Inverse Optimal $\mathcal{H}_\infty$ Disturbance Attenuation for Planar Manipulators with the Eye–in–Hand System

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Abstract

This paper deals with an inverse optimal $\mathcal{H}_\infty$ disturbance attenuation for the planar manipulators with the eye-in-hand system. The input-to-state stability control Lyapunov function (ISS-CLF) is constructed the full Lagrangian dynamics based on a potential function of the image feature parameter space. The ISS-CLF gives us an inverse optimal $\mathcal{H}_\infty$ control law. A proposed controller solves the inverse optimal $\mathcal{H}_\infty$ control problem by minimizing a cost functional, and the closed-loop system with the proposed controller is input-to-state stable. Further, we discuss that the inverse optimal $\mathcal{H}_\infty$ controller has robustness against input uncertainties.

1 Introduction

Vision is a useful robotic sensor since it mimics the human sense of vision and allows for noncontact measurement of the environment. The combination of mechanical control with visual information should become extremely important, when we consider a mechanical system working with targets whose position is unknown. In recent years, the use of the visual information in the feedback loop, called visual servo or visual feedback, has also attracted the attention [1, 2].

There seem to be a consensus that to extract high-level performance from visual-servo robotic systems, the control system must incorporate information about the dynamics/kinematics of the robot and the calibration parameters of the camera system. More recently, the Lyapunov/passivity based methods has been successfully applied to the visual feedback problem. Kelly [3, 4] and the others [5, 6, 7, 9] have derived the Lyapunov based visual feedback controller and its asymptotic stability has been shown. Maruyama and Fujita [6] proposed the robust controller for the visual feedback system against the torque disturbances and the target motions in the $\mathcal{L}_2$ gain sense. In [7, 9], the proposed controller has guaranteed the robustness against the parametric uncertainty. However, these works have not considered the optimality and have only given an upper bound of the cost functional.

In this paper, we employ the inverse optimal $\mathcal{H}_\infty$ control for the planar manipulators with the eye-in-hand system. The inverse optimality approach can avoid the task of solving the Hamilton-Jacobi equation and provide a characterization of the stability margins and the optimality with respect to a certain quadratic cost. This approach was originated by Kalman and introduced into robust nonlinear control via Freeman's robust control Lyapunov functions (CLF's) [10]. Krstić [11] addressed the inverse optimal disturbance attenuation problem from the differential game problem and established the equivalence between the solvability of the problem and the input-to-state stabilizability [12]. In [8], the inverse optimality approach was proposed for the robot manipulator systems.

We construct an ISS-CLF using the full Lagrangian dynamics based on a potential function of the image feature parameter space, called image feature parameter potential. Hence it can be shown that the inverse optimal $\mathcal{H}_\infty$ control problem is solvable for the visual feedback system. Our proposed controller solves the inverse optimal $\mathcal{H}_\infty$ control problem by minimizing the cost functional, and the closed-loop system with the proposed controller is input-to-state stable. Further the robustness against a certain class of input unmodeled dynamics is considered, and the proposed controller achieves input-to-state stability (ISS) in the presence of input uncertainties with stability margins.

This paper is organized as follows. Section 2 reviews the input-to-state stabilizability and the inverse optimal $\mathcal{H}_\infty$ control problems. In Section 3, we show the visual feedback system model and the problem formulation. Section 4 constructs an ISS-CLF and an inverse optimal $\mathcal{H}_\infty$ controller for the visual feedback system. Further, we consider the robustness of the proposed controller. Finally the numerical examples and the conclusions are shown in Section 5 and 6, respectively.

2 Input-to-State Stability and Inverse Optimal $\mathcal{H}_\infty$ Control Problem

In this section, we present preliminary results and definitions [11]. Consider the general nonlinear affine system

$$\dot{x} = f(x) + g_1(x)d + g_2(x)u \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $d \in \mathbf{R}^r$ is the time-varying, unknown but bounded disturbance, $u \in \mathbf{R}^m$ is the control and $f(0) = 0$.

The system (1) is said to be input-to-state stabilizable with respect to the disturbance d if there exists a control law, $u = \alpha(x)$ continuous away from the origin with $\alpha(0) = 0$, which guarantees that

$$|x(t)| \le \beta(|x(0)|, t) + \chi(\sup_{0\le\tau\le t} |d(\tau)|) \tag{2}$$

where β is a class $\mathcal{KL}$ function and χ is a class $\mathcal{K}$ function.

