Control for Bounded Pseudo ARMAX Stochastic Systems via Linear B-Spline Approximations

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Abstract

Following the recently developed algorithms for the control of the shape of the output probability density function for general dynamic stochastic systems ([2]-[3]), this short paper presents the modelling and control algorithms for pseudo ARMAX systems, where different from all the existing ARMAX systems the considered system is subjected to any arbitrary bounded random input and the purpose of the control input design is to make the output probability density function of the system output as close as possible to a given distribution function. At first, the relationship between the input noise distribution and the output distribution is established. This is then followed by the description on the control algorithm design.

1 Introduction

Over recent decades, research into the control for stochastic systems has been focussed on the control of the output of the system, rather than the probability density function of the system output. A number of well known algorithms have thus been developed and successfully used in many practical industrial systems. Typical examples are minimum variance control, self-tuning control ([1]) and stochastic linear quadratic control. In most existing approaches, it has been assumed that all the variables in the system obey a Gaussian-type distribution. This is based upon the fact that most input noise can be characterised as coloured noise which can be regarded as being generated by a white noise sequence. However, this assumption is restrictive for some applications. This is particularly true for many control processes in the wet end of papermaking machines, where the control of the shape of the probability density function of process variables is required.

To solve the problem of modelling and control for non-Gaussian systems, recently a number of modelling and control algorithms have been developed at UMIST ([2]-[3]). The stochastic systems considered are subjected to arbitrary bounded noise inputs whose probability density functions are defined on a known and bounded interval. For these type of non-Guassian systems, the purpose of control algorithm design is to make the probability density function of the system output to be as close as possible to a given distribution function. In this context, the information available to the controller should be

- the measured probability density function of the system output,
- the probability density function of the desired (given) distribution.

To simplify the formulation of the required algorithms, B-spline approximation has been used to represent the output probability density function of the system and the weights of such an expansion are used to link the control input dynamically. Denote u as the crisp input to the system and $\gamma_y(x,u)$ as the output probability density function defined on a known bounded interval $[a,b]$, at present the following three types of models have been used to approximate $\gamma_y(x,u)$

$$\begin{aligned}\gamma_y(x,u) &= \sum_{i=1}^{n} w_i(u)B_i(x)\\ \gamma_y(x,u) &= (\sum_{i=1}^{n} w_i(u)B_i(x))^2\\ \gamma_y(x,u) &= \frac{\sum_{i=1}^{n} w_i(u)B_i(x)}{\sum_{i=1}^{n} w_i(u)b_i}\end{aligned} \quad (1)$$

where w_i are the weights which are dynamically related to the control input u, $b_i=\int_a^b B_i(x)dx$ and $B_i(x)$ are the pre-specified basis functions. Using these approximations, the dynamic part of the system can be generally expressed by a set of differential equations which links the weights with the input u, thus realising a total decoupling of the considered stochastic system. In terms of controllability, these approximations are all full space controllable if the dynamics between the weights and the input are completely controllable.

Of these three models, the first one generally provides a simple solution to the modelling and control algorithm design. However, it cannot always guarantee the positiveness of the output probability density functions. As such, it should be used with caution when some of the weights are negative. As for the second model, it can guarantee the positiveness of the output probability density function. However, the output equation thus obtained is nonlinear, leading to some difficulties in control algorithm design. The third model is nonlinear and also relies on the positiveness of the weights so as to guarantee the positiveness of the output probability density function. Of course, for the third model, the uniqueness of the weights are not guaranteed. Indeed, if the weight set $W^* = (w_1, w_2, \cdots, w_n)$ can approximate the output probability density function $\gamma_y(x, u)$, then it can be seen that all the weight sets, which are proportional to W^*, can also approximate $\gamma_y(x, u)$.

