QFT CONTROLLER OPTIMIZATION FOR AUTOMATIC DESIGN

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1. ABSTRACT
In case of robust control, the Quantitative Feedback Theory controller design can be considered as an alternative to the μ-synthesis. Particularly, it allows to obtain controllers less conservative than other methods like H_∞. Loop-shaping algorithms have been proposed but their implementations remain complex for automatic controller design and is still an open question. This paper proposes an automatic QFT closed-loop design method based on a 2 steps optimization algorithm in case of SISO systems.

2. INTRODUCTION
The Quantitative Feedback Theory method is a robust control, developed at first by I. M. Horowitz in 1959 (1), based on a direct frequency domain design approach to satisfy objectives whatever the parameter state of the process may be. It allows to take account of perturbation and can be considered as an alternative to the μ-synthesis.
The automatic design of optimal QFT controller is the object of researches and several approaches have been proposed. Some of them are based on a non-linear problem. I. M. Horowitz and A. Gera (3), D. J. Ballance and P. J. Gawthrop (4) proposed the use of Bode Integrals within an iterative approach for loop shaping. D. J. Ballance and W. Chen (6) gave an automatic loop shaping in QFT using genetic algorithms. But convergence properties of optimization procedure, initial conditions and internal stability aspects restrict numerical implementation of these approaches.
Other approaches, using linear programming for automatic loop shaping, are interesting due to better convergence properties. A formulation based on open-loop bound approximation by convexes suggested by G. F. Bryant and G. D. Halikias (7). D. F. Thomson and O. D. I. Nwokah (5) proposed an analytical approach to loop shaping where boxes approximate the templates. Finally, Y. Zhao and S. Jayasuriya (11) introduced the Youla parameterization to transform a QFT robust performance problem into a one-dimensional search; but this allows the automatic design of only one parameter of the controller. All these methods deal with an open loop approach, which leads to optimization programs in non-convex domains.
More recently to overcome the non-convexity of the bounds on the open-loop transmission, Y. Chait (2) proposed a closed-loop optimization constrained by closed-loop bounds. The main disadvantage of this approach is the need of imposing the denominator of the closed-loop transfer function.
To improve this idea, we propose a QFT closed-loop design algorithm with two steps of optimization. The first step is the closed-loop numerator optimization similar to Chait's one (2) and the second is the closed-loop denominator optimization where the cost function is the quadratic sum of Euclidean distance between the open-loop responses and the bounds in the Nichols plane. This minimization tries to reach the optimal loop shaping defined by I. M. Horowitz and M. Sidi (9-10).
After a resume of QFT design concepts, this article proposes this solution to the automatic loop-shaping problem presented as a specific linear one. Finally, an example, the control of a DC motor, is given to demonstrate the utility of the proposed algorithm.

3. QFT DESIGN CONCEPT

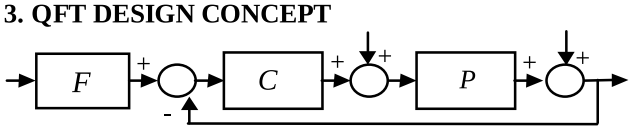

Figure 1: QFT design structure

The QFT design is the synthesis of a controller C and a pre-filter F (figure 1) to meet algebraic closed-loop specifications and to stabilize an uncertain system represented by a family $\wp$ of transfer functions:

$$\wp = \left\{ P(s) = \sum a_i s^i \Big/ \sum b_j s^j : \; [a_i, b_j] \in \Delta \right\}.$$

Closed-loop specifications for QFT controller design are:

- Robust stability margin:

$$\forall P(j\omega) \in \wp \quad \left| \frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)} \right| \le x$$

- Input disturbance attenuation:

$$\forall P(j\omega) \in \wp \quad \left| \frac{P(j\omega)}{1+C(j\omega)P(j\omega)} \right| \le W(\omega)$$

- Output disturbance attenuation:

$$\forall P(j\omega) \in \wp \quad \left| \frac{1}{1+C(j\omega)P(j\omega)} \right| \le W(\omega)$$

- Tracking performance:

$$\forall P(j\omega) \in \wp \quad T_d(j\omega) \le \left| F(j\omega) \frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)} \right| \le T_u(j\omega)$$

The basic idea in QFT is to transform these design specifications and plant uncertainties into robust stability and performance areas. An open-loop transfer function $P_0(j\omega)C(j\omega)$ of a nominal system $P_0(j\omega)$ selected in the family $\wp$ have to avoid their performance areas to meet algebraic specifications. The controller could be design using a gain-phase loop-shaping technique.

