\alpha c_1 - \lambda\beta c_2, \quad (21)$$

where c_1 and c_2 are the prices charged by the ISP for each request submitted to class 1 and class 2 respectively. The ISP's revenue is

$$V = \lambda\alpha c_1 + \lambda\beta c_2. \quad (22)$$

Same as the analysis in section 3, the ISP chooses her strategy $\vec{c} = (c_1, c_2)$ and the user chooses her strategy $\vec{r} = (\alpha, \beta)$. Then the utilities of the two parties can be denoted as $U(\vec{c}, \vec{r})$ and $V(\vec{c}, \vec{r})$. $\alpha(s)$ and $\beta(s)$ depend

on α and β . They are chosen by the user in such a way that the user's utility is maximized and so is the social welfare $S = U + V$,

$$S = \lambda \int_0^\infty (G_1(s)\alpha(s) + G_2(s)\beta(s) + G_{12}(s)\gamma(s))f(s)ds. \quad (23)$$

However, explicit equations can not be easily obtained. Thus, we need to resort to numerical approaches to solve this optimization problem. By observing that the objective function (23) and the constraints (13) and (14) are all integrals, we divide $[0, \infty)$ into small intervals and approximate the integrals by summations. Let Δ be a small positive number and N be a large integer such that the expected gains for $s > N\Delta$ is negligible. For $i = 1, 2, \dots, N$, define $s_i = i\Delta$, $a_i = G_1(s_i)f(s_i)$, $b_i = G_2(s_i)f(s_i)$, $c_i = G_{12}(s_i)f(s_i)$, $d_i = f(s_i)$ and $x_i = \alpha(s_i)$, $y_i = \beta(s_i)$, $z_i = \gamma(s_i)$. Then the optimization becomes a linear programming problem : to maximize

$$S = \lambda \sum_{i=1}^N (a_i x_i + b_i y_i + c_i z_i) \Delta \quad (24)$$

subject to the constraints

$$x_i \geq 0, y_i \geq 0, z_i \geq 0, \quad i = 1, 2, \dots, N, \quad (25)$$

$$x_i + y_i + z_i \leq 1, \quad i = 1, 2, \dots, N, \quad (26)$$

$$\sum_{i=1}^N d_i (x_i + z_i) \Delta = \alpha, \quad (27)$$

$$\sum_{i=1}^N d_i (y_i + z_i) \Delta = \beta. \quad (28)$$

This linear programming problem can be solved by the simplex method [10].

The social welfare S can be denoted as $S(\vec{r})$ since it is not relevant to \vec{c} . The linear Pareto boundary of the utility set is $U + V = S(\vec{r}^*)$, on which $S(\vec{r})$ reach the maximum $S(\vec{r}^*)$ and the solution point for cooperative game is picked up. On the other hand, the solution for the leader-follower game is the Nash Equilibrium $(U(\vec{c}^*, \vec{r}(\vec{c}^*)), V(\vec{c}^*, \vec{r}(\vec{c}^*)))$, where

$$\vec{r}(\vec{c}) = \arg[\max_{\vec{r} \in R} U(\vec{c}, \vec{r})], \quad (29)$$

$$\vec{c}^* = \arg[\max_{\vec{c} \in C} V(\vec{c}, \vec{r}(\vec{c}))]. \quad (30)$$

Normally, $\vec{r}^* \neq \vec{r}(\vec{c}^*)$, thus $S(\vec{r}^*) > S(\vec{r}(\vec{c}^*))$. Therefore, both parties can be better off by cooperation.

