On the Problem of Fault Detection and Residual Generation

M. Djemai, J.P. Barbot et O. Bethoux
Equipe Commande des Systèmes (ECS), ENSEA, 6 Av. du Ponceau, 95014 Cergy Cedex.
barbot@ensea.fr

Abstract

In this paper, the Fundamental Problem of Residual Generation (FPRG) is studied. The solution of this problem is the first step for the solution of the fault detection problem. Here a class of nonlinear perturbed input-affine system is considered. The technique proposed here is based on two steps : The first one consists in decoupling the perturbation with respect to the fault by using the technique of output injection in a new way. The second one consists in detecting, by logic decision, the fault. Nonlinear observer with linearizable estimated error dynamics decoupled from the input and the disturbance is used as a residual generator. The paper ends with an illustrative example.

Key words : Faults detection, Nonlinear systems, Disturbance decoupling, Non-linear observer.

1 Introduction

The safety is of great importance in modern control, and one of the main requirements of this problem is its task of fault detection and isolation. This problem has been widely investigated ([8, 21, 3, 9]...). A typical method of detection and fault isolation consists of three parts : *a residual generator*, *a residual evaluation module* and *decision system (detection)* and finally, *isolation of the fault*. The detection of faults is established by logic decision based on the residual which is an output signal generated by an observer. In this work, the observer-based approach is considered. Only the design of residual generator is considered.

The FPRG (Fundamental Problem of Residual Generation), was first studied for linear systems with one fault signal, the filter is required to recognize the fault signal without confusing it with the disturbance ([18]). Most of the time, the design methods for detection and diagnostics observers for nonlinear system are based on the hypothesis that the systems works in the neighbourhood of an operating point, and the linearization method is used. The disadvantage of this method is that the observation error based on the linearized system can be misinterpreted as faults by the detection algorithm and hence lead to false alarm.

The idea developed in this paper is based on the disturbance decoupling approach ([22]), (the invariance principle), a set of transformation for the nonlinear input-affine system is defined. Each transformation maps the state of the system into a subsystem that depends only on the fault and is robust to the disturbance. Also, the nonlinear observer used as a residual generator gives a linearizable estimate dynamics error independent from the disturbance and the input. The approach takes advantage of the structure of the system model which assumed to be in observable triangular form (or transformed into this form under some conditions with a diffeomorphism) ([13, 14, 10]).

The outline of the paper is the following: In section 2, recalls on the Fundamental Problem of Residual Generation are presented for nonlinear input affine systems. Section 3 is devoted to the triangular observer form and its sliding mode observer. The disturbance decoupling problem is studied in section 4 and the solution of the FPRG is given in section 5, which represent the main contribution of the paper. The paper ends with an illustrative example.

2 Recalls on FPRG for Nonlinear Systems

To study the Fundamental Problem of Residual Generation (FPRG), which is the main task of Fault Detection and Isolation (FDI), we consider nonlinear systems of the following form

$$\dot{x} = f(x,u) + l(x)m + \Sigma_{i=1}^{s} p_i(x) w_i \quad (1)$$
$$y = h(x) \quad (2)$$

where $x(t) \in \mathcal{X} := \Re^n$, $u(t) \in \mathcal{U} := \Re$, $y(t) \in \mathcal{Y} := \Re^p$, $m(.) : [0, +\infty) \to \mathcal{M} := \Re$, is an unknown input, $w_i(t)$ is an unknown disturbance, and the system is supposed to be perfectly known when $m(.) = w_i(.) = 0$. The basic problem of fault detection is to design a filter ([18]) which consists in the determination of a residual vector to detect the failure mode. In order to define the residual generator for the previous system, we consider the following filter described by the equations

$$\dot{\zeta} = f_r(\zeta(t), y(t), u(t)) \quad (3)$$
$$r(t) = h_r(\zeta(t), y(t), u(t)) \quad (4)$$

where $h_r, f_r \in \mathcal{C}^\infty$ and $r(t)$ represents the residual which is a scalar (or vector when more then one fault occur) valued signal containing information on the time and location of the occurrence of the fault. When no fault is present in the system, $r(t)$ should be near zero and deviate from zero when a fault is has accrued. The evaluation is based on logic decision $\mathcal{L}(r)$, and the threshold $\mathcal{L}_{th}$, the fault detection procedure can be stated as follow:

$$\mathcal{L}(r) < \mathcal{L}_{th} \implies m = 0$$

$$\mathcal{L}(r) \quad > \quad \mathcal{L}_{th} \quad \Longrightarrow \quad m \neq 0$$

