PERFORMANCE OPTIMIZATION OF DISCRETE EVENT SYSTEMS WITH FAILURES USING FLUID PETRI NETS

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ABSTRACT

Performance evaluation and optimization of failure-prone discrete event systems are addressed in this paper. Our analysis is based on a fluid stochastic event graph model that is a decision-free Petri net. In a fluid Petri nets, each place holds a continuous flow instead of discrete tokens of conventional Petri nets. A transition can be in operating state or in failure state. A transition in operating state can fire at its maximal speed and a transition in failure state cannot fire. Jumps between failure and operating states are independent of the firing conditions and the sojourn time in each state is a random variable of general distribution. For performance evaluation, a set of evolution equations that determines continuous state variables at epochs of discrete events is established. Based on the evolution equations, we prove that the cumulative firing of transitions are Lipschitz continuous, non-decreasing and concave functions of system parameters including maximal firing rates and the initial marking. Gradient estimators are derived and their properties established. Finally, an optimization problem that maximizes a concave function of throughput rate and the system parameters is addressed.

1. INTRODUCTION

Fluid approximation is a popular approach to cope with the state explosion, a common phenomenon in the performance evaluation and optimization of discrete-event systems. It has been applied to the performance analysis of production lines, semiconductor manufacturing systems, telecommunication systems, etc.

In this paper, we propose a generic tool for analyzing and optimizing manufacturing systems subject to failures. This tool is based on the fluid approximation of event graphs, an elementary class of Petri nets and called fluid event graphs. In a fluid event graph, places hold fluids instead of discrete tokens. Each transition fires continuously and drawing fluids out of its input places and injecting fluids into its output places. The firing speed of each transition is limited by a maximal speed. We further assume that each transition can be either in operating state or in failure state. A transition in operating state can fire at its maximal speed and a transition in failure state cannot fire at all. It is clear that the system is a hybrid system in the sense that it has both discrete-event component characterized by failures and repairs of transitions and continuous component characterized by markings (or fluids) and transition firing speeds. The continuous component can be modeled using differential equations. We assume that the continuous component is governed by the discrete-event component while the discrete-event component is independent of the continuous component. The first part of the assumption is obvious since a failure event forbids the firing of a transition and a repair event restores the firing of a transition. The independence of the discrete-event component vs. the continuous component is related to the so-called time-dependent failures often used in flow control of failure-prone manufacturing systems. The fluid stochastic event graph model includes as special cases many systems such as transfer lines, kanban controlled systems, base-stock systems, etc.

It is our belief that the fluid stochastic event graphs considered in this paper do not have analytical solutions for its performance evaluation. Simulation will be used for performance evaluation. For this purpose, we establish a set of recursive equations that determines the continuous state variables, i.e. cumulative firing quantities, at epochs of discrete events. The evolution equations allow us to prove the properties of sample-path performances, finite time average performances and long-term performances. Gradient estimators are derived. Unbiasedness and strong consistency of the gradient estimators are proved. Evolution equations are also used in solving an optimization problem with concave criterion function of the throughput rate and the system parameters including the maximal speeds and the initial marking. The concavity makes it possible to approximate the optimal solution by the optimal solution with respect to a single sample path, i.e. a single sequence of discrete events and event epochs. The sample-path optimization approach is similar to the one proposed in [6].

The fluid stochastic event graphs considered in this paper are piecewise deterministic control systems (PDCS). Piecewise deterministic control systems have been addressed by many authors. Most of the existing works are motivated by the optimal control of manufacturing flow. Among them, convergence of stochastic approximation algorithms coupled with perturbation analysis was addressed in [5] under a fairly general framework. However, conditions of this paper are difficult to check and its results difficult to apply. Perturbation analysis was also applied in [3] to the flow controller design of manufacturing systems without internal buffers and with constant demand rates. A two-machine production line with constant demand rate was considered in [10] where sample gradients of inventory cost with respect to control parameters were defined and proved to be strongly consistent. Perturbation analysis was also conducted in [2] for a single-machine/single item production system having multiple machine state.