Definition 2.1: A smooth positive definite radially unbounded function V: $\mathbf{R}^n \to \mathbf{R}_+$ is called an ISS-control Lyapunov function(ISS-CLF) for (1) if there exists a class $\mathcal{K}_\infty$ function ρ such that the following implication holds for all $x \neq 0$ and all $d \in \mathbf{R}^r$:

$$|x| \ge \rho(|d|) \Rightarrow \inf_{u\in\mathbf{R}^m} \{\mathcal{L}_f V + \mathcal{L}_{g_1} V d + \mathcal{L}_{g_2} V u\} < 0 \tag{3}$$

The following fact establishes the equivalence between the input-to-state stabilizability and the existence of an ISS-CLF. It extends Sontag's theorem in [13] to systems affine in the disturbance.

Fact 2.1: System (1) is input-to-state stabilizable if and only if there exists an ISS-CLF.

In [11], optimal ISS-stabilizers were designed for the general nonlinear system (1) in a sense similar to the nonlinear $\mathcal{H}_\infty$ control.

Definition 2.2: The inverse optimal $\mathcal{H}_\infty$ control problem for system (1) is solvable if there exist a continuous matrix-valued function $R_1(x)$ such that $R_1(x) = R_1^T(x) \ge 0$ for all x, a matrix-valued function $R_2(x)$ such that $R_2(x) = R_2^T(x) > 0$ for all x, positive definite radially unbounded functions $l(x)$ and $E(x)$, and a feedback law $u = \alpha(x)$ continuous away from the origin with $\alpha(0) = 0$, which minimizes the cost functional

$$J(u) = \sup_{d\in D}\Big\{\lim_{t\to\infty}\Big[E(x(t)) + \int_0^t \big(l(x) + u^T R_2(x) u - d^T R_1(x) d\big) d\tau\Big]\Big\} \tag{4}$$

where D is the set of locally bounded functions of x.

The solvability condition of the inverse optimal $\mathcal{H}_\infty$ control problem is given by the input-to-state stabilizability of the system (1).

Fact 2.2: The inverse optimal $\mathcal{H}_\infty$ control problem for system (1) is solvable if and only if the system is input-to-state stabilizable.

Fact 2.3: Assume that the static state feedback control law

$$u = \kappa(x) := -R_2^{-1}(x)(\mathcal{L}_{g_2} V)^T \tag{5}$$

where $R_2(x)$:$\mathbf{R}^n \to \mathbf{R}^{n\times n}$ is a positive definite matrix-valued function, stabilizes the system in (1) with respect to a positive definite radially unbounded Lyapunov function $V(x)$. Then the control law

$$u = \kappa^*(x) := \beta\kappa(x), \quad \beta \ge 2 \tag{6}$$

solves the inverse optimal $\mathcal{H}_\infty$ problem for system (1).

3 System Model and Problem Statement

3.1 Manipulator Model and Camera Model

The manipulator model considered here is the well-known Euler-Lagrange system whose inputs are joint torques and whose measurement outputs are joint positions and velocities. A pinhole camera, mounted on the hand of the manipulator, is modeled by an ideal perspective transformation.

3.1.1 Manipulator Model: The dynamics of the n-revolute joints rigid manipulator, including the presence of torque disturbances vector $d \in \mathbf{R}^n$, can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + d \tag{7}$$

where $q \in \mathbf{R}^n$ is the joint angles vector, $\tau \in \mathbf{R}^n$ is the vector of control input torques, $M(q) \in \mathbf{R}^{n\times n}$ is the manipulator inertia matrix, $C(q,\dot{q})\dot{q} \in \mathbf{R}^n$ is the Coriolis and centrifugal torques vector and $g(q) \in \mathbf{R}^n$ is the gravitational torques vector. It is well known that the inertia matrix $M(q)$ is positive definite and the matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric.

3.1.2 Camera Model: We consider a planar manipulator with the world frame $\Sigma_w = \{X_w\ Y_w\ Z_w\}$. It is assumed that the manipulator end-effector evolves in the $X_w - Y_w$ plane of Σ_w. Suppose that a camera with the frame Σ_c is mounted on the hand of the manipulator. Hence, the manipulator kinematics gives the camera position ${}^w p_c(q) := [{}^w x_c(q)\ {}^w y_c(q)]^T$ and the orientation ${}^w\theta_c(q)$ with respect to Σ_w. A frame $\Sigma_i = \{X_i\ Y_i\}$ is defined in the camera image plane and its origin is the intersection of the optical axis with the image plane. Here it is assumed that the axes X_i and Y_i parallel the axes X_c and Y_c respectively, and the planes $X_c - Y_c$ and $X_w - Y_w$ are separated by the focal length $l > 0$.