In an ideal case, a residual $r(t)$ will be zero if no fault is present, different from zero when fault is present, $\mathcal{L}_{th} = 0$, which is, however, impossible in practical case, because of noise and modelling errors. Moreover, we aim a complete decoupling of the residual from the disturbance $w(.)$ and the control $u(.)$. This means that the residual is required to be affected only by the fault. Also the observable modes of the system are asymptotically stable. If such a filter (3)-(4) exists, then a solution to the FPRG exists. We can resume in the following conditions :

Condition 2.1 *The observable modes of the system are asymptotically stable.*

Condition 2.2 *The residual $r(t)$ is not affected by the perturbation and the input.*

Condition 2.3 *The residual $r(t)$ is affected by fault.*

If the previous conditions are verified, one consider the system (1)-(2) and without loss of generality we suppose $x_0 = 0$ as an equilibrium point. The first stage of designing a residual generator consists in decoupling the fault with respect to the perturbation. This consist in making the system (1)-(2) into the form :

$$\begin{aligned} \dot{z}_1 &= \tilde{\varphi}_1(z_1, z_2, y_1, y_2, u) + \tilde{l}_1(z_1, z_2)m \\ &\quad + \Sigma_{i=1}^{s} \tilde{p}_{i,2}(z_1, z_2) w_i \end{aligned} \tag{5}$$

$$\dot{z}_2 = \tilde{\varphi}_2(z_2, y_1, y_2, u) + \tilde{l}_2(z_1, z_2)m \tag{6}$$

$$y_1 = \tilde{h}_1(z_1, z_2) \tag{7}$$

$$y_2 = \tilde{h}_2(z_2) \tag{8}$$

where $z_1 = (z_{11}, ..., z_{1n_1})^T$, and $z_2 = (z_{21}, ..., z_{2n_2})^T$. We consider the hypothesis that the subsystem (6)-(8) is observable. The problem is to find a diffeomorphism $z = \psi(x)$ with $\psi(0) = 0$ defined in U^0, a neighbourhood of $x_0 = 0$. We suppose that the filter (3)-(4) can be chosen as a nonlinear sliding mode observer characterized by a step by step convergence ([5, 2]).

3 Triangular Form

Consider the system Σ

$$\begin{aligned} \dot{x} &= F(x) + G(x, u) \\ y &= H(x) \end{aligned} \tag{9}$$

where $x \in \Re^n$ is the sate vector, $u \in \Re$ the input vector, $y \in \Re$ the output vector and F, G, H are analytic vectors functions. Also, for all $x \in \Re^n$, the system (9) est supposed to be BIBS : Bounded Input Bounded State, in finite time. In order to transform the system (9) into a triangular observable form, one use the following "degenerate" rank condition ([12]) :

Condition 3.1

- $$rank \begin{pmatrix} dH \\ dL_F H \\ \vdots \\ dL_F^{n-1} H \end{pmatrix} = n$$

where $L_F H = \frac{\partial H}{\partial x} F$ is a classical Lie derivative.

The condition 3.1 is classic for autonomous systems. We have also the next condition:

Condition 3.2 *G verify for all $u \in U \subset \Re$ where U is the set of authorized inputs*

- $dL_G L_F^i H \in \Omega^i \qquad \forall i \in \{0, ..., n-1\}$

with $\Omega^i = span\{dH, dL_F H, ..., dL_F^i H\}$

One set the following proposition ([20])

Proposition 3.1 *The system (9) may be transformed by using the diffeomorphism $\xi \triangleq \phi(x)$, into a triangular observable form (in the neighbourhood of x)*

$$\begin{aligned} &\dot{\xi}_1 = \xi_2 + g_1(\xi_1, u) \\ &\dot{\xi}_2 = \xi_3 + g_2(\xi_1, \xi_2, u) \\ &\vdots \\ &\dot{\xi}_{n-1} = \xi_n + g_{n-1}(\xi_1, ..., \xi_{n-1}, u) \\ &\dot{\xi}_n = f_n(\xi) + g_n(\xi, u) \\ &y = \xi_1 \end{aligned} \tag{10}$$

with $\bar{g}_i(., u = 0) = 0$ for all $i \in \{1, ..., n\}$, if and only if conditions 3.1 and 3.2 are verified in the neighbourhood of x.