Also related to this paper are the works [7, 8] on performance evaluation and optimization of continuous production lines with operation dependent failures. Especially, a GSMP model was proposed in [7] for representing the underlying stochastic process of a continuous production line. Perturbation analysis with respect to production rates was considered in [8].

Compared with the existing work, the most salient feature of this paper is the use of evolution equations which not only allows efficient performance evaluation but also allows thorough analysis of underlying stochastic process and various performance functions. Ergodicity of the underlying process, unbiasedness and strong consistency of gradient estimators, and convergence of the optimization algorithm will be established rigorously.

In the following, the paper is organized as follows. Section 2 introduces fluid stochastic event graphs. Notations, basic assumptions and dynamics of fluid stochastic event graphs are introduced. Section 3 establishes the evolution equations. Section 4 derives gradient estimators. Ergodic properties are examined in Section 5 and Section 6 proves the unbiasedness and strong consistency of gradient estimators. Section 7 is devoted to the optimization of the system parameters including maximal firing speeds and the initial marking. Section 8 is a conclusion.

2. FLUID EVENT GRAPHS AND FLOW CONTROL

2.1 Event graphs

A Petri net is a bipartite directed graph N = (P, T, F, m) where P is the set of places and T is the set of transitions, $F \subseteq PxT \cup TxP$ is the set of arcs from transitions to places or from places to transitions, $m : P \rightarrow IN$ is the initial marking that assigns to each place a given number of tokens. A transition can fire if each of its input places contains at least one token. The firing of a transition removes a token from each of input places and adds one token into each of its output places. Repeating the firing process leads to a firing sequence.

An event graph is a Petri net such that each place has one input transition and one output transition. As a result, each place can be represented by its input transition and its output transition. For this reason, we use in this paper the following notation:

- N = (P, T, A, m) : the event graph,
- T = (1, 2, …, i, …, I) : the set of transitions,
- (i, j) $\forall i,j \in T$: place connecting transition i to transition j if such a place exists,
- $m_{(i,j)}$ $\forall (i,j) \in P$: the initial marking of place (i,j).

2.2 Fluid event graphs

Fluid Petri nets, also called continuous Petri nets, are extension of classical Petri net models to cope with state explosion problem. In contrast to tradition Petri nets in which each place holds a discrete number of tokens, a place in fluid Petri nets holds a fluid and transitions fire continuously according to some firing speed. As a result, marking of a place is a nonnegative real number that we sometimes call the token content of a place. Furthermore, we shall say firing speed instead of firing sequence. General presentations of fluid Petri nets can be found in [4, 9] and hereafter we limit ourselves to fluid event graphs.

In a fluid event graph, we associate to each transition a maximal firing speed and the following notation will be used:

- A_i: maximal firing speed of transition i
- u_{it}: firing speed of transition i at time t
- $x_{(i,j)t}$: marking of place (i,j) at time t with $x_{(i,j)0} = m_{(i,j)}$.
- y_{it} : cumulative firing quantity of transition i up to time t.

The following assumptions are made in this paper:

(A) Transitions have finite firing speed, i.e. $A_i < \infty$.
(B) The event graph is connected.

While assumption (B) is not restrictive, assumption (A) forbids immediate transitions and the relaxation of this assumption is subject of future research.

For reasons that will become clear, we extend the notion of marking to pairs of transitions that are not connected via a place. This extension is done by means of the classical notion of token distance and is defined as follows:

$$m_{(i,j)} = \min_{\rho \in \Gamma_{ij}} \sum_{(k,l) \in \rho} m_{(k,l)}$$

$$x_{(i,j)t} = \min_{\rho \in \Gamma_{ij}} \sum_{(k,l) \in \rho} x_{(k,l)t}$$

where Γ_{ij} is the set of directed path connecting transition i to transition j. With obvious convention, $m_{(i,j)} = \infty$ and $x_{(i,j)t} = \infty$ if there is no path from i to j.