In the following, we will investigate whether the solutions for cooperative game can be improved by comparing the linear boundary of the utility set using PMP with that not using PMP. Set up a mapping between the two utility sets. In fact, for any point $(U(\vec{c}, \vec{r}), V(\vec{c}, \vec{r}))$ in the former utility set with $\vec{r} = (\alpha, \beta)$, we can find a corresponding point $(U(c, r), V(c, r))$ in the latter with $r = \alpha + \beta$. Rewrite (3), (4), (5) and (10),

$$\alpha + \beta = \int_0^\infty \alpha_0(s) f(s) ds, \quad (31)$$

$$p(s') = (\mu - \lambda\alpha - \lambda\beta) e^{-(\mu - \lambda\alpha - \lambda\beta)s'}, \quad (32)$$

$$G(s) = \int_0^\infty g(s, s')p(s')ds'. \quad (33)$$

$$S_0(r) = \lambda \int_0^\infty G(s)\alpha_0(s)f(s)ds. \quad (34)$$

Here to avoid confusion, substitute $\alpha_0(s)$ for $\alpha(s)$ and S_0 for S . Since $p(s') = p_{12}(s')$, $G(s) = G_{12}(s)$. Compare $p(s')$ with $p_1(s')$ and $p_2(s')$. It has a rate of $\mu - \lambda\alpha - \lambda\beta$ which is the summation of and thus larger than both $\mu_1 - \lambda\alpha$ and $\mu_2 - \lambda\beta$. Therefore, $p(s')$ is more concentrated on small values of s' . That reflects better service. As mentioned in the beginning of section 3, the gain function $g(s, s')$ is non-increasing with respect to s' . We know quickly $G(s) \geq G_1(s)$ and $G(s) \geq G_2(s)$. From (23), $S(\vec{r}) \leq \lambda \int_0^\infty G(s)(\alpha(s) + \beta(s) + \gamma(s))f(s)ds$. From the fact that $\alpha_0(s)$ has maximized $S_0(r)$, $\alpha(s) + \beta(s) + \gamma(s) \leq 1$ and $\int_0^\infty (\alpha(s) + \beta(s) + \gamma(s))f(s)ds \leq \alpha + \beta$, we have $S(\vec{r}) \leq S_0(r)$. Remember the linear boundary of the utility set using PMP is the straight line $U + V = S(\vec{r}^*)$ while the linear boundary not using PMP is $U + V = S_0(r^*)$. Assume $\vec{r}^* = (\alpha^*, \beta^*)$, then $S(\vec{r}^*) = S((\alpha^*, \beta^*)) \leq S_0(\alpha^* + \beta^*) \leq S_0(r^*)$. That is, the linear boundary using PMP is below the one not using PMP. In other words, for a cooperative game solution attained by using PMP which is picked up on the linear boundary, we can always find some better solutions on the linear boundary not using PMP. Thus, PMP doesn't achieve improvement for cooperative game. The intuition for the reason is that it may lose some advantages of statistical multiplexing from not aggregating all the traffic. The following example illustrates the idea in this section.

Example 2. Same as in example 1, $\lambda = \mu = 1000$ packets per second; $f(s) = e^{-s}$; $g(s, s') = 10e^{-s}[u(s') - u(s' - s)]$. But here μ is divided into two parts: μ_1 and $\mu_2 = \mu - \mu_1$. For this example, let $\mu_1 = \mu_2 = 500$. First, apply the linear programming (24) to (28) to determine the social welfare $S(\vec{r})$. Choose $\Delta = 0.0069$ and $N = 1000$. With these values the gains for $s > N\Delta$ are less than 0.1% of the maximal gain. Plot the social welfare S in Figure 3, from which we see $S(\vec{r})$ reaches the maximum $S(\vec{r}^*) = 4839.76$ when $\vec{r}^* = (\alpha^*, \beta^*) = (0.41, 0.49)$ (or $(0.49, 0.41)$ since $\mu_1 = \mu_2$). Remember in section 3, the linear boundary is $U + V = 4851.41$ which is better. Thus, PMP doesn't achieve improvement for linear boundary, or consequently, for cooperative game.