However, there are still several open problems that need to be solved. In particular, the key assumption that the control input only affects the weights of the output probability density function is strict for some applications. This is particularly true for the establishment of physical models. As such, it would be ideal if a more general model than those established in ([2] - [3]) can be developed. This forms the purpose of using the pseudo ARMAX model described in this paper, where the modelling and control of bounded and dynamic stochastic systems, represented by an ARMAX model, will be considered. Since the linear approximation (see the first equation in (1)) generally gives a simplified formulation, it will be used here to formulate the relationships between the input and the output probability density functions. Such relationships are the keys to solve the design problem for the control input.

2 Pseudo ARMAX Systems

In this paper, it is assumed that the model relates the input sequence ($\{u_k\}$), the output sequence of the system ($\{y_k\}$) and a stochastic noise term ($\{n_k\}$) through the following linear ARMAX model

$$A(z^{-1})y_k = z^{-d}B(z^{-1})u_k + n_k; \quad k = 0, 1, 2, ... \tag{2}$$

where $y_k \in R^1$ and $u_k \in R^1$ are one dimensional output and input of the system, and

$$A(z^{-1}) = 1 + \sum_{i=1}^{n} a_i z^{-1} \tag{3}$$

$$B(z^{-1}) = \sum_{j=0}^{m} b_j z^{-j} \tag{4}$$

are known polynomials of the unit back-shift z^{-1} and $d \geq 0$ is the time delay. In this paper we mainly consider the case that $A(z^{-1})$ is a stable polynomial.

Since $A(z^{-1})$ and $B(z^{-1})$ are known, equation (2) can be further expressed as a d-step-ahead predictive form to give

$$y_{k+d} = G(z^{-1})y_k + F(z^{-1})B(z^{-1})u_k + F(z^{-1})n_{k+d} \tag{5}$$

where $F(z^{-1})$ and $G(z^{-1})$ are known polynomials of the orders $d-1$ and $n-1$, and are obtained by solving the following Diophantine equation ([1])

$$1 = F(z^{-1})A(z^{-1}) + z^{-d}G(z^{-1}) \tag{6}$$

Denote

$$\omega_k = F(z^{-1})n_k \tag{7}$$

and assume that ω_k is a bounded stochastic distribution whose continuous probability density function at sample time k is denoted by $\gamma_\omega(x, k)$ which is defined on a known interval $x \in [\alpha_1, \beta_1]$ as

$$P\{\alpha_1 \leq \omega_k < x\} = \int_\alpha^x \gamma_\omega(\xi, k)d\xi \tag{8}$$

where $P\{\alpha \leq \omega_k < x\}$ is the probability for event $\{\alpha \leq \omega_k < x\}$. Thus, under the assumption that $A(q^{-1})$ is stable and u_k is bounded, the output sequence y_k of equation (2) is also a bounded stochastic process at sample time k. This means that the probability density function of y_k is defined on another bounded interval $[\alpha_2, \beta_2]$ and is of course related to the control input u_k. Denoting such a probability density function as $\gamma_y(x, u, k)$ with

$$P\{\alpha_1 \leq y_k < x\} = \int_\alpha^x \gamma_y(\xi, u, k)d\xi \tag{9}$$

and using the recently developed algorithms on the modelling and control of bounded stochastic distributions ([2]-[3]), functions $\gamma_\omega(x, k)$ and $\gamma_y(x, u, k)$ can all be approximated by the expansion of a set of pre-specified basis functions, $B_i(x)$, defined on $[\alpha, \beta]$ with

$$\alpha = \min(\alpha_1, \alpha_2) \tag{10}$$

$$\beta = \max(\beta_1, \beta_2) \tag{11}$$

This leads to the following two B-spline approximations to $\gamma_\omega(x, k)$ and $\gamma_y(x, u, k)$