Next the object point ${}^w p_o$ is located at $[{}^w x_o \ {}^w y_o \ {}^w z_o]^T$ with respect to the frame Σ_w. We assume that ${}^w z_o > l$ and the object point motion is restricted on the $X_w - Y_w$ plane, i.e. ${}^w \dot{z}_o = 0$. ${}^i p_o = [{}^i x_o \ {}^i y_o]^T$ is the image coordinate of ${}^w p_o$ through the perspective transformation with the frame Σ_i.

Taking the perspective transformation as the camera model yields [5, 6, 7]

$$ {}^i p_o := f(q) = \frac{s\lambda}{{}^w z_o} R^T({}^w\theta_c(q))({}^w p_o - {}^w p_c(q)) \quad (8) $$

where $s > 0$ is the scaling factor in pixels/m due to the camera sampling, ${}^w p_o := [x_o \ y_o]^T$ and $f : \mathbf{R}^2 \times \mathbf{R}^2 \to \mathbf{R}^2$.

Then the differential kinematics of the manipulator gives a relationship between the manipulator joint velocities $\dot{q}$ and the velocities of the camera mounted on the end-effector. The relation is represented using the manipulator Jacobian $J_p(q) \in \mathbf{R}^{n\times n}$:

$$ {}^w \dot{p}_c(q) = J_p(q)\dot{q}. \quad (9) $$

Now, differentiating (8) yields,

$$ \dot{f} = -\frac{s\lambda}{{}^w z_o} R^T J_p \dot{q} - R^T \dot{R} f. \quad (10) $$

3.2 Visual Feedback Control Problem

In this subsection, we will state the visual feedback control problem. The visual feedback control objective is to design a control input τ such that the camera position ${}^w p_c$ coincides with the object position ${}^w p_o$. ${}^w p_c = {}^w p_o$ is equivalent to $f = 0$, because the scaling factor $\frac{s\lambda}{{}^w z_o}$ is positive and $R({}^w\theta_c(q))$ is nonsingular in the camera model (8).

The following assumptions will be made throughout the paper:

- **A1** There exists a manipulator joint configuration achieved $f = 0$.
- **A2** The Jacobian J_p is the nonsingular matrix.

Under the assumptions A1 and A2, we can formulate the visual feedback control problem as follows:

Visual Feedback Control Problem: For the eye-in-hand system described by the equations (7) and (10) and a static object point, design a control law τ such that $f(q) \to 0$ and $\dot{q} \to 0$ as $t \to \infty$.

4 Inverse Optimal $\mathcal{H}_\infty$ Control of the Visual Feedback System

4.1 ISS-CLF of the Visual Feedback System

In Section 2, we have introduced that the solvability of the inverse $\mathcal{H}_\infty$ problem reduces to the existence of the ISS-CLF. In this section, we will show that the inverse optimal $\mathcal{H}_\infty$ control problem for the visual feedback system is solvable.

Firstly, we define a nominal visual feedback controller [7] as

$$ \tau = u + M(q)(\alpha\dot{\eta}) + C(q,\dot{q})(\alpha\eta) + g(q) \quad (11) $$

where $\eta := J_p^T R f$ and $\alpha \in \mathbf{R}_+$. u is the control input that will be designed to achieve the visual feedback control objective.

Substituting the control law (11) into (7) gives us the following closed loop system

$$ M(q)\dot{\xi} + C(q,\dot{q})\xi = u + d \quad (12) $$

where $\xi := \dot{q} - \alpha\eta$.

Hence, by the variable transformation $x := [\xi^T \ f^T]^T$, the equations (10) and (12) will be transformed as

$$ \begin{aligned} \dot{x} &= f(x) + g_1(x)d + g_2(x)u \quad (13) \\ f(x) &= \begin{bmatrix} -M^{-1}C\xi \\ -\frac{s\lambda}{{}^w z_o} R^T J_p \eta - \frac{s\lambda}{{}^w z_o} R^T J_p \xi - R^T \dot{R} f \end{bmatrix} \\ g_1(x) &= g_2(x) = \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix}. \end{aligned} $$

Remark 4.1: By the assumptions A1 and A2, $x = [\xi^T \ f^T]^T = 0$ is equivalent to $[\dot{q}^T \ f^T]^T = 0$.

