LMI-Based Reliable Robust Tracking Control Against Actuator Faults with Application to Flight Control[1]

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Abstract

In this paper, we consider the reliable robust tracking controller design problem against actuator faults and control surface damages for a LTI system with input disturbance. First, models of actuator faults and control surface damages are presented. Then a reliable tracking controller design method is developed, which guarantees the closed-loop system stability, optimizes the tracking performance of the system in normal operations, and maintains an acceptable low-level tracking performance of the system in the event of actuator faults and/or control surface damages. This method is based on LQ/H_∞ tracking performance indices and multi-objective optimization in terms of linear matrix inequalities. A numerical example of an F-16 aircraft model and its simulation results are given.

Keywords: reliable control; flight control; tracking; LMI; actuator faults; control surface damages.

1 Introduction

In the design of either classical or modern control systems, it is often presumed that all system actuators and sensors are in good working conditions. As a result, a majority of control systems designed using conventional techniques may not be able to maintain a satisfactory performance in the presence of sensor/actuator faults or control surface damages, especially for aircraft. In some cases, even the closed-loop system stability can not be guaranteed. However, there are some inherent redundancies in multi-input and multi-output systems, which often makes it possible to design controllers such that faults in some system components do not cause immediate threats to the safety of the overall system. Such controller design methodologies are particularly important in safety-critical systems, such as air vehicles *etc.*[11].

A control system designed to tolerate faults of sensors or actuators, while maintaining an acceptable level of the closed-loop system stability/performance, is called a *reliable* control system[8]. Presently there are two main methodologies that can improve system reliability. The first one relies solely on the existing system redundancies to achieve a tolerable performance degradation in the event of component faults, i.e. robustness against component faults. In this type of systems, once a controller is designed, it remains fixed. The second one involves such procedures as real-time fault detection, isolation, and control system reconfiguration. Compared to the reconfigurable reliable controllers, one of the advantages of fixed reliable controllers is that it guarantees satisfactory system performance not only during normal operations but also under various component faults without spending time on fault detection, isolation, and controller reconfiguration. This is very important in the case that the available reaction time for the system is very little after the occurrence of a severe fault.

In recent years, there are many researchers working on robust tracking problems. For example, *Shaked et al.*[6] used a game theory approach to solve a tracking problem, *Takaba*[7] developed a design method for state feedback control with integral and preview actions to achieve the robust LQ tracking performance by using the LMI approach, and so on. However, there are very limited results for reliable tracking problems. A method based on robust pole region assignment techniques has been proposed by *Zhao, et al.*[10] to realize the reliable tracking control in the presence of actuator faults.

A problem to be solved in the reliable tracking problem is how to optimize the tracking performance during normal system operation, while guarantee an acceptable low-level tracking performance in the event of component faults. In this paper, we study the reliable robust tracking problem of linear time-invariant systems with input disturbance in the presence of actuator faults and/or control surface damages. The multi-objective optimization methodology is used to ensure the designed tracking controller not only guarantees the stability of the closed-loop system and optimal tracking performance during normal system operation, but also maintains an acceptable low-level tracking performance in the event of actuator faults and/or control surface damages.

This paper is organized as follows: Section 2 gives the fault models of actuators and control surfaces. Section 3 discusses the problem formulation. Section 4 presents the design method of the reliable robust tracking controller. In section 5, a numerical example is used to illustrate the design, followed by some concluding remarks in Section 6.

2 Fault Model

Consider a LTI aircraft model described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \quad x(0) = x_0 \\ y(t) = Cx(t) \end{cases} \tag{1}$$

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where $x(t) \in R^n$ is the state, $y(t) \in R^p$ is the output, $u(t) \in R^m$ is the nominal control input, and $w(t) \in R^h$ is the derivative-bounded input disturbance. To investigate the reliable control and tracking problem in the event of actuator faults or control surface damages, the fault model must be established first. Common actuator faults include outage and stuck, while the usual control surface fault is the control surface damage. Let $u^F(t)$ represent the control vector after failures have occurred, then the following actuator fault model is adopted for this study:

$$u^F(t)=\omega_L u(t)+\alpha_a\beta_a^F, \quad L=0,1,\cdots,l_p, \quad l_p\leq 2^m-1 \tag{2}$$

where $\beta_a^F=[\beta_{a1}^F,\cdots,\beta_{am}^F]^T$ with β_{aj}^F $(j=1,2,\cdots,m)$ being a positive bounded shift that represents the maximum outputs of the jth actuator in the presence of an actuator stuck fault. The scaling factors ω_L $(L=0,1,\cdots,l_p, l_p\leq 2^m-1)$ and α_a satisfy

$$\begin{array}{lll}\omega_L\in\Omega & \triangleq & \{\omega_L=diag[\omega_{L1},\omega_{L2},\cdots,\omega_{Lm}],\\ & & \quad \omega_{Lj}=0 \;\; or \;\; 1, \;\; j=1,2,\cdots,m\}\\ \alpha_a\in\Lambda & \triangleq & \{\alpha_a=diag[\alpha_{a1},\alpha_{a2},\cdots,\alpha_{am}],\\ & & \quad -1\leq\alpha_{aj}\leq 1, \;\; j=1,2,\cdots,m\}\end{array} \tag{3}$$

Define $\omega_0 = I$ when $L = 0$.