$$\gamma_\omega(x, k) = \sum_{i=1}^{M} w_i(k)B_i(x) \tag{12}$$

$$\gamma_y(x, u, k) = \sum_{i=1}^{M} v_i(u, k)B_i(x) \tag{13}$$

where, similar to ([2], [3]), $B_i(x)$ are the **pre-specified** basis functions, $w_i(k)$ and $v_i(u, k)(i = 1, 2, ...)$ are the weights of the B-spline expansions in (12) and (13). The aim of this type of approximation is to use B-spline expansions to approximate the considered output probability density function. For this approximation, another constraint is that the following equalities

$$\int_{\alpha}^{\beta} \gamma_\omega(x,k)dx = 1 \quad (14)$$

$$\int_{\alpha}^{\beta} \gamma_y(x,u,k)dx = 1 \quad (15)$$

should hold for both $\gamma_\omega(x,k)$ and $\gamma_y(x,u,k)$ as they are both probability density functions. This means that the weights in each approximation are dependent. Denote

$$C_o(x) = (B_1(x), B_2(x), \cdots, B_{M-1}(x)) \quad (16)$$

$$W(k) = (w_1(k), w_2(k), \cdots, w_{M-1}(k))^T \quad (17)$$

$$V(k) = (v_1(u,k), \cdots, v_{M-1}(u,k))^T \quad (18)$$

then it can be seen from equations (12) and (13) that

$$\gamma_\omega(x,k) = C_0(x)W(k) + w_M(k)B_M(x) \quad (19)$$

$$\gamma_y(x,u,k) = C_0(x)V(k) + v_M(u,k)B_M(x) \quad (20)$$

By substituting equations (19) and (20) into the two constraints in equations (14) and (15), it can be found out that $w_M(k)$ and $v_M(u,k)$ are linearly dependent on $W(k)$ and $V(k)$ as follows

$$\begin{aligned} w_M(k) &= \frac{1}{\int_\alpha^\beta B_n(x)dx}(1 - \int_\alpha^\beta C_o(x)dx)W(k) \\ &= h(W(k)) \quad (21) \\ v_M(u,k) &= \frac{1}{\int_\alpha^\beta B_n(x)dx}(1 - \int_\alpha^\beta C_o(x)dx)V(k) \\ &= l(V(k)) \quad (22) \end{aligned}$$

where $h(.)$ and $l(.)$ denote such linear relationships. By substituting equations (21) and (22) into equations (19) and (20), it can be further obtained that

$$\gamma_\omega(x,k) = C(x)W(k) + L(x) \quad (23)$$

$$\gamma_y(x,u,k) = C(x)V(k) + L(x) \quad (24)$$

$$L(x) = \frac{B_n(x)}{\int_\alpha^\beta B_n(x)dx}$$

$$C(x) = C_o(x) - \frac{\int_\alpha^\beta C_o(x)dx}{\int_\alpha^\beta B_n(x)dx}B_n(x) \quad (25)$$

Since the system is represented by equation (2) which says that the system output y_k is related to both u_k and ω_k, there are certain mathematical relationships between $\gamma_y(x,u,k)$ and $\gamma_\omega(x,k)$. Also, because after the B-spline approximations all the stochastic characters of the noise ω_k are determined by the weights $w_i(k)$, this means that the expansion weights, $v_i(u,k)$ should be regarded as a function of both $w_i(k)$ and u_k. As a result, it can be seen that at sample time k the shape of the output probability density function of the system (2) is controlled by the input and the noise pattern characterised by weights $w_i(k)$.

Different from all the existing stochastic control systems documented so far, here the dynamic part of the system is still described by the ARMAX model. However, this is not a normal ARMAX model as the noise here is an arbitrarily bounded random signal, rather than the widely used Gaussian noise. As a result, this model is called pseudo ARMAX model. Based on the discussions so far, it can be seen that pseudo ARMAX model is a more general expression for dynamic stochastic systems as there is no specific assumption on the input noises. This model is a starting point in seeking more general solution to the modelling and control of dynamic stochastic systems subjected to arbitrary bounded random input. In this paper the control of such a system will be considered where the purpose of the controller design is to achieve the shape control of the output probability density function of the system, rather than the output itself.