Proof : See [5] for details of proof.

3.1 Triangular Observer

From the work [7] and [1] we propose the following type of sliding observer for the system (10) :

$$\begin{aligned} &\dot{\hat{\xi}}_1 = \hat{\xi}_2 + g_1(\xi_1, u) + \lambda_1 sign_1(\xi_1 - \hat{\xi}_1) \\ &\dot{\hat{\xi}}_2 = \hat{\xi}_3 + g_2(\xi_1, \tilde{\xi}_2, u) + \lambda_2 sign_2(\tilde{\xi}_2 - \hat{\xi}_2) \\ &\vdots \\ &\dot{\hat{\xi}}_{n-1} = \hat{\xi}_n + g_{n-1}(\xi_1, \tilde{\xi}_2 ..., \tilde{\xi}_{n-1}, u) \\ &\quad + \lambda_{n-1} sign_{n-1}(\tilde{\xi}_{n-1} - \hat{\xi}_{n-1}) \\ &\dot{\hat{\xi}}_n = f_n(\xi_1, \tilde{\xi}_2 ..., \tilde{\xi}_n) + g_n(\xi_1, \tilde{\xi}_2 ..., \tilde{\xi}_n, u) \\ &\quad + \lambda_n sign_n(\tilde{\xi}_n - \hat{\xi}_n) \end{aligned} \tag{11}$$

where $\tilde{\xi}_i = \hat{\xi}_i + \lambda_{i-1} sign_{i-1}(\xi_{i-1} - \hat{\xi}_{i-1})$ for $i = 2, ..., n-1$, and the $sign_i(\xi)$ function denotes the usual sign function but with a low pass filter of the ξ variable ([7]) and anti-peaking structure ([15]). This anti-peaking structure issues from the idea that we do not inject the observation error information before reaching the sliding manifold linked with this information. Moreover we reach the manifold one by one. Doing this we obtain a sub-dynamics of dimension one and consequently we do not have peaking phenomena. More precisely $sign_i(.)$ is equal to zero if there exists $j \in \{1, i\}$ such that $\tilde{\xi}_j - \hat{\xi}_j \neq 0$ (by definition $\tilde{\xi}_1 = \xi_1$), else $sign_i(.)$ is equal to the usual $sign(.)$ function.
In the observer structure, this particular *sign* function allows $\tilde{\xi}_i - \hat{\xi}_i$ to converge to zero if all the $\tilde{\xi}_j - \hat{\xi}_j$ with $j < i$ have converged to zero before. This particular *sign* function which allows to converge in finite time step by step and at each step a dynamic of dimension one is a key point of our antipeaking technique and give the following result

Theorem 3.1 *Considering the Bounded Input Bounded State (BIBS) system (10) and observer (11), for any initial state $\xi(0)$, $\hat{\xi}(0)$ and any bounded input u, there exists a choice of λ_i such that the observer state $\hat{\xi}$ converges in finite time to ξ.*

Proof: From (10) and (11) and considering the initial state condition such that $\xi_1(0) \neq \hat{\xi}_1(0)$ (if this is not the case, we directly move on to the next step of the proof). In fact this observer is characterized by its step by step convergence (see [5, 2] for details).

4 Disturbance Decoupling

In this section we propose to study the problem of decoupling of the disturbance and the defect. The problem consists in breaking up the total system into two subsystems of which one of both do not depend on the disturbance but depends on the defect because of the observability conditions. This subsystem will have an output which will depends only on the part of the state insensitive with the disturbance. It will be used thereafter to develop an observer for fault detection. Now let us consider of rejection of disturbance for the following nonlinear system (the inexistence of the control input and the defect in this equation does not change anything with the general form seen in the preceding section) :

$$\begin{aligned} \dot{x} &= f(x) + P(x)p(t) && (12) \\ y_1 &= h_1(x) && (13) \\ y_2 &= h_2(x) && (14) \end{aligned}$$

we seek a diffeomorphism allowing to put this system in the following decoupled form :

$$\begin{aligned} \dot{\zeta}_1 &= g_1(\zeta_1, \zeta_2) + \tilde{P}_1(\zeta_1, \zeta_2)p(t) && (15) \\ \dot{\zeta}_2 &= g_2(\zeta_2) && (16) \\ \tilde{y}_1 &= \tilde{h}_1(\xi_1, \xi_2) && (17) \\ \tilde{y}_2 &= \tilde{h}_1(\xi_2) && (18) \end{aligned}$$