Remark 1: When $\omega_{Lj} = \alpha_{aj} = 0$, it corresponds to the case of the jth actuator outage. When $\omega_{Lj} = 0$ and $-1 \leq \alpha_{aj} \leq 1$, $\alpha_{aj} = 0$, it covers the case of the jth actuator stuck fault. When $\omega_{Lj} = 1$ and $\alpha_{aj} = 0$, it corresponds to the case of no fault in the jth actuator. Therefore when $\omega_L = I$ and $\alpha_a = 0$, $u^F(t) = u(t)$ represents the nominal control input vector.

Unlike actuator faults of the form (2), aircraft control surface damages will change the aerodynamic characteristics of the aircraft (i.e, A, B and G matrices). Such faults can be modeled by polytopic uncertainties in the matrices A, B and G as follows

$$\bar{A}(\theta,i)=A\theta_1+\bar{A}_i\theta_2,\ \bar{B}(\theta,i)=B\theta_1+\bar{B}_i\theta_2,\ \bar{G}(\theta,i)=G\theta_1+\bar{G}_i\theta_2 \tag{4}$$

where for $i = 0$ (corresponding to the nominal case), $\bar{A}_0 = A$, $\bar{B}_0 = B$, $\bar{G}_0 = G$, and the uncertain constant parameter vector $\theta = [\theta_1 \;\; \theta_2]^T \in R^2$ satisfies

$$\theta\in\Theta \triangleq \{\theta\in R^2 : \theta_1\geq 0, \;\; \theta_2\geq 0 \;\; and \;\; \theta_1+\theta_2=1\} \tag{5}$$

The matrices $\bar{A}_i$, $\bar{B}_i$ and $\bar{G}_i$ $(i = 1, 2, \cdots, q)$ are known constant matrices of appropriate dimensions, which represent the vertices of possible control surface damages.

Hence, the aircraft dynamics with both actuator faults (2) and control surface damages (4) is described by

$$\left\{\begin{array}{l}\dot{x}(t)=\bar{A}(\theta,i)x(t)+\bar{B}(\theta,i)u^F(t)+\bar{G}(\theta,i)w(t)\\ y(t)=Cx(t), \;\; x(0)=x_0, \qquad i=0,1,\cdots,q\end{array}\right. \tag{6}$$

Remark 2: It is noted that, in the faulty system description (4) - (6), when $i = 0$, we have $\bar{A}(\theta, 0) = A$, $\bar{B}(\theta, 0) = B$ and $\bar{G}(\theta, 0) = G$. When $q \geq 1$, the index $i = 1, 2, \cdots, q$ correspond to the models with control surface damage faults. The parameter θ provides interpolation between nominal and the fault cases, hence providing approximate models for intermediate faults.

3 Problem Formulation

Consider the aircraft dynamics (6) with both actuator faults (2) and control surface damage faults (4). The design problem considered in this paper is to find a controller such that:

- The closed-loop system is robustly stable for all $\omega_L \in \Omega$, $\theta \in \Theta$, $L = 0, 1, \cdots, l_p$ and $i = 0, 1, \cdots, q$.
- In the normal operation, the output $Sy(t)$ tracks the reference signal $r(t)$ without steady-state error, that is,
$$\lim_{t\to\infty} e(t) = 0, \qquad e(t) = r(t) - Sy(t) \tag{7}$$
and with optimal closed-loop performance. $S \in R^{l\times p}$ is a known constant matrix used to form the output required to track the reference signals.
- In the event of actuator faults or control surface damages, the output $Sy(t)$ tracks the reference signal $r(t)$ without steady-state error and with an acceptable low-level performance.