A set of frequency: $S_\omega = \{\omega_i \; i = 1, \ldots, w\}$ is taken around the desired crossover frequency. For each frequency, a bound delimits a region in the Nichols plane that the nominal open-loop frequency response has to avoid to meet one algebraic specification. These bounds could be closed (figure 2) or opened (figure 3), according to the specification type.

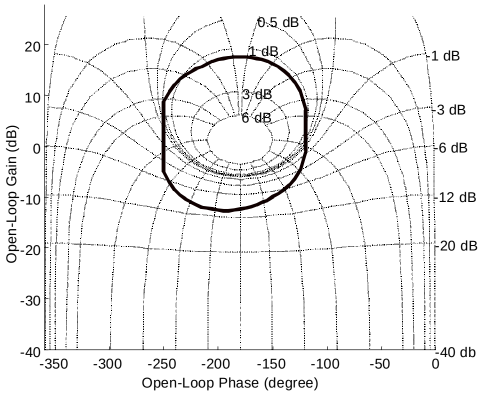

Figure 2: QFT area in Nichols plane for robust stability margin

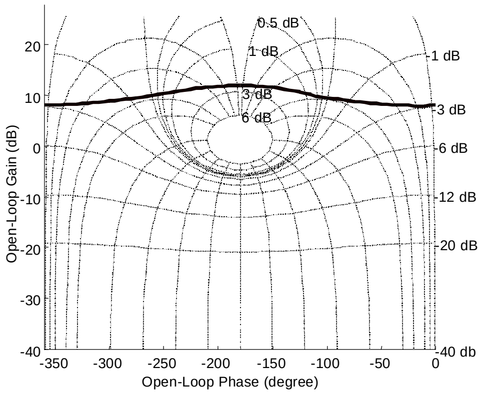

Figure 3:QFT area in Nichols plane for tracking performance

The number of bound is equal to the number of algebraic specification. For each frequency $\omega_i \in S_\omega$, we construct the intersection of all the regions delimited by a dotted line in figure 4. We obtain an area, which meets all the algebraic specifications. The border of this area is called the QFT bound represented by a solid line in figure 4.

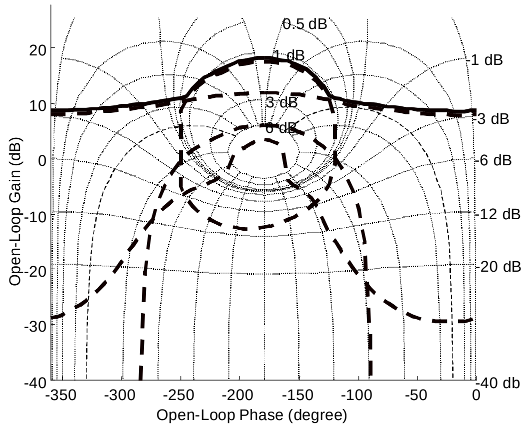

Figure 4: QFT borders and QFT bound at ω_i

4. CONTROLLER DESIGN METHOD

A fact posing a real problem in automatic loop-shaping is the unconvexity of open-loop domains due to ineffectiveness of non-convex optimization. The convexity of the regions is improved when we translate open-loop specifications into closed-loop specifications in the complex plane, as shown in Figure 5.

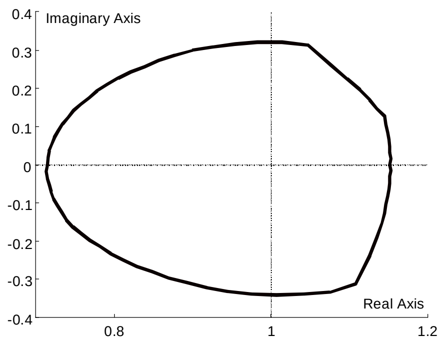

Figure 5: Closed-loop QFT bound in complex plane

To transform each closed-loop bound in a convex one, an algorithm, based on an iterative double step search, determines a set of points limiting a convex included in the QFT bound. The new formulation imposes that the closed-loop response has to be in the closed-loop QFT bound at ω_i to answer all the algebraic specifications.