The conditions so that there is such a diffeomorphism are well-known (see [13]) that is to say Δ = smallest involutive distribution containing P and invariant with respect to f if this distribution is of dimension d then there exists $\zeta_1 = (z_1, ..., z_d)^T$ and $\zeta_2 = (z_{d+1}, ..., z_n)^T$ and (15)-(18). Moreover, one knows (see [11] pp. 43) that $z_{d+1} = \phi_{d+1}, ..., z_n = \phi_n$ with $d\phi_i \in \Delta^{\perp}$ for all $i \in \{d+1, n\}$. The conditions of transformation into outputs injection form are in [16]. One rewrites the system (12) in the form.

$$\dot{x} = \bar{f}(x) + \rho(y) + P(x)p(t) \quad (19)$$

and $\bar{\Delta}$ = the smallest involutive distribution is containing P and invariant with respect to $\bar{f}(x) = f(x) - \rho(y)$ with $\bar{\Delta}$ of dimension $\bar{d} < d$.

Remark 4.1 *The idea consists in using the function $\rho(y)$ to falls the dimension of $\bar{\Delta}$ with respect to Δ.*

As the vector $\rho(y)$ can be decomposed as the following (one choose the orthonormed basis $d\phi_i$ for $i > d$, which is equivalent to choose τ_i nilpotent of order zero for $i > d$):

$$\rho(y) = \sum_{i=1}^{n} a_i(y)\tau_i \quad (20)$$

where $a_i(y)$ are a scalar output functions, with $span\{\tau_1, ..., \tau_d\} = \Delta$ and for $i, j > d$, we have $d\phi_i \tau_i = 1$ and $d\phi_i \tau_j = 0$ for $i \neq j$. One obtain immediately that there exists a diffeomorphism such that

$$\begin{aligned} \dot{\zeta}_1 &= g_1(\zeta_1, \zeta_2) + \tilde{\rho}_1(y) + \tilde{P}(\zeta_1, \zeta_2)p(t) \\ \dot{\zeta}_2 &= g_2(\zeta_2) + \tilde{\rho}_2(y) \\ \tilde{y}_1 &= \tilde{h}_1(\zeta_1, \zeta_2) \\ \tilde{y}_2 &= \tilde{h}_2(\zeta_2) \end{aligned}$$

this is a direct consequence of the decomposition of $f = \bar{f} + \rho(y)$ and (20).

THEOREM 4.1 *Consider the system (19) such that $\bar{\Delta}$ is the small involutive distribution invariant with respect to $\bar{f}$ contained P and of dimension $\bar{d}$, then, there exists a diffeomorphism ϕ such that the system can be rewritten as :*

$$\begin{aligned} \dot{\zeta}_1 &= g_1(\zeta_1,\zeta_2) + \tilde{\rho}_1(y) + \tilde{P}(\zeta_1,\zeta_2)p(t) \\ \dot{\zeta}_2 &= g_2(\zeta_2) + \tilde{\rho}_2(y) \\ \tilde{y}_1 &= \tilde{h}_1(\zeta_1,\zeta_2) \\ \tilde{y}_2 &= \tilde{h}_2(\zeta_2) \end{aligned}$$

where ζ_2 is of dimension $n - \bar{d}$. □

REMARK 4.2 *In order to have detection and rejecting perturbation, we need that the fault acts on ζ_2, thus the condition $l(x) \notin \bar{\Delta}$ must be verified.*

REMARK 4.3 *In order to have $\tilde{y}_2$ only function of the original outputs y_1, y_2 we reclame that there exists a function $\psi(y_1, y_2)$; such that $d\psi \in \bar{\Delta}^{\perp}$ ($\bar{\Delta}^{\perp}$ is the annihilator of $\bar{\Delta}$) and $L_l L_g^i \psi \neq 0$ for at least one $i \in \{0, .., n\}$.*