3 Controller Design

The purpose of the controller design is to select a sequential inputs u_k so that the probability density function of y_k tracks a given distribution function $g(y)$. This will be achieved by minimising the following performance index at sample time k

$$J = \int_{x\in[\alpha,\beta]} (\gamma_y(x,u,k+d) - g(x))^2 dx + Ru_k^2 \quad (26)$$

where the first term provides a metric to measure the difference between $\gamma_y(x,u,k+d)$ and $g(y)$, and the second term reflects the constraints of the input energy. This means that the actual output distribution is made as close as possible to its desired distribution whilst the energy of the input will be minimised.

To derive the required control sequence, it is imperative that the relationships between v_i, u_k and w_i are established. For this purpose, denote

$$G(z^{-1}) = \sum_{i=0}^{n-1} g_i z^{-i} \quad (27)$$

$$H(z^{-1}) = F(z^{-1})B(z^{-1}) = \sum_{i=0}^{m+d-1} h_i z^{-i} \quad (28)$$

and assume that the current sample time is k, then the following term

$$\eta_k = G(z^{-1})y_k + (H(z^{-1}) - h_0)u_k \quad (29)$$

is known. As such, it can be further obtained from equation (5) that

$$y_{k+d} = \eta_k + h_0 u_k + \omega_{k+d} \quad (30)$$

which reveals the relationship between the two random variables, y_k and ω_k at sample time $k+d$. In this equation, u_k is the control input to be designed. This relationship links the two probability density functions, $\gamma_y(x,u,k+d)$ and $\gamma_\omega(x,k+d)$ in the following way

$$\gamma_y(x,u,k+d) = \gamma_\omega(x-\eta_k-h_0u_k, k+d) \qquad (31)$$

Using equations (12) and (13), it can be seen that

$$\begin{aligned}\gamma_\omega(x-\eta_k-h_0u_k,k+d) &= \sum_{i=1}^{M} w_i(k+d) \\ &\times\ B_i(x-\eta_k-h_0u_k) \\ &. \qquad (32)\end{aligned}$$

From equation (32), it can be seen that the control input only affects the shift of the output probability density function $\gamma_y(.)$ in response to the shape of the probability density function $\gamma_\omega(x)$. As a result, the controller thus obtained can only control the major shift rather than the arbitrary shape of the output density function. This is simply because the existence of nonlinear transform shown in equation (32), whereby the controllable space of the system is not, in any sense, the whole space spanned by weights, v_j. In fact, since the shift is a nonlinear operation, only a subspace spanned by v_j is controllable. This is a vital difference in comparison to the models used in ([2]-[3]), where the system is always controllable. However, the advantage of using pseudo ARMAX model here is that some possible dynamics of the real systems can be expressed in a normal way.

For each index i, $B_i(x)$ is a continuous function defined on $[\alpha,\beta]$. This means that functions $B_i(x-\eta_k-h_0u_k)$, as shown in Fig. 1, can be further expressed by B-spline expansion with the same set of basis function to give

$$B_i(x-\eta_k-h_0u_k) = \sum_{j=1}^{M} \sigma_{ij}(\eta_k+h_0u_k)B_j(x) \qquad (33)$$

where $\{\sigma_{ij}(\eta_k+h_0u_k)\}$ are another set of weights for the B-spline expansion (33). Since

$$\eta_k = \theta^T\phi(k) \qquad (34)$$

with

$$\begin{aligned}\theta &= (g_0,g_1,\cdots,g_{n-1}, \\ &\quad h_1,h_2,\cdots,h_{m+d-1})^T \\ &\in R^{n+m+d-1} \qquad (35)\\ \phi(k) &= (y_k,\cdots,y_{k-n},u_{k-1}, \\ &\quad \cdots,u_{k-d-m+1})^T \in R^{n+m+d-1} \qquad (36)\end{aligned}$$