Next, we consider the positive definite function (14) as an ISS-CLF candidate which is important for the Lyapunov/passivity based control design.

$$ V(x) = \frac{1}{2}\xi^T M(q)\xi + \frac{{}^w z_o}{2s\lambda}\|f\|^2 \quad (14) $$

Evaluating the time derivative of V along the trajectories to the system (13) gives us

$$ \begin{aligned} \dot{V} &= \mathcal{L}_f V(x) + \mathcal{L}_{g_1}(x)d + \mathcal{L}_{g_2}V(x)u \quad (15) \\ \mathcal{L}_f V(x) &= -\alpha f^T R^T J_p J_p^T R f - f^T R^T J_p \xi \quad (16) \\ \mathcal{L}_{g_1}V(x) &= \mathcal{L}_{g_2}V(x) = \xi^T. \end{aligned} $$

The equation (16) is obtained from the skew-symmetry of the matrix $\dot{M}(q) - 2C(q,\dot{q})$ and $R^T\dot{R}$.

In the case when there are no disturbances, consider the following control input

$$ u = -K_1\xi + \eta \quad (17) $$

where $K_1 = K_1^T > 0$. Then $\dot{V}$ satisfies the following

$$\dot{V} = -\xi^T K_1 \xi - \alpha \eta^T \eta. \tag{18}$$

Hence the asymptotic stability can be confirmed.

For $\|x\| \geq \rho(\|d\|)$, the above equations give us

$$\begin{aligned} &\inf_u \Big\{ \mathcal{L}_f V + \mathcal{L}_{g_1} V d + \mathcal{L}_{g_2} V u \Big\} \\ &\leq \inf_u \Big\{ -\alpha f^T R^T J_p^T J_p R f - f^T R^T J_p \xi \\ &\qquad + \|\xi\| \rho^{-1}(\|x\|) + \xi^T u \Big\} \\ &= \begin{cases} -\alpha f^T R^T J_p J_p^T R f < 0 & \xi = 0 \\ -\infty & \xi \neq 0 \end{cases}. \end{aligned} \tag{19}$$

This fact implies that the positive definite function V is an ISS-CLF. The above discussion can be summarized as follows.

Theorem 4.1: System (13) is input-to-state stabilizable.

Remark 4.2: The above theorem leads to the solvability of the inverse $\mathcal{H}_\infty$ problem.

4.2 Inverse Optimal $\mathcal{H}_\infty$ Control of the Visual Feedback System

The ISS-CLF (14) gives us a solution to the inverse optimal $\mathcal{H}_\infty$ control problem of the system (13). We suppose the control law

$$u = -\beta \mathcal{R}^{-1}(x) (\mathcal{L}_{g_1} V)^T = -\beta \mathcal{R}^{-1}(x) \xi \tag{20}$$

where $\mathcal{R}(x) > 0$. The equation (15) shows that

$$\begin{aligned} 4\dot{V} = -4x^T &\begin{bmatrix} \frac{1}{2}\beta\mathcal{R}^{-1} - I & \frac{1}{2} J_p^T R \\ \frac{1}{2} R^T J_p & \alpha R^T J_p J_p^T R \end{bmatrix} x \\ &+ 4\xi^T d + 4\xi^T u - 4\xi^T \xi + 2\beta \xi^T \mathcal{R}^{-1} \xi. \end{aligned}$$

By the completing-the-squares, we obtain

$$\begin{aligned} -4x^T P(x) x = 4\dot{V} + \|d - 2\xi\|^2 - \|u + 2\mathcal{R}^{-1}\xi\|^2_{\mathcal{R}} \\ -d^T d + u^T \mathcal{R} u + 2(2-\beta) \xi^T \mathcal{R}^{-1} \xi \end{aligned}$$

where

$$P(x) = \begin{bmatrix} \frac{1}{2}\beta\mathcal{R}^{-1} - I & \frac{1}{2} J_p^T R \\ \frac{1}{2} R^T J_p & \alpha R^T J_p J_p^T R \end{bmatrix}.$$