5 Resolution of the FPRG with sliding mode observer

Let us consider the observable system of the form (1)-(2), we initially seek a diffeomorphism allowing to put this system in the form uncoupled given by (5)-(8). We then propose an observer for under system given by (6)-(8) or we do not consider the defect. In order to facilitate the study of the stability of the observer and its convergence, we propose a triangularisation of the sub system, thus we obtain.

$$\begin{aligned} \frac{dz_2}{dt} &= \tilde{\varphi}_2(z_2, y, u) + \tilde{l}_2(z_1, z_2)m \\ \tilde{y}_2 &= \tilde{h}_2(z_2) \end{aligned}$$

If the expression of the system is developed, on obtain

$$\tilde{\varphi}_2(z_2, y, u) = \bar{f}_2(z_2) + \rho_2(y) + \bar{g}_2(z_2)u$$

$\rho_2(y) = \rho_2(y_1, y_2)$. Then under conditions 3.1 and 3.2 there is a diffeomorphism $\xi_2 = \phi(z_2)$ with $\dim(\xi_2) = n - d$, we make a triangularisation only for $\bar{f}_2(z_2)$ and $\bar{g}_2(z_2)$, we obtain the following form

$$\begin{aligned} \frac{d\xi_2}{dt} &= \bar{A}_2\xi_2 + \rho_2(y) + \bar{g}_2(\xi_2, u) \\ &\quad + \tilde{l}_2(z_1, z_2 = \phi^{-1}(\xi_2))m \\ y_2 &= (1, 0, ..., 0)\xi_2 = \xi_{21} \end{aligned}$$

$$\begin{aligned} \bar{A}_2 &= \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix} \\ \bar{g}_2(\xi_2, u) &= \begin{pmatrix} \bar{g}_{21}(\xi_{21}, u) \\ \bar{g}_{22}(\xi_{21}, \xi_{22}, u) \\ \vdots \\ \bar{g}_{2,n-d-1}(\xi_{21}, ..., \xi_{2,n-d-1}, u) \\ \bar{f}_{2,n-d}(\xi_2) + \bar{g}_{2,n-d}(\xi_2, u) \end{pmatrix} \\ \xi_2 &= \begin{pmatrix} \xi_{21} & \xi_{22} & \dots & \xi_{2,n-d} \end{pmatrix}^T \end{aligned}$$

Thus the observer (fault detector) is made up like this

$$\begin{aligned} \dot{\hat{\xi}}_{21} &= \hat{\xi}_{22} + g_{21}(\xi_{21}, u) + \tilde{l}_{21}(z_1, \hat{\xi}_2)m \\ &\quad + \lambda_{21} sign_{21}(\xi_{21} - \hat{\xi}_{21}) \\ \dot{\hat{\xi}}_{22} &= \hat{\xi}_{23} + g_{22}(\xi_{21}, \tilde{\xi}_{22}, u) + \tilde{l}_{22}(z_1, \hat{\xi}_2)m \\ &\quad + \lambda_{22} sign_{22}(\tilde{\xi}_{22} - \hat{\xi}_{22}) \\ &\quad \vdots \\ \dot{\hat{\xi}}_{2,n-d-1} &= \hat{\xi}_{2,n-d-1} + g_{2,n-d-1}(\xi_{21}, \tilde{\xi}_{22} ..., \tilde{\xi}_{2,n-d-1}, u) \\ &\quad + \tilde{l}_{2,n-d-1}(z_1, \hat{\xi}_2)m \\ &\quad + \lambda_{2,n-d-1} sign_{2,n-d-1}(\tilde{\xi}_{2,n-d-1} - \hat{\xi}_{2,n-d-1}) \\ \dot{\hat{\xi}}_{2,n-d} &= f_{2,n-d}(\xi_{21}, \tilde{\xi}_{22} ..., \tilde{\xi}_{2,n-d}) \\ &\quad + g_{2,n-d}(\xi_{21}, \tilde{\xi}_{22} ..., \tilde{\xi}_{2,n-d}, u) + \tilde{l}_{2,n-d}(z_1, \hat{\xi}_2)m \\ &\quad + \lambda_{2,n-d} sign_{2,n-d}(\tilde{\xi}_{2,n-d} - \hat{\xi}_{2,n-d}) \end{aligned}$$