It immediately follows that:

$$dx_{(i,j)t}/dt = u_{it} - u_{jt}, \ \forall (i,j) \quad (1)$$

$$y_{it} = \int_0^t u_{i\tau} d\tau, \qquad \forall i \in T \quad (2)$$

$$0 \le u_{it} \le A_i, \ \forall i \in T \quad (3)$$

$$x_{(i,j)t} \ge 0, \ \forall (i,j). \quad (4)$$

We notice that relations (1) and (4) hold not only pairs of transitions (i, j) connected via a place but also for any pairs of transitions (i, j).

We say that a control policy u_{it} is feasible if relations (3)-(4) hold. Of course, a transition can fire at its maximal firing speed if each of its input place has positive marking. Otherwise, it can be easily shown that, for any place, $x_{(i,j)t} = 0$ implies that $u_{it} \geq u_{jt}$ and transition j cannot fire at its maximal speed if $A_j < u_{it}$. It naturally leads to the following firing policy:

$$u_{it} = \min\left\{A_i, \min_{(j,i)\in P / x_{(j,i)t}=0} u_{jt}\right\}, \forall i \in T. \qquad (5)$$

The following assumption will be considered throughout the paper :

(C) The total number of tokens in each elementary circuit γ is positive, i.e. $\Sigma_{(i,j)\in\gamma}\, m_{(i,j)} > 0$.

As a result, firing policy (5) can be iteratively determined since assumption (C) implies $\Sigma_{(i,j)\in\gamma} x_{(i,j)t} = \Sigma_{(i,j)\in\gamma} m_{(i,j)} > 0$. It can be proved that:

Theorem 1: Given the marking at time t, the firing policy (5) is feasible and can be explicitly determined as follows:

$$u_{it} = \min_{(j,i) / x_{(j,i)t}=0} A_j, \forall i \in T. \qquad (6)$$

We notice that only transitions immediately preceding transition i via an empty place are considered in relation (5), while all transitions connecting to transition i through a token-free path are considered in (6). From the result,

Corollary 1: Given the marking at time t, the firing policy (5) maximises the firing speed of all transitions. Further, it maximises the cumulative firing quantities y_{it}.

This is not surprising since firing policy (5) corresponds to the earliest firing policy of discrete event graph.

2.3 Fluid stochastic event graphs

The extension to fluid stochastic event graphs is motivated by the need of a general tool to representing failure-prone manufacturing systems. For this purpose, each transition can be either in operating state or in failure state. When a transition i is in operating state, its maximal firing speed is A_i. When it is in failure state, it cannot fire at all. The following notations will be used:

- t_n : epoch of n^{th} event including failures and repairs of all transitions with $t_0 = 0$,
- $\tau_n = t_{n+1} - t_n$: inter-arrival time of events,
- α_{it} : state of transition i at time t with $\alpha_{it} = 1$ if it is in operating state and $\alpha_{it} = 0$ otherwise,
- $A_{it} = A_i\, \alpha_{it}$: maximal firing speed of transition i at time t.

The dynamics of fluid stochastic event graph is similar to that of fluid event graphs. All results of section 2.2 hold with A_i replaced by A_{it}.

Concerning the underlying stochastic process, we assume that:

(D) the failure and repair process, represented by $\{(\alpha_{1t},\alpha_{2t}), \forall t\geq 0\}$ and $\{t_n, \forall n \geq 0\}$, does not depend on the control policy, i.e. u_{it}.

This assumption corresponds to the assumption of time-dependent failures. It implies that the failure and repair process does not depend on the machine utilisation. This assumption is usually made in flow control of failure-prone manufacturing systems.

From the above, it is clear that the fluid stochastic system is hybrid system composed of a discrete-event component characterised by $\{(\alpha_{1t},\alpha_{2t}), \forall t\geq 0\}$ and $\{t_n, \forall n \geq 0\}$ and a continuous component characterised by relations (1)-(6). Assumption (D) implies that the continuous component is governed by the discrete-event component but it has no impact on the discrete-event component.