In the leader-follower game, after the ISP chooses some prices $\vec{c} = (c_1, c_2)$, the user must choose $\vec{r}(\vec{c})$ which can be calculated by (29) to maximize her own utility U . Likewise, given different prices (c_1, c_2) , the user will react accordingly. See Table 1 and 2. The the ISP must choose the strategy which will make herself earn the most. In the end, the Nash equilibrium is calculated to be $(U, V) = (1228.69, 2500)$ when $\vec{c}^* = (5.0, 5.0)$ by (30) and $\vec{r}(\vec{c}^*) = (0.08, 0.42)$ (or $\vec{r}(\vec{c}^*) = (0.42, 0.08)$). The social welfare is only 3728.69 which is much smaller than 4839.76 that can be achieved by cooperative game. The result also indicates that the leader-follower game

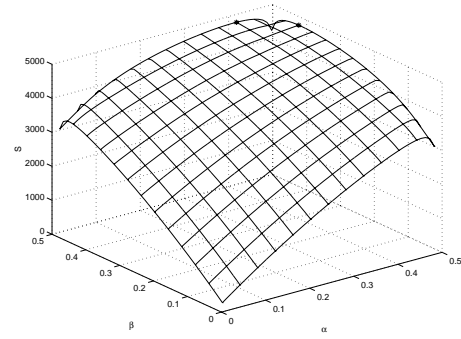


Figure 3: The social welfare vs the user's strategy.

may lead to an optimal solution with the same price for both classes. This contradicts to the intention of the original design of the scheme.

c_1 c_2	3.0	4.0	5.0	6.0	7.0
3.0	2414.10	2240.36	2135.32	2074.85	2035.55
4.0	2240.36	1772.90	1673.80	1620.55	1586.66
5.0	2135.32	1673.80	1228.69	1181.53	1156.84
6.0	2074.85	1620.55	1181.53	783.09	760.17
7.0	2035.55	1586.66	1156.84	760.17	436.86

Table 1: The user's utility U under optimal user's strategy at different prices.

c_1 c_2	3.0	4.0	5.0	6.0	7.0
3.0	2070	1930	1810	1680	1560
4.0	1930	2360	2190	2040	1970
5.0	1810	2190	2500	2280	2240
6.0	1680	2040	2280	2400	2360
7.0	1560	1970	2240	2360	2100

Table 2: The ISP's revenue V under optimal user's strategy at different prices.

5 Pricing With Priority

In this section, we investigate pricing with priority. Some other related works include [12] and [13]. Here, to avoid loss of statistical multiplexing gain, we divide the user's requests into two classes - class H with higher priority and class L with lower priority - but let them share the total bandwidth in a preemptive way. That is, if a class H request arrives when a class L request is being served, then the class H request gets served immediately. The suspended service for the class L request is resumed later once there is no class H request to be served. The preemptive way is not the only manner to realize priorities [11], but this assumption will not impair the generality of our conclusion as well as will simplify the analysis.

With total arrival rate λ and service rate μ , the user's requests can be submitted to class H with a probability of $\alpha_H(s)$ or submitted to class L with $\alpha_L(s)$ or discarded with $1 - \alpha_H(s) - \alpha_L(s)$. There's no point to be submitted to both classes. Now for class H requests, the arrival rate is $\lambda\alpha_H$, where,

$$\alpha_H = \int_0^\infty \alpha_H(s)f(s)ds. \quad (35)$$

From the preemptive assumption, the system just works like an M/M/1 queue with arrival rate $\lambda\alpha_H$ and service rate μ , thus

$$p_H(s') = (\mu - \lambda\alpha_H)e^{-(\mu - \lambda\alpha_H)s'}, \quad (36)$$

$$G_H(s) = \int_0^\infty g(s, s')p_H(s')ds'. \quad (37)$$

For class L requests,

$$\alpha_L = \int_0^\infty \alpha_L(s)f(s)ds, \quad (38)$$

$$G_L(s) = \int_0^\infty g(s, s')p_L(s')ds'. \quad (39)$$

Explicit form of $p_L(s')$ is not easy to be written out. Overall, the social welfare is

$$S = \lambda \int_0^\infty (G_H(s)\alpha_H(s) + G_L(s)\alpha_L(s))f(s)ds. \quad (40)$$