$$\hat{y}_2 = \hat{\xi}_{21}$$

Without the presence of the defect ($m = 0$), all the $\tilde{\xi}_{2i}$ converge towards ξ_{2i}, but with the defect, there is at least $\tilde{\xi}_{2i}$ which does not converge towards ξ_{2i} then the idea to use a second observer

$$\frac{d\hat{\xi}}{dt} = \bar{A}_2\hat{\xi}_2 + \rho(y_1, y_2) + \bar{g}_2(\tilde{\xi}_2, u) + \Gamma_2(y_2 - y_2^r)$$

with $y_2^r = \hat{\xi}_{21}$. This observer converges towards zero if $\xi_2 = \tilde{\xi}_2$ i.e. if there is no a defect, if not one has $(y_2 - y_2^r) \neq 0$. In practice, one has $|r| > \varepsilon$, with ε which is predetermined, the defect introduced a skew on the observer.

6 Illustrative Example

With an aim of showing the utility of the injection of output proposed, we chose a linear and a simple non-linear model in order to generate the residue.

6.1 Linear example

Let us consider the linear example

$$\begin{aligned} \dot{x} &= Ax + Bu + Pw + Lm \\ y &= Cx \end{aligned} \tag{21}$$

Where $x \in \Re^n$ is the state, $u \in \Re$ is the input, $y \in \Re^2$ is the output, $w \in \Re$ is the perturbation, $m \in \Re$ is the fault to be detected and $A, B, C, P,$ and L are matrix of appropriate dimensions. The application of the developed method to this example gives

$$\begin{array}{l} \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u + P_1w + L_1m \\ \dot{x}_2 = A_{22}x_2 + B_2u + L_2m \\ y_1 = C_{11}x_1 + C_{12}x_2 \\ y_2 = C_{22}x_2 \end{array} \qquad (22)$$

The condition of theorem 4.1, concerning the distribution Δ is given as the following :

$$\Delta = \{P, AP, ..., A^{n-1}P\}$$

the idea is to find a function $\rho(y) = Qy = QCx$ such that the distribution

$$\bar{\Delta} = \{P, \tilde{A}P, ..., \tilde{A}^{n-1}P\}$$

where $\tilde{A} = A - QC$. The idea is to find an appropriate matrix Q in order to obtain $Dim(\bar{\Delta}) < Dim(\Delta)$.

More particularly, let us suppose

$$\begin{array}{rcl} \dot{x} &=& \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} m \\ &=& Ax + Pw + Lm \\ y &=& \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x = Cx \end{array}$$

which gives

$$\Delta = \{P, AP, A^2P\} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

of dimension 3. If we consider

$$\dot{x} = Ax + Pw + Lm + \rho(y)$$

and suppose that: $\rho(y) = Qy = QCx$, and $Q = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ one obtain

$$\dot{x} = \tilde{A}x + Pw + Lm + QCx$$

with $\tilde{A} = A - QC$. Then

$$\bar{\Delta} = \{P, \tilde{A}P, \tilde{A}^2P\} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

which is of dimension 1. We note that $L \notin \bar{\Delta}$ and for $\psi(y_1, y_2) = (y_1 - y_2)$ we have $d\psi \in \bar{\Delta}^{\perp}$ (ψ is chosen as a new output $\tilde{y}_2$) and $L_L\tilde{y}_2 \neq 0$.

6.2 Nonlinear example

Let us consider the nonlinear system Σ

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 - x_1^3 - l_1m \\ \dot{x}_2 &=& x_3 - x_2^3 + p_2w \\ \dot{x}_3 &=& x_2 - x_3^3 + l_3m \end{array} \qquad (23)$$

with $y_1 = x_1$ and $y_2 = x_3$. We choose as initial conditions $(x_1, x_2, x_3) = (1, 1, 1)$ for our simulations. In the same way, w is a scalar disturbance and we take $p = (0, p_2, 0)$, $p_2 = 1$ and $L = (-l_1, 0, l_3)^T = (-1, 0, 1)^T$ and m is the defect to be detected. In order to detect the defect using an observer, let us start by decoupling the disturbance $w(t)$ and the defect $m(t)$. For that, we split the system in two sub systems where one should not depend on the disturbance but only on the defect. Cocequentely, we must find $\rho(y)$ such that $\bar{\Delta}$ falls of dimension compared to Δ. Remmember that for $\rho(y) = 0$ one finds Δ of dimension 3 and the disturbance is not decoupled. Now we set