it can be seen that

$$\sigma_{ij}(\eta_k+h_0u_k) = \sigma_{ij}(\theta^T\phi(k)+h_0u_k) \qquad (37)$$

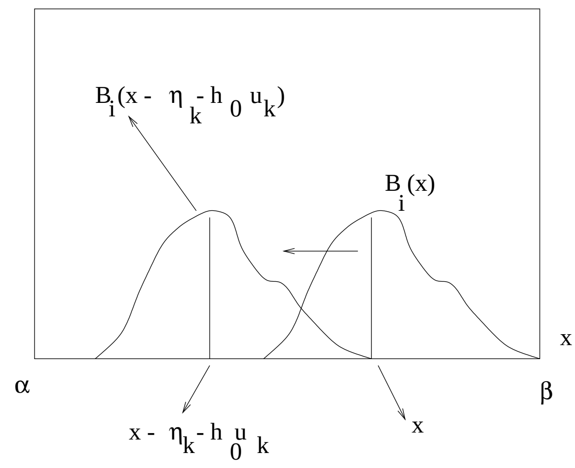

Figure 1: A shift controlled by $\eta_k+h_0u_k$.

As a result, by substituting equation (33) into equation (32), we have

$$\begin{aligned}\gamma_y(x,u,k+d) &= \sum_{i=1}^{M} w_i(k+d) \\ &\quad \sum_{j=1}^{M}\sigma_{ij}(\eta_k+h_0u_k)B_j(x) \\ &= \sum_{j=1}^{M}\Big(\sum_{i=1}^{M} w_i(k+d)\sigma_{ij}(\eta_k+h_0u_k)\Big) \\ &\quad \times B_j(x) \qquad (38)\end{aligned}$$

This means that the relationships between $v_j(u,k+d)$ and $w_i(k+d)$ can be expressed as

$$v_j(u,k+d) = \sum_{i=1}^{M} w_i(k+d)\sigma_{ij}(\theta^T\phi(k)+h_0u_k) \qquad (39)$$

Indeed, this is an important equation which relates the weights v_j of the output probability density function with all the measurable past inputs, the outputs and the characteristics of the input noise which has been represented by the weights $w_i (i=1,2,...)$ in equation (12). Indeed, this equation can be written in the following matrix format

$$V(k+d) = \Sigma(\theta^T\phi(k)+h_0u_k)W(k+d) \qquad (40)$$

where

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \cdots & \sigma_{(M-1)1} \\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{(M-1)2} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{1(M-1)} & \sigma_{2(M-1)} & \cdots & \sigma_{(M-1)(M-1)} \end{bmatrix} \qquad (41)$$

and

$$\Sigma = \Sigma(u_k) \in R^{(M-1)\times(M-1)} \qquad (42)$$

is a square matrix function of $\theta^T\phi(k) + h_0u_k$. Also, the constraints on the dependency of $v_M(k+d)$ and $w_M(k+d)$ on $V(k+d)$ and $W(k+d)$ are determined by equations (21) and (22). By substituting equation (39) into the performance index function (26), J can be expressed explicitly and nonlinearly in terms of u_k to give

$$\begin{aligned} J &= \int_{x\in[\alpha,\beta]} (\gamma_y(x,u,k+d) - g(x))^2 dx + Ru_k^2 \\ &= V(k+d)^T (\int_\alpha^\beta C(x)C^T(x)dx))V(k+d) \\ &+ 2(\int_\alpha^\beta C(x)(L(x)-g(x))dx)V(k+d) \\ &+ \int_\alpha^\beta (L(x)-g(x))^2 dx + Ru_k^2 \qquad (43) \\ &= W^T(k+d)\Sigma^T(u_k)\Lambda_o\Sigma(u_k)W(k+d) \\ &+ \Omega\Sigma(u_k)W(k+d) + Ru_k^2 \qquad (44) \end{aligned}$$