If the ISP charges c_H for each class H request and c_L for each class L request ($c_H > c_L$), then the utilities of the two parties become

$$U = \lambda \int_0^\infty (G_H(s)\alpha_H(s) + G_L(s)\alpha_L(s))f(s)ds - \lambda\alpha_H c_H - \lambda\alpha_L c_L, \quad (41)$$

$$V = \lambda\alpha_H c_H + \lambda\alpha_L c_L. \quad (42)$$

Denote the social welfare here as $S(\vec{r})$ where $\vec{r} = (\alpha_H, \alpha_L)$ and that in section 1 as $S_0(r)$ where $r = \alpha$ to avoid confusion. When $r = \alpha^*$, $S_0(r)$ reaches the maximum $S_0(\alpha^*)$. Now let $\vec{r} = (\alpha^*, \alpha_L)$, then $p_H(s') = p(s')$ and $G_H(s) = G(s)$. By comparing (40) with (10), we have $S((\alpha^*, \alpha_L)) \geq S_0(\alpha^*)$. Since the maximum $S(\vec{r})$ is $S(\vec{r}^*) \geq S((\alpha^*, \alpha_L)) \geq S_0(\alpha^*)$, this scheme improves the linear boundary of the utility set. The intuition is without degrading the service for requests with higher priority, it also fully takes the advantage of statistical multiplexing by serving requests with lower priority in between the higher priority services.

6 Conclusion and Discussion

In this paper, we derived some further results for Internet pricing with a game theoretical framework. In aid of the concept of social welfare, we explained clearly the linear property of the Pareto boundary and proved that both parties can be better off by cooperation. Different pricing schemes were studied by this approach and the results showed that PMP does not provide a better solution based on the cooperative game while the pricing with priority scheme may achieve so. In

particular, detailed quantitative analysis was provided for PMP. The numerical examples indicated that the leader-follower game leads to an optimal solution with both parts charging the same price. This contradicts to the intention of the original design of the PMP scheme.

Many of our results are based on numerical solutions, because the analytical results are hard to obtain. More examples are needed to explore the issue. And the work should also be extended to multiple ISPs and multiple users. Nevertheless, the results provide some insights and the approach proposed here can be applied to other Internet pricing schemes.

References

- [1] X.-R. Cao, R. Miletto, and P. Wirth, "Internet Pricing with a Game Theoretical Approach: Concepts and Examples", submitted.
- [2] A. Odlyzko, "Paris Metro Pricing: the Minimalist Differentiated Services Solution", *Proc. 1999 Seventh International Workshop on Quality of Service (IWQoS '99)*, IEEE, pp. 159-161, 1999.
- [3] A. Odlyzko, "A Modest Proposal for Preventing Internet Congestion", AT & T labs, Sept. 1997, <http://www.research.att.com/~amo>.
- [4] X.-R. Cao, "Preference Functions and Bargaining Solutions", *Proceedings of IEEE Conference on Decision and Control*, pp. 164-171, 1982.
- [5] S. Shenker, D. Clark, D. Estrin, and S. Herzog, "Pricing in Computer Networks: Reshaping the Research Agenda", *Telecommunication Policy*, Vol. 20, No. 3, pp. 183-201, 1996.
- [6] L.W. McKnight and J.P. Bailey (ed), *Internet Economics*, The MIT Press, Cambridge Massachusetts, 1995.
- [7] J.K. MacKie-Mason and H.R. Varian, "Pricing the Internet", *Public Access to the Internet*, pp. 269-314, the MIT press, 1995.
- [8] L. Kleinrock, *Queueing Systems, Volume I: Theory*, John Wiley, New York, 1975.
- [9] A. Gupta, D.O. Stahl and A.B. Whinston, "A Stochastic Equilibrium Model of Internet Pricing", *Journal of Economic Dynamics & Control*, vol. 21, no. 4-5, pp. 697-722, May 1997.
- [10] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, Second Edition, pp. 430-443, Cambridge University Press, NY, 1992.
- [11] P. Ferguson and G. Huston, *Quality of Service: Delivering QoS on the Internet and in Corporate Networks*, Wiley, New York, 1998.
- [12] R. Cocchi, D. Estrin, S. Shenker and L. Zhang, "A Study of Priority Pricing in Multiple Service Class Networks", *Proceedings of SIGCOMM 91*, pp. 123-130, 1991.
- [13] H. Mendelson and S. Whang, "Optimal Incentive-Compatible Priority Pricing for the M/M/1 Queue", *Operations Research*, vol. 38, no. 5, pp. 870-83, Sept.-Oct. 1990.