It is well known that the tracking error integral action of a controller can effectively eliminate the steady-state tracking error. In order to obtain a robust tracking controller with state feedback plus tracking error integral, we introduce the following augmented state-space description of the system (6).

$$\begin{bmatrix}\dot{e}(t)\\ \frac{d\dot{x}(t)}{dt}\end{bmatrix}=\begin{bmatrix}0 & -SC\\ 0 & \bar{A}(\theta,i)\end{bmatrix}\begin{bmatrix}e(t)\\ \dot{x}(t)\end{bmatrix}+\begin{bmatrix}0\\ \bar{B}(\theta,i)\end{bmatrix}\dot{u}^F(t)+\begin{bmatrix}I & 0\\ 0 & \bar{G}(\theta,i)\end{bmatrix}\begin{bmatrix}\dot{r}(t)\\ \dot{w}(t)\end{bmatrix} \quad i=0,1,\cdots,q \tag{8}$$

Define the augmented state vector $\zeta(t)=[e^T(t) \;\; \dot{x}^T(t)]^T$ and disturbance vector $v(t)=[\dot{r}^T(t) \;\; \dot{w}^T(t)]^T$. The augmented system (8) can be rewritten as

$$\dot{\zeta}(t) = A_F(\theta,i)\zeta(t) + B_F(\theta,i)\dot{u}^F(t) + G_F(\theta,i)v(t) \quad i=0,1,\cdots,q \tag{9}$$

where

$$\begin{array}{l}A_F(\theta,i)=\begin{bmatrix}0 & -SC\\ 0 & \bar{A}(\theta,i)\end{bmatrix}\in R^{(l+n)\times(l+n)}\\ B_F(\theta,i)=\begin{bmatrix}0\\ \bar{B}(\theta,i)\end{bmatrix}\in R^{(l+n)\times m}\\ G_F(\theta,i)=\begin{bmatrix}I & 0\\ 0 & \bar{G}(\theta,i)\end{bmatrix}\in R^{(l+n)\times(l+h)}\end{array} \tag{10}$$

It is easy to see from (4) and (10) that this augmented faulty system can be expressed in the polytopic form as follows

$$\begin{array}{l}A_F(\theta,i)=A_o\theta_1+A_{Fi}\theta_2\\ B_F(\theta,i)=B_o\theta_1+B_{Fi}\theta_2\\ G_F(\theta,i)=G_o\theta_1+G_{Fi}\theta_2\end{array} \tag{11}$$

where

$$\begin{array}{lll}A_o=\begin{bmatrix}0 & -SC\\ 0 & A\end{bmatrix}, & B_o=\begin{bmatrix}0\\ B\end{bmatrix}, & G_o=\begin{bmatrix}I & 0\\ 0 & G\end{bmatrix}\\ A_{Fi}=\begin{bmatrix}0 & -SC\\ 0 & \bar{A}_i\end{bmatrix}, & B_{Fi}=\begin{bmatrix}0\\ \bar{B}_i\end{bmatrix}, & G_{Fi}=\begin{bmatrix}I & 0\\ 0 & \bar{G}_i\end{bmatrix}\end{array} \tag{12}$$

Since $\dot{u}^F(t) = \omega_L\dot{u}(t)$ according to (2), the augmented faulty system can be rewritten as

$$\dot{\zeta}(t) = A_F(\theta,i)\zeta(t) + B_F(\theta,i)\omega_L\dot{u}(t) + G_F(\theta,i)v(t) \quad L=0,1,\cdots,l_p, \quad i=0,1,\cdots,q \tag{13}$$

Remark 3: **(a)** Since $\bar{A}_0 = A$, $\bar{B}_0 = B$, $\bar{G}_0 = G$, it is easy to see $A_{F0} = A_o$, $B_{F0} = B_o$, $G_{F0} = G_o$ in (11). **(b)** It is obvious that if we obtain a controller robustly stabilizing the augmented system (13), the controller also stabilizes the original system (6) and achieves the output regulation $\lim_{t\to\infty} e(t) = 0$.

In the following design, we choose the performance indices of normal and fault cases as

$$\begin{array}{c} V_{Li} = \int_0^t [e^T(t)Qe(t) + \dot{u}^T(t)\omega_L R\omega_L \dot{u}(t)]dt \\ L = 0,1,\cdots,l_p, \quad i = 0,1,\cdots,q \end{array} \tag{14}$$

where $Q \in R^{l\times l}$ is symmetric positive semi-definite and $R \in R^{m\times m}$ is symmetric positive definite. Our objective is to minimize the performance index V_{00} of the nominal system (1), and simultaneously guarantee the performance indices $V_{Li} \leq \varphi_{Li}$ $(i = 0,1,\cdots,q,\ \ L = 0,1,\cdots,l_p,\ \ i + L = 0)$ for each of the probable faults.

4 Reliable Robust Tracking Controller Design

Before presenting the design, the following two hypotheses are made:

[H1] $(\bar{A}_i,\ \bar{B}_i\omega_L)$ is stabilizable for $i = 0,1,\cdots,q,\ L = 0,1,\cdots,l_p$.

[H2] $\mathrm{rank}\begin{bmatrix} \bar{A}_i & \bar{B}_i\omega_L \\ C & 0 \end{bmatrix} = n + p,\ i = 0,1,\cdots,q,\ L = 0,1,\cdots,l_p$.

Remark 4: Hypotheses [H1] and [H2] guarantee that the augmented system (13) corresponding to the normal operation and each of the probable faults is stabilizable via state feedback.