where

$$\Lambda_o = \int_\alpha^\beta C(x)C^T(x)dx \qquad (45)$$

$$\Omega = \int_\alpha^\beta C(x)(L(x)-g(x))dx \qquad (46)$$

are known matrices once all the basis functions are prespecified. Since $V(k+d)$ is nonlinearly related to the control input u_k through equation (40), it can be seen from equation (43) that J is nonlinearly related to u_k as well. Using equations (43) and (46), it can be seen that

$$J = J_0(\theta^T\phi(k) + h_0u_k) + Ru_k^2 \qquad (47)$$

where $J_0(.)$ groups all the nonlinear terms related to $V(k+d)$ in equation (43). As a result, the optimal control sequence ([1]) can be obtained from

$$\frac{\partial J}{\partial u_k} = 0 \qquad (48)$$

This leads to

$$2Ru_k + \frac{\partial J_0}{\partial u_k} = 0 \qquad (49)$$

or

$$\begin{aligned} 0 &= W^T(k+1)\frac{\partial\Sigma(u_k)}{\partial u_k}\Lambda_o\Sigma(u_k)W(k+1) \\ &+ \Omega\frac{\partial\Sigma(u_k)}{\partial u_k} + Ru_k \qquad (50) \end{aligned}$$

In this context, the main aims and objectives of finding out an optimal control sequence u_k is to solve the nonlinear equation (48). If this equation can be solved, it can be seen that the resulting control input at sample time k (i.e., u_k) is a function of $W(k)$, the given distribution function $g(x)$ and all the past inputs and outputs of the system. However, due to the nonlinear relationship between J and u_k, such a solution would always lead to a nonlinear control law. Indeed, an analytical solution to equation (48) is difficult to obtain in most cases as the involvement of term $\theta^T\phi(k) + h_0u_k$ is originally started from the expansion of the basis functions $B_i(x - \eta_k - h_0u_k)$ in equation (33). Since η_k controls the shift of the basis function $B_i(x)$ (see Fig. 1), it can be seen that normally the relationships between $\sigma_{ij}(.)$ $(i,j = 1,2,...,M)$ and $\eta_k + h_0u_k$ are nonlinear. As such, it is natural to use the well known gradient rule to minimise J in equation (43), this leads to the following recursive calculation for an optimal or at least a local optimal control input u_k at the current sample time k

$$u_k^{p+1} = u_k^p - \lambda\frac{\partial J}{\partial u_k}|_{u_k=u_k^p} \qquad (51)$$

where $p = 0,1,2,...,N$ is an integer and the initial condition for the updating of u_k at sample time k is

$$u_k^0 = u_{k-1} \qquad (52)$$

In this case, N is a pre-specified number indicating the total iteration at the current sample time k.

4 Discussions

4.1 Control of unstable system

Although the system considered here is assumed to be stable, similar control algorithm can also be formulated for unstable and known systems. In this case, assume that the polynomial $A(z^{-1})$ is unstable, then since both $A(z^{-1})$ and $B(z^{-1})$ are known, there are two polynomials $P(z^{-1})$ and $Q(z^{-1})$ such that the following polynomial

$$T(z^{-1}) = A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) \qquad (53)$$

becomes stable. As such, by using the following preliminary feedback controller

$$P(z^{-1})u_k = -Q(z^{-1})(y_k + v_k) \qquad (54)$$

it can be seen, in terms of y_k and v_k, that we have

$$T(z^{-1})y_k = B(z^{-1})Q(z^{-1})v_k + P(z^{-1})n_k \qquad (55)$$

which has the same properties as equation (2). Since $P(z^{-1})n_k$ is also a bounded noise input, the control algorithm design in sections 2 - 3 can still be applied to design signal v_k. This means that the similar principle to these used in sections 2 - 3 can also be used to design the output probability density function control for unstable systems. The detailed structure of such a control system is shown in Fig. 2.