2.4 Modelling flow control strategies

Before presenting the analysis of fluid stochastic event graphs, let us show how to model flow control strategies of manufacturing systems using this tool. For this purpose, we consider a production line composed of three machines M1, M2, M3. Products flow from M1 to M2 and then to M3. Different production control mechanisms will be considered.

Figure 1 is the model of a production line separated by buffers of limited size. Transitions t1, t2 and t3 represent the machines. Places p1 and p3 represent the buffers while places p2 and p4 represent their remaining capacity. Fig. 2 corresponds to a production line with a global buffer represented by p2.

Figure 3 corresponds to a Kanban system. Transitions LUi are load/unload operations. t0 represents the arrival of demand. Places fi are buffers of free kanbans of stage i, places wi represent parts waiting for machine Mi, places pi represent parts ready to move to the next stage. Place p0 model backlogged demand.

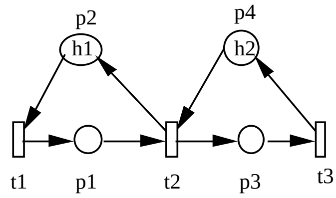

Fig. 1: A transfer line with limited buffers

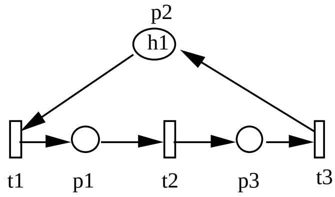

Fig. 2: A production line with a global buffer

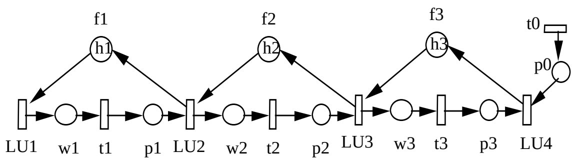

Fig. 3: A production line controlled by kanbans

Figure 4 models a production line controlled by the so-called base-stock policy, also called surplus control. The base-stock policy allows each machine to produce more than the true demand of the system. This surplus also called echelon-stock, including the parts in the output buffer of the machine and those located in the downstream stages, is limited by a base-stock level. In the Petri net model, the base-stock levels of M1, M2 and M3 are represented respectively by the initial markings of places p2, p4 and p6. Place p0 represents unsatisfied demand and place p5 the inventory of finished products.

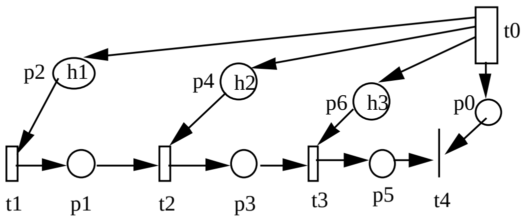

Fig. 4: Base-stock control

The control policy represented by figure 5 is similar to the base-stock policy except that in the new policy the buffers among machines have limited sizes represented by places b1 and b2. This policy is called generalised kanban systems or two-boundary control policy (surplus control and buffer control).

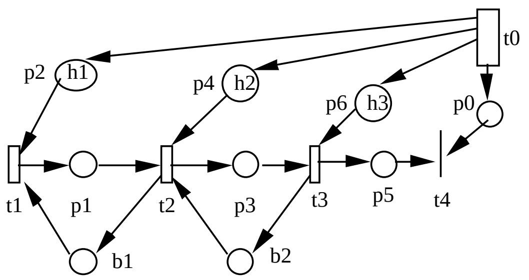

Fig. 5: A generalised kanban system

Machine failures and random demand can be modelled easily by appropriately specified the failure/repair process of each transition.

3. EVOLUTION EQUATIONS

Theorem 2: Between any two events, i.e. $t_n \le t \le t_{n+1}$, the following evolution equations hold:

$$y_{it} = \min_{j \in T} \{Y_{jn} + A_{jn}(t - t_n) + m_{(j,i)}\}, \ \forall i \in T \qquad (7)$$

where $Y_{in} = y_{i,tn}$, $A_{in} = A_i \alpha_{i,tn}$.