Theorem 1 *Consider the closed-loop system of the augmented faulty system (13) with the state-feedback controller $\dot{u}(t) = K\zeta(t)$. Assume that the hypotheses [H1] and [H2] hold, and for $t < 0$, $x(t) = 0$, $u(t) = 0$, and $r(t) = 0$. For every given positive constant γ_{Li} $(L = 0,1,\cdots,l_p$, $i = 0,1,\cdots,q)$, suppose that there exist a matrix $Z_{Li} \in R^{m\times(l+n)}$ and a symmetric matrix $X_{Li} \in R^{(l+n)\times(l+n)}$ satisfying, for all $i = 0,1,\cdots,q$ and $L = 0,1,\cdots,l_p$,*

$$X_{Li} > 0$$
$$\begin{bmatrix} \begin{array}{c} X_{Li}A_{Fi}^T + A_{Fi}X_{Li} \\ +Z_{Li}^T\omega_L B_{Fi}^T + B_{Fi}\omega_L Z_{Li}\end{array} & G_{Fi} & X_{Li}Q_a^{\frac{1}{2}} & Z_{Li}^T\omega_L R^{\frac{1}{2}} \\ G_{Fi}^T & -\gamma_{Li}I & 0 & 0 \\ Q_a^{\frac{1}{2}}X_{Li} & 0 & -I & 0 \\ R^{\frac{1}{2}}\omega_L Z_{Li} & 0 & 0 & -I \end{bmatrix} < 0 \tag{15}$$

where

$$Q_a = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

Then, the following controller robustly stabilizes the closed-loop system.

$$u(t) = K_e \int_0^t e(t)dt + K_x x(t) \tag{17}$$

$$K = [K_e \quad K_x] = Z_{Li}X_{Li}^{-1} \tag{18}$$

Furthermore, upper bounds of performance indices V_{Li} are given by

$$\begin{array}{c} V_{Li} \leq \zeta(0)^T X_{Li}^{-1}\zeta(0) + \gamma_{Li}\int_0^t v^T(t)v(t)dt, \\ \forall\theta \in \Theta,\ \ i = 0,1,\cdots,q,\ \ \omega_L \in \Omega,\ \ L = 0,1,\cdots,l_p \end{array} \tag{19}$$

Proof: Substituting $\dot{u}(t) = K\zeta(t)$ into (13) and (14) yield, for all $L = 0,1,\cdots,l_p$, $i = 0,1,\cdots,q$

$$\dot{\zeta}(t) = [A_F(\theta,i) + B_F(\theta,i)\omega_L K]\zeta(t) + G_F(\theta,i)v(t) \tag{20}$$

$$V_{Li} = \int_0^t \zeta^T(t)[Q_a + K^T\omega_L R\omega_L K]\zeta(t)dt \tag{21}$$

By the Schur complement formula, (15) is equivalent to

$$\begin{array}{c} X_{Li} > 0, \\ X_{Li}A_{Fi}^T + A_{Fi}X_{Li} + X_{Li}K^T\omega_L B_{Fi}^T + B_{Fi}\omega_L KX_{Li} \\ + \frac{1}{\gamma_{Li}}G_{Fi}G_{Fi}^T + X_{Li}Q_aX_{Li} + X_{Li}K^T\omega_L R\omega_L KX_{Li} < 0 \end{array} \tag{22}$$

Post and pre multiplying the inequality (22) by $P_{Li} = X_{Li}^{-1}$, we obtain $P_{Li} > 0$ and

$$\begin{array}{c} [A_{Fi} + B_{Fi}\omega_L K]^T P_{Li} + P_{Li}[A_{Fi} + B_{Fi}\omega_L K] \\ + \frac{1}{\gamma_{Li}}P_{Li}G_{Fi}G_{Fi}^T P_{Li} + Q_a + K^T\omega_L R\omega_L K < 0 \end{array} \tag{23}$$

Since $\gamma_{Li} > 0$, $Q_a \geq 0$, $Q_a = Q_a^T$ and $R > 0$, $R = R^T$, then

$$[A_{Fi} + B_{Fi}\omega_L K]^T P_{Li} + P_{Li}[A_{Fi} + B_{Fi}\omega_L K] < 0 \tag{24}$$

According Lyapunov stability theorem, controller $\dot{u}(t) = K\zeta(t)$ which satisfies (15) robustly stabilizes the augmented system (13). Furthermore,

$$\begin{aligned} V_{Li} \leq & -\int_0^t \zeta^T(t)\left\{[A_{Fi} + B_{Fi}\omega_L K]^T P_{Li} + P_{Li}[A_{Fi}\right. \\ & \left. + B_{Fi}\omega_L K] + \frac{1}{\gamma_{Li}}P_{Li}G_{Fi}G_{Fi}^T P_{Li}\right\}\zeta(t)dt \\ \leq & -\int_0^t d[\zeta^T(t)P_{Li}\zeta(t)] + \gamma_{Li}\int_0^t v^T(t)v(t)dt \\ \leq & \ \zeta^T(0)P_{Li}\zeta(0) + \gamma_{Li}\int_0^t v^T(t)v(t)dt \end{aligned} \tag{25}$$