4.2 Nonlinear ARMAX model

Instead of linear pseudo ARMAX model, we can also

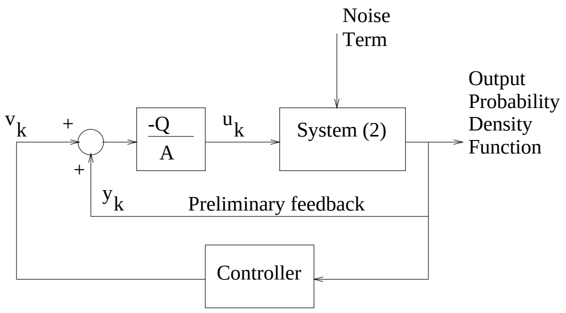

Figure 2: Control of Unstable System via Preliminary feedback loop.

consider nonlinear pseudo ARMAX system represented by

$$\begin{aligned} y_k &= f(y_{k-1}, y_{k-2}, \cdots, y_{k-n}, \\ &\quad u_{k-d}, u_{k-d-1}, \cdots, u_{k-d-m}, \\ &\quad n_k, n_{k-1}, \cdots, n_{k-l}) \end{aligned} \tag{56}$$

where $f(.)$ is a known nonlinear function and n_k is a bounded noise term whose stochastic character is represented by its probability density function $\gamma_n(x)$. For such systems, the control problem is still to find a control input u_k so that the probability density function of the system output y_k is made as close as possible to a give distribution. In this case, since the relationship between y_k and n_k is nonlinear, the control should be more than just a simple shift of the probability density functions as shown in Fig. 1. For example, let us consider the following case

$$\begin{aligned} y_k &= f(y_{k-1}, y_{k-2}, \cdots, y_{k-n}, \\ &\quad u_{k-1}, u_{k-2}, \cdots, u_{k-m}, n_k) \end{aligned} \tag{57}$$

Assuming that the nonlinear function $f(.)$ is monotone with respect to n_k, then there is a unique inverse function $f^{-1}(.)$ such that at sample time k,

$$\begin{aligned} n_{k+1} &= f^{-1}(y_k, y_{k-1}, \cdots, y_{k-n+1}, \\ &\quad u_k, u_{k-1}, \cdots, u_{k-m}, y_{k+1}) \end{aligned} \tag{58}$$

As a result, the output probability density function for y_{k+1} can be formulated to give

$$\gamma_y(x, u_k) = \gamma_n(f^{-1}(\pi_k, x)) \left| \frac{df^{-1}(\pi_k, x)}{dx} \right| \tag{59}$$

where it has been defined that

$$\begin{aligned} \pi_k &= (y_k, y_{k-1}, \cdots, y_{k-n+1}, \\ &\quad u_k, u_{k-1}, \cdots, u_{k-m}) \end{aligned} \tag{60}$$

which is dependent on all the up-to-k measurements of the system and the control input u_k to be decided. This means that the output probability density function of the system is nonlinearly related to the control input. Let the desired distribution function of the system be $g(y)$, then the controller design can be realised by solving the following problem

$$g(y) = \gamma_n(f^{-1}(\pi_k, x)) \left| \frac{df^{-1}(\pi_k, x)}{dx} \right| \tag{61}$$

It can be seen that once the weights of the B-spline approximation to $\gamma_n(x)$ are known, the design procedure is straightforward.

5 Conclusions

In this paper, bounded distribution control problem has been addressed for a class of dynamic stochastic systems represented by ARMAX model which is subjected to arbitrary bounded random input. Using the B-spline functions to approximate the probability density functions of the system random input and the output an optimal control strategy has been developed which controls the shape of the output probability density function so as to make it as close as possible to a given distribution $g(x)$. It has been shown that

- the weights of the B-spline expansion of the output probability density function are nonlinearly related to the measured input and the output of the system, and
- the solution to this problem is equivalent to a nonlinear optimization problem where only a local optimal solution can be obtained.

Discussions on the control of unstable systems and general nonlinear pseudo ARMAX systems have been made, where it has been shown that similar procedure for the control design can be applied.

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