5. CONVEXITY RESEARCH

A closed loop bound is defined by an ordered set of points. The first step of the algorithm extracts a set of point from the initial closed-loop QFT bound which defines a convex. It verifies if it is included in the bounds. The second step increases its size in adding points verifying the convexity and if it is included in the bound. At each step, the bigger convex is memorized.

At last the controller must reach robust the closed loop specifications, translated by QFT bounds, and also insure the internal stability of the system. Consequently, the QFT controller is now designed searching a stable closed-loop transfer function which meets closed-loop QFT bounds and whose controller does not cancel any right half-plane pole and zero of all the uncertain systems.

I. M. Horowitz (10) proposes that a stabilizing open-loop, which satisfies its bounds and has minimum high-frequency gain, gives an optimal QFT controller. The closed-loop design is based on this assertion knowing that the minimization of the high-frequency gain of the open-loop is the same as the minimization of the high-frequency gain of the closed-loop.

The closed-loop transfer function $T(s)$ is linear in its numerator coefficients and it is not linear in its denominator coefficients. So, we propose an iterative closed-loop optimization made up of 2 steps:

- A linear numerator coefficients convex optimization.
- A nonlinear denominator coefficients optimization.

Once closed-loop transfer function obtained, the controller C is recovered from the nominal-plant transfer function P_0 via:

$$C = \frac{1}{P_0}\frac{T}{1-T}$$

6. NUMERATOR OPTIMIZATION

In the numerator optimization, we define the closed-loop transfer function via:

$$T(s) = \frac{\sum_{i=0}^{n} \alpha_i s^i}{D(s)}$$

where the denominator has to be a priori defined but supposed stable. The closed-loop denominator is a priori chosen as a stable one. To increase the effectiveness of the algorithm, the frequency response of $1/D(s)$ is chosen to respect the tracking specification $T_d(j\omega)$ and $T_u(j\omega)$. In order to minimize the high frequency gain of $T(s)$ the optimization has to focus on the minimization of the coefficient α_n of the highest power in s of the closed-loop numerator.

To insure internal stability, we guarantee not only the stability of the closed-loop but also that the underlying controller C does not share any RHP poles and zeros with the plant family $\wp$. One way to achieve this is to add linear constraints on the closed loop transfer function of the nominal plant. These are given in the following theorem (8).

Theorem

We define p_i $(i=1,\dots,k)$ as unstable poles with multiplicity n_i $(i=1,\dots,k)$ and z_j $(j=1,\dots,l)$ as unstable zeros with multiplicity m_j $(j=1,\dots,l)$ of the nominal-plant P_0. If the closed-loop $T(s)$ is internally stable and the closed-loop denominator degree is higher than the nominal-plant denominator one then $T(s)$ must satisfy the following conditions.

$$\begin{cases} T(p_i)=1, T^{(1)}(p_i)=0,\dots,T^{(n_i+1)}(p_i)=0 \ (i=1,\dots,k) \\ T(z_j)=0, T^{(1)}(z_j)=0,\dots,T^{(m_j+1)}(z_j)=0 \ (j=1,\dots,l) \end{cases}$$

The second series of equality imposes that the underlying controller C does not share any right-half plane zero with the plant family $\wp$. It can be replaced by a factorization of the closed-loop numerator letting appear the unstable part $[N_{p_0}(s)]^-$ of the numerator of the nominal-plant P. Then, the closed-loop is define via:

$$T(s) = \frac{\left(\sum_{i=1}^{n} \alpha_i s^i\right)[N_{p_0}(s)]^-}{D(s)}$$

To resume this part, the numerator coefficients are obtained solving the following optimization problem by linear programming:

determine α_i $i \in [0, n]$ minimizing α_n

such that $T(j\omega_b) \subset \text{QFT bound } \omega_k, \forall k \in [1, w]$

$$T^{(p)}(z_j) = 0, (p = 0,\dots,m_j+1)(j=1,\dots,l)$$

Once the optimal numerator obtained, we have to cancel pole zero between P_0 and T to obtain a minimum order controller.