Further, partitioning K as in (18) , and by using the assumptions that for $t < 0$, $x(t) = 0$, $u(t) = 0$ and $r(t) = 0$, we obtain (17). This completes the proof. □

Remark 5: The upper bounds of the above performance indices consist of two parts. The first part corresponds to the upper bounds of the performance indices under the condition of zero-input. The second part corresponds to the upper bounds of the performance indices under the condition of zero-initial state. The performance parameter γ_{Li} $(L = 0,1,\cdots,l_p,\ i = 0,1,\cdots,q)$ corresponds to the H_∞ norm $||T_{zv}||_\infty$ of the transfer function from the input $v(t)$ in (20) to the performance output $z(t) = Q_a^{\frac{1}{2}}\zeta(t) + \begin{bmatrix} 0 & (R^{\frac{1}{2}})^T \end{bmatrix}^T \omega_L\dot{u}(t)$. It is noted that the above Theorem 1 is a sufficient condition guaranteeing the close-loop system (20) is robustly stable and the quadratic performance indices V_{Li} $(L = 0,1,\cdots,l_p,\ i = 0,1,\cdots,q)$ have the upper bounds.

Now the reliable tracker design problem can be rewritten as $J := Minimize[Tr(Y)]$ subject to (15) and

$$\begin{bmatrix} Y & I \\ I & X_{00} \end{bmatrix} > 0 \tag{26}$$

In other words, a common controller K must ensure the above performance criteria. Namely, $K = Z_{Li}X_{Li}^{-1}$. Due to this constraint, the above reliable tracker design problem is not jointly convex. In order to get around this problem, a general method is to use a common LMI solution: $X_{00} = X_{Li}$, $i = 0, 1, \cdots, q$, $L = 0, 1, \cdots, l_p$. However, this method is at the cost of high conservativeness. To overcome the conservativeness of the common LMI solution, an iterative method is adopted here.

Theorem 2 *Consider the closed-loop system of the augmented systems (13) with the state-feedback controller $\dot{u}(t) = K\zeta(t)$. Assume that the hypotheses [H1] and [H2] hold, and for $t < 0$, $x(t) = 0$, $u(t) = 0$, $r(t) = 0$. For given positive constants γ_{Li} ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$) as well as given positive-definite matrices $P_{Li0} \in R^{(l+n)\times(l+n)}$, suppose that there exist a feedback gain matrix $K \in R^{m\times(l+n)}$ and symmetric matrices $P_{Li} \in R^{(l+n)\times(l+n)}$ satisfying, for all $i = 0, 1, \cdots, q$ and $L = 0, 1, \cdots, l_p$*

$$P_{Li} > 0$$

$$\begin{bmatrix} Z_{pLi} & P_{Li}G_{Fi} & (\omega_L K + R^{-1}B_{Fi}^T P_{Li})^T \\ G_{Fi}^T P_{Li} & -\gamma_{Li} I & 0 \\ \omega_L K + R^{-1}B_{Fi}^T P_{Li} & 0 & -R^{-1} \end{bmatrix} < 0 \quad (27)$$

with Q_a as in (16) and

$$Z_{pLi} = A_{Fi}^T P_{Li} + P_{Li}A_{Fi} + Q_a - P_{Li0}B_{Fi}R^{-1}B_{Fi}^T P_{Li} - P_{Li}B_{Fi}R^{-1}B_{Fi}^T P_{Li0} + P_{Li0}B_{Fi}R^{-1}B_{Fi}^T P_{Li0}$$

Then, the following controller robustly stabilizes the closed-loop system of the augmented system (13).

$$u(t) = K_e \int_0^t e(t)dt + K_x x(t) \quad (28)$$

$$K = [K_e \quad K_x] \quad (29)$$

Furthermore, the upper bounds of performance indices V_{Li} are given by

$$V_{Li} \le \zeta(0)^T P_{Li}\zeta(0) + \gamma_{Li}\int_0^t v^T(t)v(t)dt \quad \forall\theta \in \Theta,\ i = 0, 1, \cdots, q,\ \omega_L \in \Omega,\ L = 0, 1, \cdots, l_p \quad (30)$$

Proof: The proof is omitted due to lack of space. □

Remark 6: Theorem 2 converts non-convex problem into convex problem and its conservativeness can be minimized when P_{Li0} trends to P_{Li} ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$). It should be noted that the existence of a solution X_{Li} to (15) is equivalent to the existence of solutions P_{Li} and P_{Li0} to (27). Therefore, Theorem 2 does not introduce any additional conservativeness other than those in Theorem 1.