Consider that $\beta = 2$, $\mathcal{R}^{-1}(x) = K_1 + (1 + \frac{1}{4\alpha}) I$, where $K_1 \in \mathbf{R}^{2\times 2} > 0$, $\mathcal{R}^{-1}(x) > 0$, in this case, $P(x)$ satisfies the following inequality

$$\begin{aligned} P(x) &= \begin{bmatrix} K_1 + \frac{1}{4\alpha} I & \frac{1}{2} J_p^T R \\ \frac{1}{2} R^T J_p & \alpha R^T J_p J_p^T R \end{bmatrix} > 0 \\ -4x^T P(x) x &= 4\dot{V} + \|d - 2\xi\|^2 - \|u + 2\mathcal{R}^{-1}\xi\|^2_{\mathcal{R}} \\ &\quad -d^T d + u^T \mathcal{R} u. \end{aligned} \tag{21}$$

Therefore, the following result is obtained.

Theorem 4.2 (Main Result): A control law

$$\begin{aligned} u &= -2\mathcal{R}^{-1}(x) \xi \\ \mathcal{R}^{-1}(x) &= K_1 + (1 + \frac{1}{4\alpha}) I \end{aligned} \tag{22}$$

solves the inverse optimal $\mathcal{H}_\infty$ control problem for (13) by minimizing the cost functional

$$\begin{aligned} J(u) &= \sup_d \Big\{ \lim_{t\to\infty} \big[4V(x(t)) \\ &\qquad + \int_0^t (l(x) + u^T \mathcal{R} u - d^T d) d\tau \big] \Big\} \end{aligned} \tag{23}$$

$$l(x) := 4x^T P(x) x > 0. \tag{24}$$

Further its optimal value is given by

$$J(u) = 4V(x(0)). \tag{25}$$

Proof: The above-mentioned leads to

$$\begin{aligned} l(x) = -4\dot{V} - \|d - 2\xi\|^2 \\ + \|u + 2\mathcal{R}^{-1}\xi\|^2_{\mathcal{R}} - u^T \mathcal{R} u + d^T d. \end{aligned} \tag{26}$$

Substituting (26) into the functional $J(u)$ yields

$$\begin{aligned} J(u) = \sup_d \Big\{ \lim_{t\to\infty} \big[4V(x(0)) \\ - \int_0^t \big(\|d - 2\xi\|^2 - \|u + 2\mathcal{R}^{-1}\xi\|^2_{\mathcal{R}} \big) d\tau \big] \Big\} \end{aligned} \tag{27}$$

It is clearly that the worst case disturbance is given by $d = 2\xi$ which leads to

$$J(u) = 4V(x(0)) + \int_0^\infty \big(\|u + 2\mathcal{R}^{-1}\xi\|^2_{\mathcal{R}} \big) d\tau. \tag{28}$$

Since the positive definiteness of the matrix $\mathcal{R}$, the inverse optimal control law $u = -2\mathcal{R}^{-1}(x)\xi$ minimizes the cost functional (28). The value function of (23)

$$J(u) = 4V(x(0)) \tag{29}$$

is obtained. □

A reason why Theorem 4.2 can be established is that the system (13) is input-to-state stabilizable. Therefore the following proposition shows that the proposed inverse optimal $\mathcal{H}_\infty$ controller achieves the input-to-state stabilizing for the system (13).

Proposition 4.1: The close-loop system, which consists of the proposed controller (22) and the system (13), is input-to-state stable.

Proof: We regard the positive definite function (14) as the candidate of the ISS Lyapunov function. The equation (26) with the control input u provides

$$\begin{aligned} \dot{V} = -\frac{1}{4} l(x) - \frac{1}{4} \|d - 2\xi\|^2 \\ + \frac{1}{4} \|u + 2\mathcal{R}^{-1}\xi\|^2_{\mathcal{R}} - \frac{1}{4} u^T \mathcal{R} u + \frac{1}{4} d^T d. \end{aligned} \tag{30}$$

Substituting (22) and (24) into (30) yields

$$\dot{V} = -x^T P(x)x - \frac{1}{4}\|d - 2\xi\|^2 - \xi^T \mathcal{R}^{-1}(x)\xi + \frac{1}{4}d^T d$$
$$\leq -x^T Q(x)x + \frac{1}{4}d^T d \qquad (31)$$

where

$$Q(x) := P(x) + \begin{bmatrix} \mathcal{R}^{-1}(x) & 0 \\ 0 & 0 \end{bmatrix} > 0 \qquad (32)$$

for all x, since $P(x)$ and $\mathcal{R}^{-1}(x)$ are positive definite. The positive definiteness of $Q(x)$ derives

$$\dot{V} \leq -\inf_x(\sigma(Q(x)))\|x\|^2 + \frac{1}{4}\|d\|^2. \qquad (33)$$

Since $\|x\|^2$ and $\|d\|^2$ are both radially unbounded, V is the ISS Lyapunov function. In other words, the closed-loop system is input-to-state stable. □

4.3 Stability Margins

In order to characterize the class of allowable input uncertainties, we show the definition of strict passivity.