7. DENOMINATOR OPTIMIZATION

The previous method of numerator optimization can only determine optimal zero placements with a denominator that is a priori defined. A second step of optimization is implemented to perform the choice of closed-loop poles. This section is based on the optimal controller property (9,10) to compute the denominator of the closed-loop. It consists in the nonlinear optimization of the real denominator coefficients d_j of the closed-loop transfer function.

I. M. Horowitz and M. Sidi (9,10) demonstrate that if an optimal controller exists, it's unique and the associated open-loop or closed-loop lies on the boundary at each value of ω_k, $k \in [1 \quad w]$. To take into account this property, we consider a cost function based on the quadratic sum of the Euclidean distance between the open-loop and QFT bound in the Nichols plane at each frequency ω_k:

$$f(d_1, d_2, \dots) = \sum_{k=1}^{w} \left|\text{dist}(C(j\omega_k)P_0(j\omega_k), \text{ QFT bound } \omega_k)\right|^2$$

This function depends a priori on the numerator and denominator coefficients of T. But the first step of optimization links numerator variables to the denominator.

To insure the closed-loop stability, we add a second term to the cost function, which imposes that all the closed-loop poles are in the left-half plane. Not to degrade efficiency of the numerical implementation of the optimization, we choose a C^∞ monotone function of the real parts of the closed-loop poles. It's equal to zero when every real part of poles is in the left-half plane and close to infinity otherwise. The function to minimize becomes:

$$\hat{f}(d_{1,}d_2,...)= f(d_{1,}d_2,...)+\sum_{r=\text{real part of unstable pole}} e^{-\frac{1}{r^2}}\, e^{cr}$$

Another term can be added to the cost function in order to insure smoothness of the frequency responses between the bounds specified at given frequencies. This additional term imposes in fact a pole placement region R illustrated in figure 6.

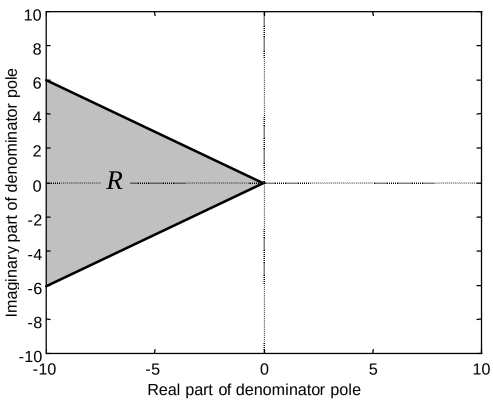

Figure 6: Additional smoothy function

For the same reasons previously defined, this new term is chosen as a C^{∞} monotone function of the pole-damping ratio ξ. It's equal to zero if variables are higher than a priori defined damping factor and close to infinity otherwise. The function to minimize becomes:

$$\hat{f}(d_{1,}d_2,...)= f(d_{1,}d_2,...)+\sum_{\xi<\xi_0} e^{-\frac{1}{(\xi_0-\xi)^2}}\, e^{c(\xi_0-\xi)}$$

Then, the optimization QFT controller design is an iterative algorithm. Two steps compose one iteration. The first step is a linear convex optimization of the closed-loop numerator with constant denominator, and the second step is an optimization of the closed-loop denominator parameters which minimizes the Euclidean distance between the open-loop and QFT bounds and the high-frequency gain of the open-loop. It also imposes closed-loop stability and the internal stability.

8. EXAMPLE

In this section, the control design of a DC motor whose uncertain transfer function is equal to:

$$P(j\omega)=\frac{K}{s\,(1+j\tau_m\omega)(1+j\tau_e\omega)}$$

with $K\in[150\quad 300]$, $\tau_m\in[0.012\quad 0.02]$ s, $\tau_e=0.001$ s.

is considered so as to compare our two step algorithm with Chait's approach.