Theorem 2 leads to the following iterative algorithm to get an optimal tracking controller:

1. Choose proper γ_{Li} ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$), and minimize $trace(Y)$, subject to (15) and (26) with $X_{Li} = X_{00}$ and $Z_{Li} = Z_{00}$. Let $K_{opt} = Z_{00opt}X_{00opt}^{-1}$.
2. Respectively minimize $trace(X_{Li}^{-1})$ ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$) subject to the inequalities (15) with $K = K_{opt}$. Then we obtain $P_{Liopt} (= X_{Liopt}^{-1})$.
3. Choose a proper $\Delta > 0$ and $\rho_{Li} > 0$ ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$, $L + i = 0$). Let $P_{Li0} = P_{Liopt}$ ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$).
4. Minimize $trace(P_{00})$ subject to the inequalities (27) as well as $trace(P_{Li}) < \rho_{Li}$ ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$, $L + i = 0$). Then we obtain P_{Liopt}.
5. If $|trace(P_{Liopt} - P_{Li0})| > \Delta$, let $P_{Li0} = P_{Liopt}$ ($L = 0, 1, \cdots, l_p$, $i = 0, 1, \cdots, q$) and go to step 4. Otherwise, set $K = K_{opt}$ and stop.

5 Numerical Examples

In this section, an example of tracking control for a linear F-16 aircraft model is given to demonstrate the proposed methods. The trimmed values of F-16 aircraft equation are:

Thrust	T	=	0.1464,
Center-of-gravity location	X_{cg}	=	0.35,
Velocity	V_t	=	$152.4m/s$,
Altitude	H	=	$500m$,
Angle of attack	α	=	2.3703^o,
Angle of sideslip	β	=	0.0^o,
Horizontal stabilator deflection	δ_h	=	-0.5831^o,
Aileron deflection	δ_a	=	0.0^o,
Rudder deflection	δ_r	=	0.0^o.

After linearization and allowing the left/right control surfaces move independently, the aircraft model is described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ y(t) = Cx(t) \end{cases} \quad (31)$$

where $x(t) = [u\ w\ q\ v\ p\ r]^T$, $u(t) = [\delta_{hr}\ \delta_{hl}\ \delta_{ar}\ \delta_{al}\ \delta_r]^T$, $y(t) = [q\ \dot{\mu}_{rot}\ r_{stab}\ \alpha\ \beta]^T$, $w(t)$ represents the vertical gust disturbance. The numerical values of the matrices are given in the Appendix. The following possible actuator faults are considered:

1. Left elevator actuator outage corresponding to $\omega_1 = diag\{1, 0, 1, 1, 1\}$;
2. Right elevator actuator outage corresponding to $\omega_2 = diag\{0, 1, 1, 1, 1\}$;
3. Right aileron actuator outage corresponding to $\omega_3 = diag\{1, 1, 0, 1, 1\}$;
4. Left aileron actuator outage corresponding to $\omega_4 = diag\{1, 1, 1, 0, 1\}$;
5. Left elevator actuator and right aileron actuator outage corresponding to $\omega_5 = diag\{1, 0, 0, 1, 1\}$;
6. Right elevator actuator and right aileron actuator outage corresponding to $\omega_6 = diag\{0, 1, 0, 1, 1\}$;
7. Right elevator actuator and left aileron actuator outage corresponding to $\omega_7 = diag\{0, 1, 1, 0, 1\}$;
8. Left elevator actuator and left aileron actuator outage corresponding to $\omega_8 = diag\{1, 0, 1, 0, 1\}$;

Since we don't consider the control surface damage in the example, then $q = 0$, hence $i = 0$. Let $\gamma_{L0} = 1$, $L = 0, 1, \cdots, 8$ and

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

where the matrix S determines the outputs required to track, i.e., $\dot{\mu}_{rot}$, α and β.

Case 1: Consider only fault 2 (ω_2) where right elevator actuator outage has to be tolerated. By solving the convex multi-objective optimization problem using the proposed iterative algorithm, we obtain two reliable trackers under different design requirement of the fault performance indices 71 and 45.5. Table 1 compares the performance indices obtained from the standard tracker design method (without

Table 1: Performance indices comparison for fault 2 (ω_2) with $\Delta = 0.01$

System status	Stand'd tracker	Reliable robust tracker*			
		designed	achieved	designed	achieved
Nominal	16.7063	16.7261	16.7227	17.7921	17.7918
δ_{hr}	71.5299	< 71.0	63.8079	< 45.5	45.4945
Simu'ion	Fig. 1	–		Fig. 2	

**The two sets of "achieved" values are obtained from the different upper bounds (ρ_{Li}) of the fault performance indices, respectively.*

considering actuator fault) and the reliable robust tracker design method.