From Theorem 2,

$$Y_{i,n+1} = \min_{j \in T} \{Y_{jn} + A_{jn}\tau_n + m_{(j,i)}\}, \ \forall i \in T. \qquad (8)$$

From the above evolution equations, it is clear that $x_{(i,j)t}$ and y_{it} are piece-wise linear functions of A_i. Furthermore,

Theorem 3: y_{it} for all $i \in T$ are Lipschitz continuous, non-decreasing and concave in (A, m).

It is worth noticing that Theorem 3 implies that y_{it} for all i are as well Lipschitz continuous, non-decreasing and concave component-wise in (A, m).

Corollary 2: Assume that (A, m) is a function of a real number θ denoted by $F(\theta)$. y_{it} for all $i \in T$ are Lipschitz continuous in θ if $F(\theta)$ is Lipschitz continuous. y_{it} are non-decreasing in θ if $F(\theta)$ is non-decreasing. y_{it} are concave in θ if $F(\theta)$ is concave.

4. GRADIENT ESTIMATION

Theorem 4: Assume that (A, m) is a function of a real number θ denoted by $F(\theta)$. If $F(\theta)$ is continuous and differentiable at θ, then the right and left derivatives of Y_{in} exist and

$$\frac{\partial^+ Y_{1n+1}}{\partial\theta} = \min_{j \in E_{in}} \left\{ \frac{\partial^+ Y_{jn}}{\partial\theta} + \frac{\partial^+ A_j}{\partial\theta}\alpha_{jn}\tau_n + \frac{\partial^+ m_{(j,i)}}{\partial\theta} \right\}$$

$$\frac{\partial^- Y_{1n+1}}{\partial\theta} = \max_{j \in E_{in}} \left\{ \frac{\partial^- Y_{jn}}{\partial\theta} + \frac{\partial^- A_j}{\partial\theta}\alpha_{jn}\tau_n + \frac{\partial^- m_{(j,i)}}{\partial\theta} \right\}$$

where $E_{in} = \{j \in T \mid Y_{i,n+1} = Y_{jn} + A_{jn}(t_{n+1} - t_n) + m_{(j,i)}\}$.

We notice that Theorem 4 can be easily extended to y_{it} and $x_{(i,j)t}$. Furthermore, from Corollary 2,

Theorem 5: Under the conditions of Theorem 4, if $F(\theta)$ is concave, then $\partial^+ Y_{in}/\partial\theta \le \partial^- Y_{in}/\partial\theta$ for $i \in T$ and for all n

≥ 0. Further, any g_{in} such that $\partial^+ Y_{in}/\partial\theta \leq g_{in} \leq \partial^- Y_{in}/\partial\theta$ is a subgradient of Y_{in}.

Theorems 4 and 5 lead to the following subgradients of Y_{in} for concave $F(\theta)$:

$$g_{i,n+1} = g_{jn} + a_j \alpha_{jn} \tau_n + \mu_{(j,i)}$$

for any $j \in E_{in}$ where a_j is a subgradient of A_j and $\mu_{(i,j)}$ a subgradient of $m_{(i,j)}$.

Since $x_{(i,j)t} = m_{(i,j)} + y_{it} - y_{jt}$ for all t, $\partial^+ m_{(i,j)}/\partial\theta + \partial^+ y_{it}/\partial\theta - \partial^- y_{jt}/\partial\theta$ is a subgradient of $x_{(i,j)t}$ if $F(\theta)$ is concave and differentiable.