The closed-loop objectives are:

- Margin specifications:

$$\forall P(j\omega)\in\wp, \forall\omega>0\quad \left|\frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)}\right|\le 1.1$$

- Tracking specifications:

$$\forall P(j\omega)\in\wp, \forall\omega>0\quad T_d(\omega)\le\left|F\,\frac{C\,P}{1+C\,P}\right|\le T_u(\omega)\ \text{with}$$

$$T_d(\omega)=\left|1\Big/\left(1+\frac{3(j\omega)}{50}+\frac{3(j\omega)^2}{50^2}+\frac{(j\omega)^3}{50^3}\right)\right|\ \text{and}$$

$$T_u(\omega)=\left|1\Big/\left(1+\frac{(j\omega)}{120}+\frac{(j\omega)^2}{120^2}\right)\right|$$

- Input disturbance attenuation:

$$\forall P(j\omega)\in\wp, \forall\omega>0\quad \left|\frac{P(j\omega)}{1+C(j\omega)P(j\omega)}\right|\le\left|\frac{\frac{(j\omega)}{10}}{1+\frac{(j\omega)}{10}}\right|$$

- Output disturbance attenuation:

$$\forall P(j\omega)\in\wp, \forall\omega>0\quad \left|\frac{1}{1+C(j\omega)P(j\omega)}\right|\le\left|\frac{(j\omega)}{100}\right|$$

Here, we focus on the design of the controller C. To use the QFT controller design, we have to choose a nominal process:

$$P_0(j\omega)=\frac{150}{s\,(1+j\tau_e\omega)(1+j0.02\omega)}$$

and a set of frequencies around the closed-loop cut-off frequency where QFT bounds are defined $\omega=[0.01,\,10,\,40,\,80,\,160,\,320,\,1000]$.

The open-loop QFT bounds are shown in the Nichols chart in figure 7, their closed-loop-associated bounds in the complex plane figure 8. To used the first step optimization, which is a convex optimization, an iterative algorithm determines the bigger convex included in each QFT bound (figure 9).

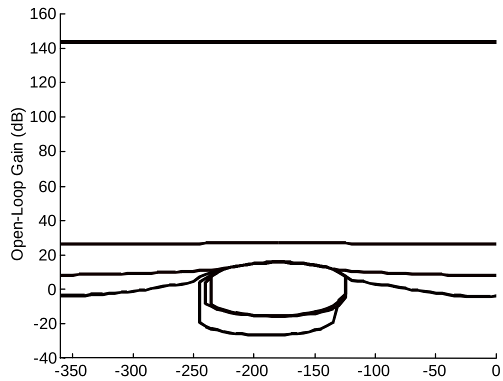

Figure 7: Open-loop QFT bounds

To initialise the optimization, we choose to define a stable closed-loop denominator which respects the tracking specification $T_d(j\omega)$ and $T_u(j\omega)$. We impose the denominator via:

$$D(j\omega)=\left(1+\frac{2(j\omega)}{80}+\frac{(j\omega)^2}{80^2}\right)\left(1+\frac{(j\omega)}{1000}\right)\left(1+\frac{(j\omega)}{10000}\right)$$

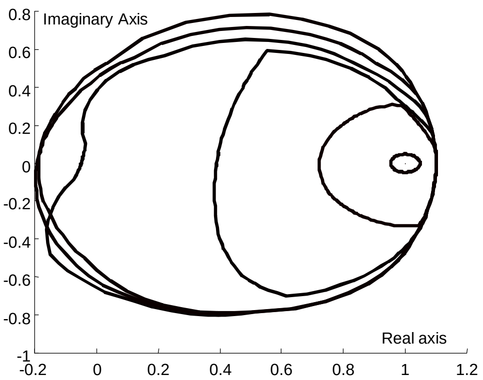

Figure 8: Closed-loop QFT bounds

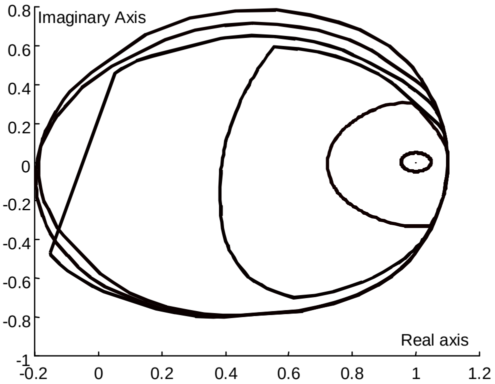

Figure 9: Convex QFT bounds

Imposing a two degrees of freedom on the closed-loop numerator:

- The optimisation only composed by the first step leds to results shown in figure 10 to 14 in term of frequency and temporal responses. Tracking and output disturbance attenuation specifications are respected, but the input disturbance attenuation is not contained in its template.
- The two step optimization allows to obtain a controller satisfying all specifications as shown in figure 15 to 19. We can also verify that the input perturbation rejection is included in the desired template.