In this particular example, the standard controller (without considering the fault) can still stabilize the system when the ω_2 fault occurs. From Table 1, it can be seen that for the first reliable controller, by imposing a fault case performance of 71 (which is about the same as that achieved by the standard controller), our method gives an optimal guaranteed nominal performance of 16.7261 and an actually achieved nominal performance of 16.6227. The guaranteed and the actually achieved nominal performance values are very close, and both are just slightly larger (0.1%) than that achieved by the standard controller. This indicates that the conservativeness of our method is very small. In the second reliable controller, by sacrifacing the nominal performance slightly (6.5% from 16.7063 to 17.7918), the performance when the fault occurs can be drastically improved by 36.4% (from 71.5299 to 45.4945). Simulation results using the standard controller are given in Figure 1, and the results using the second reliable controller are given in Figure 2. Results for both without and with ω_2 fault are given. It can be seen from the simulation results that, while the standard and the reliable controllers give above the same response in the normal case, the reliable controller performs much better when ω_2 fault occurs.

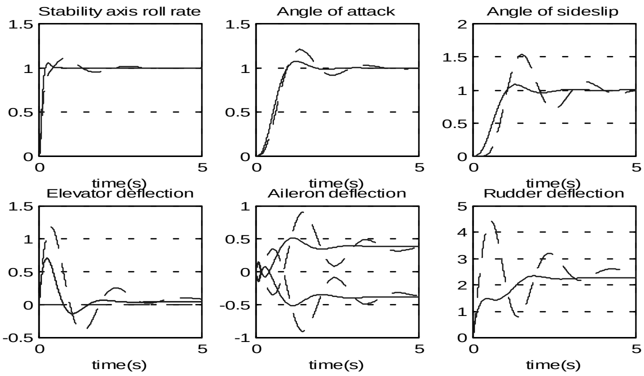

Figure 1: The nominal case (solid) and fault 2 case (dash) with standard tracker K

Case 2: Consider fault 6 (ω_6) where both right elevator actuator and right aileron actuator outage are to be tolerated. The design results are given in Table 2. Closed-loop stability is also maintained by the standard controller in this case. Similar to Case 1 (ω_2 fault), our method gives litter conservativeness and greatly improves the system performance when the ω_6 fault occurs. Simulation results given in Figures 3 and 4 also confirm this observation.

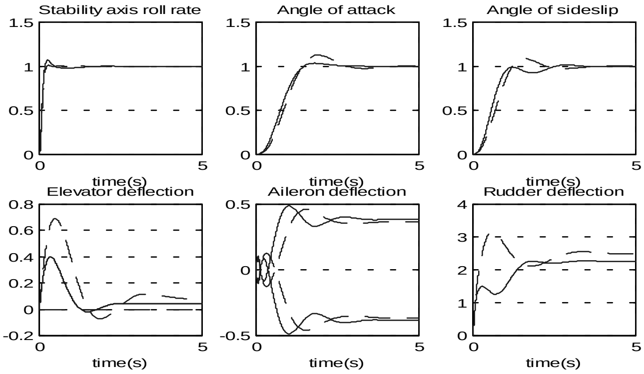

Figure 2: The nominal case (solid) and fault 2 case (dash) with reliable tracker K_{opt12}

Table 2: Performance indices comparison for fault 6 (ω_6) with Δ=0.01

System status	Stand'd tracker	Reliable robust tracker			
		designed	achieved	designed	achieved
Nominal	16.7063	16.7120	16.7078	18.0976	18.0940
$\delta_{hr}+\delta_{ar}$	75.9259	< 75.0	64.8140	< 43.0	42.9435
Simu'ion	Fig. 3	–		Fig. 4	

Case 3: Consider simultaneously all faults from 1 to 8 (ω_1 to ω_8). By using the iterative algorithm, reliable tracking controllers satisfying all the performance requirements of the eight kinds of fault cases are obtained. The results for two reliable controllers and for the standard controller are given in Table 3.

In this case, the standard controller is also able to stabilize the system in all fault cases. It can be seen from Table 3 that sacrificing the nominal performance slightly (7.4% from nominal 16.7063 to 17.9368), the performance indices for the 8 fault cases can all be improved significantly (ranging from 28.4% to 38.0%, except for cases of ω_3 and ω_4 where about the same level of performance as that of the standard controller is achieved.

Summarizing all the three cases of faults, it is noted that the reliable tracker design method can significantly improve the system performance in the event of fault cases than the standard design method, under minimal sacrifice of nominal performance. it is easy to see that we can obtain various reliable tracker according to the upper bound requirements ρ_{Li} of fault performance indices in the design of the reliable robust tracker. This provides the design of the reliable tracker design the flexibility of trading off between the nominal performance and the performance when faults occur. It can also be observed that, as more and more fault cases are considered in the design, the conservativeness of our method also tends to increase.