Definition 4.1 [11]: A nonlinear system,

$$\dot{\chi} = \hat{f}(\chi) + \hat{g}(\chi)\hat{u}$$
$$\hat{y} = \hat{h}(\chi) \qquad (34)$$

is said to be strict passive, if there exist a strange function $\hat{V}(\chi)$ which satisfies the following dissipative inequality, for all $\hat{u}$, $\chi(0)$, $t \geq 0$,

$$\int_0^T \hat{y}^T \hat{u} d\tau \geq \hat{V}(\chi(t)) - \hat{V}(\chi(0)) + \int_0^t \psi(\|\chi(\tau)\|)d\tau \qquad (35)$$

where ψ is a class $\mathcal{K}_\infty$ function.

Theorem 4.3: Consider the visual feedback system with input uncertainties in Fig. 1, so that

$$\dot{x} = f(x) + g_1(x)d + g_2(x)\nu \qquad (36)$$
$$\nu = a(u + \mathcal{R}^{-\frac{1}{2}}\hat{y})$$
$$\dot{\chi} = \hat{f}(\chi) + \hat{g}(\chi)\hat{u}, \ \ \hat{y} = h(\chi)$$
$$\hat{u} = \mathcal{R}^{\frac{1}{2}}u$$

where $a \in [\frac{1}{2}, \infty)$ and the ξ-system, $\mathcal{P}$ is strict passive. Then the proposed controller (22) is input-to-state stabilizing for the above closed system (36).

Proof: The visual feedback system (13) is ISS and the system $\mathcal{P}$ is strict passive, there exist the positive definite functions $V(x)$ and $\hat{V}(\chi)$ that satisfy (30) and (35) respectively. Consider the following ISS-Lyapunov function candidate

$$V_c = V + \frac{a}{2}\hat{V}. \qquad (37)$$

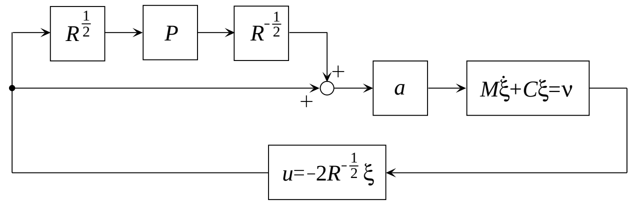

Figure 1: Input uncertainties

Differentiating V_c with respect to time along the solutions of the closed loop systems (22) and (36) gives

$$\dot{V}_c \leq -x^T Px + 2(\frac{1}{2} - a)\xi^T \mathcal{R}^{-1}\xi + \frac{1}{4}d^T d - \frac{a}{2}\psi(\|\chi\|).$$

Observing that $a > \frac{1}{2}$ and $\mathcal{R}^{-1} > 0$, $\dot{V}_c$ satisfies

$$\dot{V}_c \leq -x^T Px - \frac{a}{2}\psi(\|\chi\|) + \frac{1}{4}\|d\|^2. \qquad (38)$$

Since $x^T P(x)x$ and $\psi(\|\chi\|)$ are radially unbounded, V_c is the ISS Lyapunov function. □

In short there exists a control law that achieves input-to-state stability in the presence of input uncertainties with $a \geq \frac{1}{2}$(Gain Margin) and $\mathcal{P}$ strict passive(Phase Margin) as depicted in Fig. 1.