In each case, the obtained controller is a 3 zeros/3 poles transfer function which can be reduced to a 2 zeros/2 poles one in the frequency band of interest.

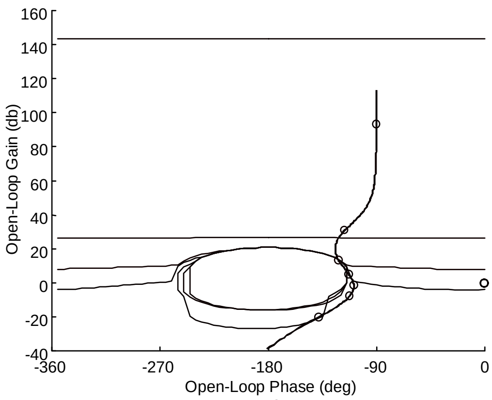

Figure 10: Black diagram for one step open loop optimization

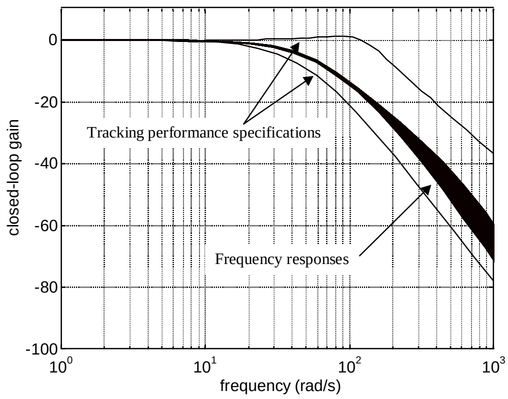

Figure 11 : Closed-loop response frequency

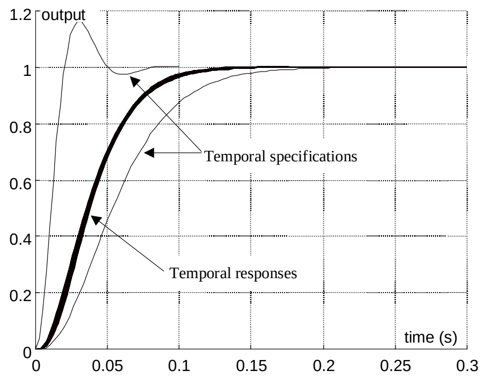

Figure 12 : Closed-loop step response

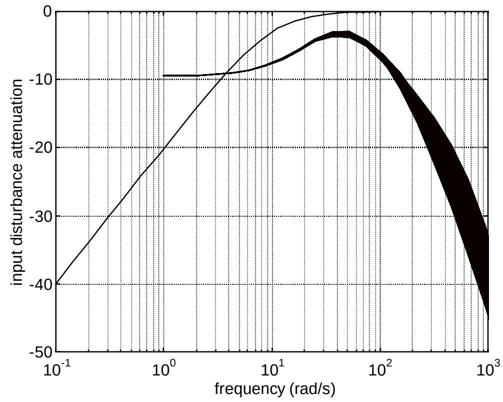

Figure 13 : Input disturbance response frequency

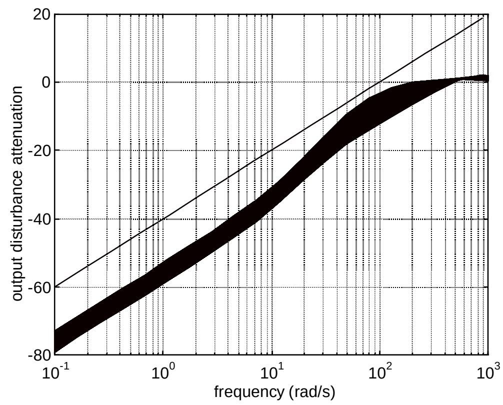

Figure 14 : Sensitivity response frequency

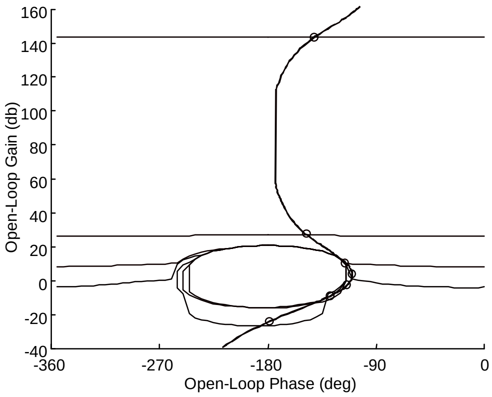

Figure 15: Black diagram for two step open loop optimization

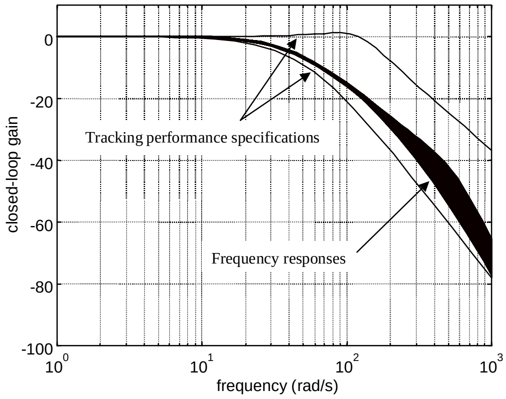