6 Conclusions

In this paper, we have investigated a reliable robust tracking problem for a linear time-invariant system against actuator faults and control surface damages. Based on the multi-objective robust performance analysis of the system in the nominal case and the faulty cases using LMIs method, we

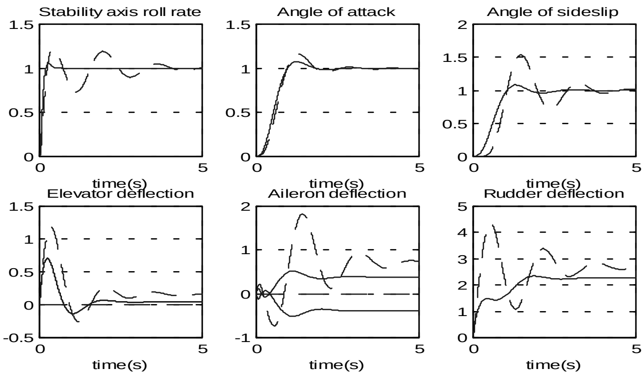

Figure 3: The nominal case (solid) and fault 6 case (dash) with Standard tracker K

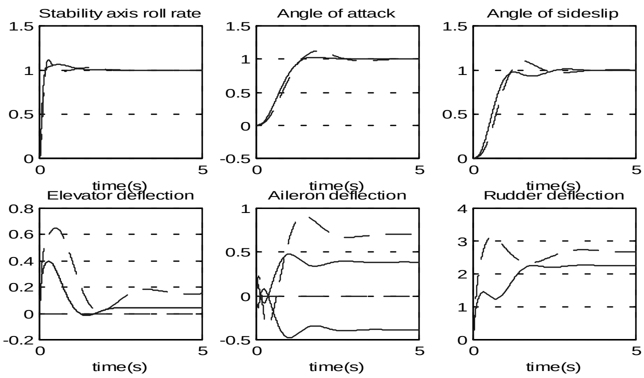

Figure 4: The nominal case (solid) and fault case 6 (dash) with Reliable tracker K_{opt22}

have derived a reliable robust tracking controller with state feedback plus tracking errors integral. The simulation result of an example of F-16 shows its effectiveness.

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Table 3: Performance indices comparison for fault 1 - 8 (ω_1 - ω_8) with Δ=0.01

System status	Stand'd tracker	Reliable robust tracker			
		designed	achieved	designed	achieved
Nominal	16.7063	17.3245	17.3196	17.9410	17.9368
δ_{hl}	71.3870	< 95.0	55.8404	< 54.0	50.6761
δ_{hr}	71.5299	< 95.0	55.8404	< 54.0	51.2459
δ_{ar}	19.5262	< 95.0	19.7188	< 24.0	19.5544
δ_{al}	19.5197	< 95.0	19.7383	< 24.0	19.4848
$\delta_{hl}+\delta_{ar}$	94.9750	< 95.0	66.7316	< 59.0	58.9276
$\delta_{hr}+\delta_{ar}$	75.9259	< 95.0	56.6173	< 51.0	50.4769
$\delta_{hr}+\delta_{al}$	95.4682	< 95.0	66.6672	< 60.0	59.9109
$\delta_{hl}+\delta_{al}$	75.2653	<95.0	56.4005	< 51.0	49.7324

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7 Appendix

System Matrices:

$$A = \begin{bmatrix} -0.015 & 0.048 & -5.942 & 0.002 & 0 & 0 \\ -0.091 & -0.957 & 138.361 & 0.016 & 0 & 0 \\ 0.000 & 0.005 & -1.022 & -0.001 & 0 & -0.003 \\ 0 & 0 & 0 & -0.280 & 6.267 & -151.144 \\ 0 & 0 & 0.000 & -0.182 & -3.419 & 0.640 \\ 0 & 0 & 0.003 & 0.045 & -0.030 & -0.454 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.024 & 0.024 & -0.025 & -0.025 & 0 \\ 0.172 & 0.172 & -0.180 & -0.180 & 0 \\ 0.087 & 0.087 & -0.008 & -0.007 & 0 \\ -0.315 & 0.315 & 0.023 & -0.023 & 0.121 \\ 0.189 & -0.189 & -0.346 & 0.346 & 0.124 \\ -0.168 & 0.168 & -0.015 & 0.015 & -0.059 \end{bmatrix}, G = \begin{bmatrix} 0.048 \\ -0.957 \\ 0.005 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 57.296 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.247 & 2.370 \\ 0 & 0 & 0 & 0 & -2.370 & 57.247 \\ -0.016 & 0.376 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.376 & 0 & 0 \end{bmatrix}$$