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 + \rho_1(y) - lm \\ \dot{x}_2 &=& x_3 + \rho_2(y) + pw \\ \dot{x}_3 &=& x_2 + \rho_3(y) + lm \end{array}$$

and by choosing $\rho_1(y) = -y_1^3$, $\rho_2 = 0$ and $\rho_3 = -y_2^3$ one find the previous system but the function $\bar{f} = (x_2, x_3 - x_2^3, x_2)^T$, generates us the distribution $\bar{\Delta} = span\{p, \bar{f}\}$ which is of dimension 2. As $\bar{\Delta}^T = span\{dh = (1, 0, -1)\}$, one can then choose the coordinates $z_1 = x_1$, $z_2 = x_2$ and $z_3 = x_1 - x_3$. One deduces the following representation of state from it:

$$\begin{array}{rcl} \dot{z}_1 &=& z_2 - y_1^3 - lm \\ \dot{z}_2 &=& -z_2^3 + y_2 + pw \\ \dot{z}_3 &=& y_2^3 - y_1^3 - 2lm \\ \tilde{y}_1 &=& y_1 = z_1 \\ \tilde{y}_2 &=& y_1 - y_2 = z_1 - z_3 \end{array}$$

As $\bar{\Delta}$ is in $ker\{d\bar{h}_2\}$ with $\bar{h}_2 = y_1 - y_2$, one makes an observer on the variable z_3 starting from the combinations of outputs $\tilde{y}_2$. Moreover we have $d(\tilde{y}_2) \in \bar{\Delta}^{\perp}$ and $L_L\bar{h}_2 \neq 0$. We note well that we have two pennies systems decoupled; the first under system is formed of (z_1, z_2) and second is made of z_3. Our observer will be realized on this last under system completely insensitive with the disturbance.

$$\frac{d\hat{z}_3}{dt} = y_2^2 - y_1^2 + \lambda_3(z_3 - \hat{z}_3)$$

The outputs y_1 and y_2 being measurable, we will not pose a problem. On the other hand this observer is likely to crush the defect and nothing to detect because the error is :

$$\dot{e}_3 = -2lm + \lambda_3(z_3 - \hat{z}_3)$$

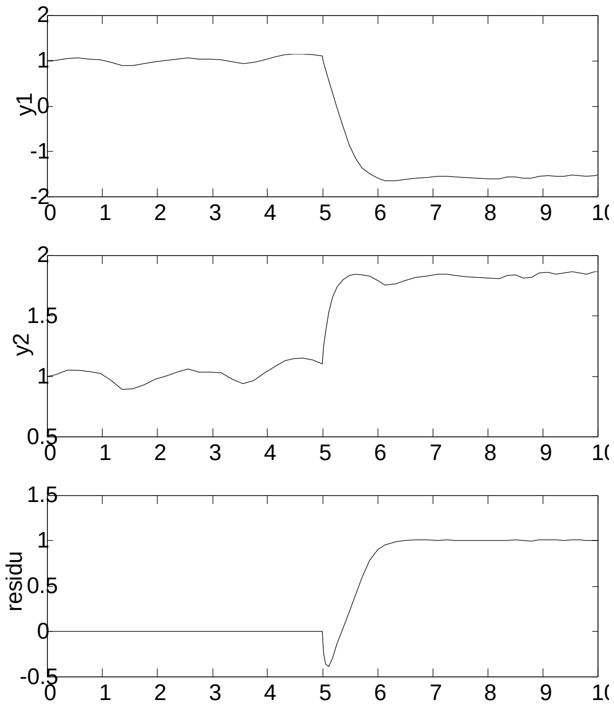

Figure 1: The outputs y_1 et y_2 and the residual r which occur at $5s$.

a good choice of $\lambda_3 = 10$ makes it possible to converge the error towards zero in the absence of the defect. When the defect appear, the error will move away from zero. The results of simulation obtained (1) show outputs y_1 and y_2 which are disturbed and the residual which indicates that at the moment $t = 5s$ a fault occur.

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