Figure 16 : Closed-loop response frequency

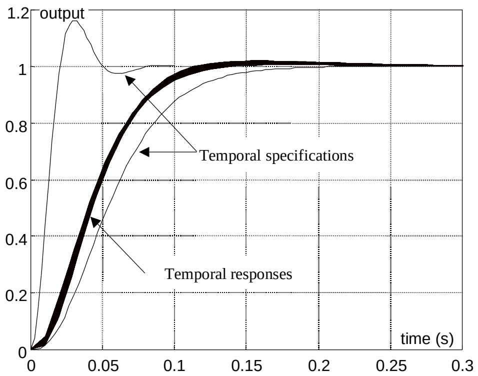

Figure 17 : Closed-loop step response

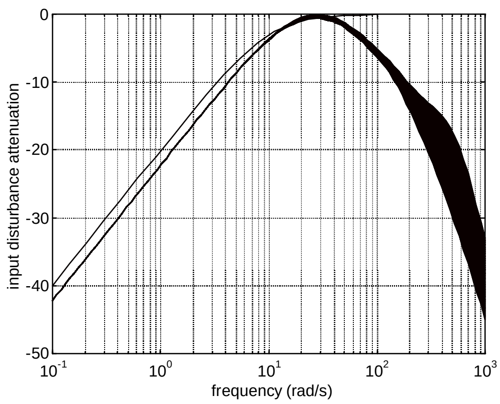

Figure 18 : Input disturbance response frequency

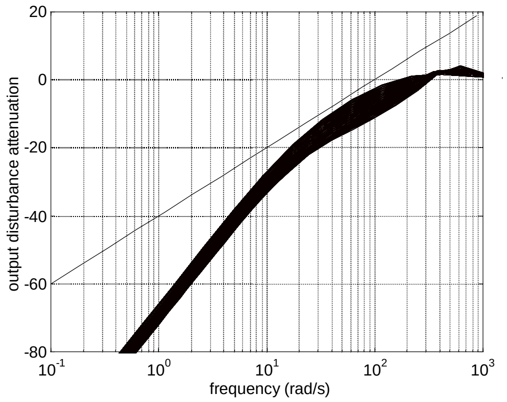

Figure 19 : Sensitivity response frequency

9. CONCLUSION

This paper proposes an automatic QFT controller design with a 2-step optimization algorithm tracking the optimal controller define by I. Horowitz. The first step is a convex closed-loop numerator optimization and the second one is a closed-loop denominator optimization. This method tested on a DC motor gives good results, the extension of this concept will be considered for MIMO systems.

10. REFERENCES

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