5 Numerical Example

Computer simulations have been carried out to confirm the effectiveness of the proposed visual feedback control design. Suppose a 2 DOF planar manipulator with a CCD camera. The planar manipulator has the following regressor [14].

$$Y(q, \dot{q}, v, a) = \begin{bmatrix} a_1 & a_1 + a_2 & Y_{13}(q, \dot{q}, v, a) \\ 0 & a_1 + a_2 & Y_{23}(q, \dot{q}, v, a) \end{bmatrix.$$
$$\begin{matrix} \cos q_1 & \cos q_1 & \cos(q_1 + q_2) \\ 0 & 0 & \cos(q_1 + q_2) \end{matrix}\Bigg]$$

where $v = [v_1 \ v_2]^T \in \mathbf{R}^2$, $a = [a_1 \ a_2]^T \in \mathbf{R}^2$, $Y_{13} = (2a_1 + a_2)\cos q_2 - (\dot{q}_2 v_1 + \dot{q}_1 v_2 + \dot{q}_2 v_2)\sin q_2$, and $Y_{23} = a_1 \cos q_2 + \dot{q}_1 v_1 \sin q_2$. Let us assume that the inertia parameters θ are given by

$$\theta = \begin{bmatrix} 0.25 & 0.028 & 0.033 & 7.57 & 4.08 & 1.63 \end{bmatrix}^T.$$

It is assumed that the focal length λ multiplied by the scale factor s, i.e. $s\lambda$, is equal to 2180, the controller is running at 1000 Hz, and the image processing unit computes the centroid of the object point at 120 Hz.

The initial camera position vector and orientation is $[{}^w x_c(0) \ {}^w y_c(0)]^T = [0.20 \ 0.20]^T$[m]. The static object point is located at $[{}^w x_o \ {}^w y_o]^T = [0.34 \ \ -0.14]^T$[m].

The true depth parameter in this simulation is ${}^{w}z_o = 1.86$[m]. The controller gains were chosen as $K_1 = \mathrm{diag}\{10, 10\}$ and $\alpha = 1$. Fig. 2 shows the response of the image position errors without input uncertainties, i.e. $d = 0$, $a = 1$, and $\mathcal{P} = 0$. After the transient, the image position errors tend asymptotically to zero.

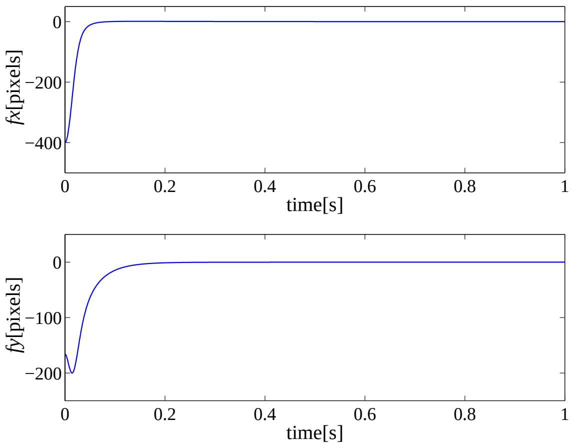

Figure 2: Image Position Errors without Input Uncertainties

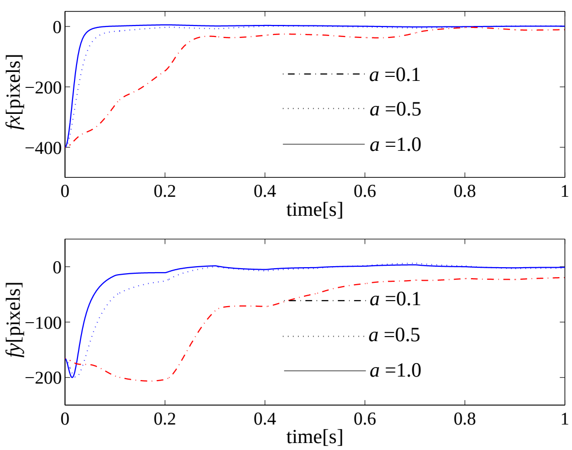

Figure 3: Image Position Errors with Input Uncertainties

Fig. 3 shows the response of the image position errors with input uncertainties. Consider that d is white noise, $a = 0.1 \sim 1$, and $\mathcal{P}$ is the strictly positive real transfer function $\frac{5}{\varsigma+10.0}I$, where ς stands for Laplace operator. In this case, the image position errors tend asymptotically to a neighborhood of zero.

6 Conclusions

In this paper, the inverse optimal $\mathcal{H}_\infty$ disturbance attenuation of the planar manipulators with the eye-in-hand system has been discussed. In particular, we showed that the visual feedback system has the ISS-CLF and proposed the inverse optimal $\mathcal{H}_\infty$ control law. Moreover, it was shown that the proposed controller guarantees the robustness against input uncertainties.

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