5. ERGODICITY

This section analyses the ergodic properties of $\{y_{it}\}$ and $\{x_t\}$ by using the results of [1]. For this purpose, let us rewrite equations (8) as follows:

$$Y_{n+1} = W(n) \otimes Y_n$$

where the the (min, +) algebra is used, $\oplus$ denotes the "min" operator and $\otimes$ the "+" operator, $Y_n = (Y_{1n}, \ldots, Y_{In})^T$, and $W(n)$ is $I \times I$ matrix with $W(n)_{ij} = A_{jn}(t_{n+1} - t_n) + m_{(j,i)}$. According to the terminology of [1], $W(n)$ for any n is a monotone-homogeneous operator since (i) it is homogeneous, i.e. $W(n) \otimes (Y + \lambda 1) = \lambda 1 + W(n) \otimes Y$ for all Y in IR^I and λ in IR, (ii) it is monotone, i.e. $W(n) \otimes X \leq W(n) \otimes Y$ if $X \leq Y$.

Theorem 6: Assume that the event graph is strongly connected. If $\{W(n), n \in IN\}$ is a stationary ergodic sequence, then there exists a constant $\gamma > 0$ such that: $\lim_{n\to\infty} Y_{in}/n = \gamma$ with probability 1 (or w.p. 1 for short) and $\lim_{n\to\infty} E[Y_{in}]/n = \gamma$.

Let us notice that the stationary ergodicity of $\{T_n, n \in IN\}$ is equivalent to the stationary ergodicity of $\{(\alpha_{1n}, \alpha_{2n}, r_{1n}, r_{2n}), n \in IN\}$ where r_{in} is the remaining life time of machine M_i at time t_n+ as $t_{n+1} - t_n = \min\{r_{1n}, r_{2n}\}$.

Corollary 3: Assume that the event graph is strongly connected. If $\{T_n, n \in IN\}$ is a stationary ergodic sequence and if $E[t_{n+1} - t_n] = \mu > 0$, then $\lim_{n\to\infty} y_{it}/t = \Gamma$ w.p. 1 and $\lim_{n\to\infty} E[y_{it}]/t = \Gamma$ where $\Gamma = \gamma/\mu$.

From Theorem 3:

Corollary 4: Assume that the event graph is strongly connected. If $\{T_n, n \in IN\}$ is a stationary ergodic sequence and if $E[t_{n+1} - t_n] = \mu > 0$, then γ and Γ Lipschitz continuous, non-decreasing and concave in (A, m).

6. UNBIASEDNESS AND CONSISTENCY OF GRADIENT ESTIMATORS

Theorem 7: If (A, m) is a differentiable and concave function of θ and if $E[y_{it}]$ is differentiable at θ, then $E[g_{it}] = \partial E[y_{it}]/\partial\theta$ for any subgradient g_{it} of y_{it}.

Theorem 8: Under conditions of Theorem 6, if (A, m) is a differentiable and concave function of θ and if γ is differentiable at θ, then $\lim_{n\to} \partial g_{in}/n = \partial\gamma/\partial\theta$, w.p.1 for any subgradient g_{in} of Y_{in}.

Corollary 5: Under conditions of Corollary 3, if (A, m) is a differentiable and concave function of θ and if Γ is differentiable at θ, then $\lim_{t\to\infty} g_{it}/t = \partial\Gamma/\partial\theta$ for any subgradient g_{it} of y_{it}.

Since $x_{(i,j)t} = m_{(i,j)} + y_{it} - y_{jt}$, Theorem 7 implies that $\mu_{(i,j)} + g_{it} - g_{jt}$ is an unbiased estimator of $\partial E[x_{(i,j)t}]/\partial\theta$. Unfortunately, we were not able to derive strongly consistent estimators of $\partial E[x_{(i,j)\infty}]/\partial\theta$.

7. OPTIMISATION OF CONTROL POLICIES

In this section, we consider the following optimisation problem. Let z = (A, m). The problem consists in maximising a criterion function of $\Gamma(z)$ and z denoted as $J(\Gamma(z), z)$. The following assumptions will be considered:

(E) $J(\Gamma, z)$ is non-decreasing in Γ and is concave in (Γ, z).
(F) $\lim_{z\to\infty} J(\Gamma, z) = -\infty$.

An example of such criteria is $J(\Gamma, z) = \Gamma - \langle p, z\rangle$ with $p > 0$ where $\langle p, z\rangle$ corresponds to the cost of the firing speeds and markings.

In the remainder of this paper, we propose a simulation-based approach for solving the optimisation problem. The approach makes use of a single sample path $\omega = \{t_0=0, \alpha_0, t_1, \alpha_1, \ldots, t_N, \alpha_N\}$. Equations (2) are used to compute Y_{in} $\forall i \in T$ and $1 \leq n \leq N$. We then choose $\Gamma_n(z) = Y_{in}(z)/t_n$ as estimator of $\Gamma(z)$ and maximise $J(\Gamma_n(z), z)$. The new sample function has the following property.

Theorem 9: Under assumption (E), $J(\Gamma_n(z), z)$ is concave in z. If assumption (F) holds as well, then $J(\Gamma_n(z), z)$ reaches its maximum at a finite z_n.

The asymptotic optimality of z_n is ensured by the following theorem and motivates the use of a single sample path of finite length for solving our optimisation problem.

Theorem 10: Under assumptions (E)-(F) and conditions of Corollary 3, w.p. 1,

$$\lim_{n\to\infty} J(\Gamma_n(z_n),z_n) = \lim_{n\to\infty} J(\Gamma(z_n),z_n) = J(\Gamma(z^*),z^*).$$

Example: To illustrate the approach, let us consider a production line composed of three machines M1, M2 and M3 separated by two buffers B1 and B2. The machines are identical with maximal production rate equal to 1 unit per unit of time. The time to failure of each machine is exponentially distributed with mean equal 10 and the time to repair is also exponentially distributed with mean equal to 5. The buffer capacity of B1 is h_1 and that of B2 is h_2. We assume that $h_1 + h_2 = 10$ and determine h_1 and h_2 that maximize the throughput rate.

The Petri net model of this system is given in Figure 1. Clearly, transition ti has maximal firing speed $A_i = 1$ and its failure and repair process represent that of machine Mi. We restrict ourselves to marking $\mathbf{m} = [0, h_1, 0, h_2]$. A gradient-based method is used to solve problem: $\max\{J(\Gamma_n(\mathbf{z}), \mathbf{z})$. The simulation results are given in Table 1. From the symmetry of the production line, the optimal solution is $h_1 = h_2 = 5$. Table I shows that $\mathbf{z}_n$ quickly converges to the optimal solution. Further the finite-horizon solution $\mathbf{z}_n$ is already in the neighborhood of the optimal solution even with a very small number of events n. For comparison, the throughput rate with $h_1 = 1$ and $h_2 = 9$ is 0.4572.

Table I: Simulation results

n	h_1	h_2	$\Gamma_n(\mathbf{z})$
50	5.495	4.505	0.6801
100	5.495	4.505	0.5360
500	4.933	5.067	0.4829
1000	5.556	4.224	0.4695
2000	5.355	4.645	0.4577
5000	5.214	4.786	0.4618
10000	5.074	4.926	0.4692
100000	4.933	5.067	0.4737

8. CONCLUSION

In this paper, we have proposed fluid stochastic event graphs in which places contain fluids and transitions can be either in operating state or in failure state. The most important results are the evolution equations that determine the cumulative firing quantities at epochs of failures and repairs. Based on these equations, we established the Lipschitz continuity, monotonicity and concavity of cumulative firing quantities with respect to maximal firing speeds of the transitions and the initial marking. Unbiased and strongly consist gradient estimators were obtained. Finally a sample path optimization approach was proposed for maximizing a concave criterion function throughput rate and system parameters. The most important future research issue within the framework of this paper is the analysis of marking process. This analysis is necessary for evaluation and optimization of waiting times and inventory levels of manufacturing systems. It is not clear whether the results of this paper can be used to analyze the marking process. Another research issue is the analysis of general fluid Petri net models.

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