Global Adaptive Compensation of Noises with Unknown Frequency [1]

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Abstract

Given an asymptotically stable, observable linear system affected by an additive sinusoidal disturbance with unknown magnitude, phase and frequency, entering where the control input enters (matching case), the problem of designing an output feedback compensator which drives the output to zero for any initial condition is addressed.

Keywords: adaptive compensation, noise cancellation, adaptive observers.

1 Introduction

Consider the linear single-input single-output observable system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -a_{n-1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_1 & 0 & \cdots & 1 \\ -a_0 & 0 & \cdots & 0 \end{bmatrix} x + \begin{bmatrix} b_{n-1} \\ \vdots \\ b_0 \end{bmatrix} (u+w) \\ &\triangleq Fx + g(u+w) \\ y &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} x \triangleq hx \end{aligned} \tag{1}$$

whose transfer function is given by

$$y(s) = \frac{b_{n-1}s^{n-1} + \ldots s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0}(u(s) + w(s)) \tag{2}$$

in which the disturbance $w(t)$ is a sinusoid

$$w(t) = A\sin(\omega t + \phi) \tag{3}$$

of fixed but unknown magnitude A, frequency $\omega > 0$ and phase ϕ.

We assume that: a_i, b_i, $0 \le i \le n-1$ are known; the system is asymptotically stable; there are no zeros in the imaginary axis. We pose the global adaptive disturbance compensation problem, that is the design of a dynamic output feedback control which for any initial condition of the plant and of the compensator and for any unknown constant values A, ω, ϕ guarantees that $\lim_{t\to\infty} y(t) = 0$, for the closed loop system.

If the frequency is known, the posed problem has a classical solution by modeling the disturbance as a linear exosystem and by using an observer which provides an asymptotic estimate of the disturbance so that it can be cancelled. If the frequency is unknown, the posed problem has been given a local solution in [1] when the initial estimated frequency is close to the true value. In this paper we provide a global solution using the adaptive observers developed in [2]: we propose a compensator of order $2n+4$ which solves the global adaptive disturbance compensation problem.

2 Adaptive observer

The sinusoidal noise $w(t) = A\sin(\omega t + \phi)$ is modeled by the linear exosystem

$$\begin{aligned} \dot{w}_1 &= w_2 \\ \dot{w}_2 &= -\omega^2 w_1 \triangleq -\theta w_1 \\ y &= w_1 \end{aligned} \tag{4}$$

with $\theta = \omega^2 > 0$ an unknown positive parameter and $w_1(0) = A\sin\phi$, $w_2(0) = A\omega\cos\phi$ unknown initial conditions. The overall system (1), (4) is transformed into the adaptive observer form ($z \in R^{n+2}$)

$$\begin{aligned} \dot{z} &= A_c z - \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \\ 0 \\ 0 \end{bmatrix} y + \begin{bmatrix} b_{n-1} \\ \vdots \\ b_0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 \\ d_n \\ \vdots \\ d_0 \end{bmatrix} \mu\theta \\ y &= C_c z \end{aligned} \tag{5}$$

with $d = [1, d_n, \ldots, d_0]^T \in R^{n+2}$ any vector such that the zeros of $d(s) = s^{n+1} + d_n s^n + \cdots + d_0$ have negative real part, by the nonsingular transformation

$$\begin{aligned} \zeta_1 &= x_1 \\ \zeta_2 &= x_2 + b_{n-1} w_1 \end{aligned}$$

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$$
\begin{aligned}
\zeta_i &= \theta x_{i-2} + x_i + b_{n-i+1}w_1 + b_{n-i+2}w_2, \\
&\qquad 3 \le i \le n \\
\zeta_{n+1} &= \theta x_{n-1} + b_0 w_1 + b_1 w_2 \\
\zeta_{n+2} &= \theta x_n + b_0 w_2 \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\xi} &= \begin{bmatrix} -d_n & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -d_1 & 0 & \cdots & 1 \\ -d_0 & 0 & \cdots & 0 \end{bmatrix} \xi \\
&\quad + \left(- \begin{bmatrix} 1 \\ a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} y + \begin{bmatrix} 0 \\ b_{n-1} \\ \vdots \\ b_0 \end{bmatrix} u \right) \\
\mu &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \xi \qquad (7) \\
z_1 &= \zeta_1 = y \\
z_j &= \zeta_j - \xi_{j-1}\theta, \quad j = 2, \ldots, n+2 \qquad (8)
\end{aligned}
$$

Following [2], an adaptive observer for (5) is given by

$$
\begin{aligned}
\dot{\hat{z}} &= A_c \hat{z} - \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \\ 0 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ b_{n-1} \\ \vdots \\ b_0 \end{bmatrix} u \\
&\quad + \begin{bmatrix} 1 \\ d_n \\ \vdots \\ d_0 \end{bmatrix} \mu \hat{\theta} + K(y - C_c \hat{z}) \\
\dot{\hat{\theta}} &= \gamma \mu (y - C_c \hat{z}) \qquad (9)
\end{aligned}
$$

in which $K = (A_c + \lambda I)d$ with $\lambda > 0$ a design parameter and $\gamma > 0$ the adaptation gain. Now, consider the filter (7) which generates the regressor μ, whose transfer function is

$$
\mu(s) = \frac{-a(s)y(s) + b(s)u(s)}{d(s)} = \frac{-b(s)}{d(s)} w(s) \qquad (10)
$$

The regressor μ given in (7) is equivalently generated by

$$
\begin{aligned}
\dot{\eta} &= \begin{bmatrix} -d_n & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -d_1 & 0 & \cdots & 1 \\ -d_0 & 0 & \cdots & 0 \end{bmatrix} \eta - \begin{bmatrix} 0 \\ b_{n-1} \\ \vdots \\ b_0 \end{bmatrix} w \\
\mu &= \begin{bmatrix} 1 & \cdots & 0 \end{bmatrix} \eta \qquad (11)
\end{aligned}
$$

which implies that μ is bounded and consequently ξ in (7) is bounded as well. Since the system (11) is asymptotically stable and has no zeros on the imaginary axis and $w = A\sin(\omega t + \phi)$, it follows that the regressor $\mu(t)$ is persistently exciting in the error dynamics

$$
\begin{aligned}
\dot{\tilde{z}} &= (A_c + KC_c)\tilde{z} + d\mu\tilde{\theta} \\
\dot{\tilde{\theta}} &= \gamma \mu C_c \tilde{z} \ . \qquad (12)
\end{aligned}
$$

Hence, $\tilde{z}(t)$ and $\tilde{\theta}(t)$ tend exponentially to zero as t goes to infinity, for any initial condition $\hat{z}(0)$, $\hat{\theta}(0)$, $\xi(0)$.

From (8) we define

$$
\begin{aligned}
\hat{\zeta}_1 &= \hat{z}_1 \\
\hat{\zeta}_j &= \hat{z}_j + \xi_{j-1}\hat{\theta}, \quad j = 2, \ldots, n+2 \ . \qquad (13)
\end{aligned}
$$

Since $\lim_{t\to\infty} \|z(t) - \hat{z}(t)\| = 0$, $\lim_{t\to\infty} \|\tilde{\theta}(t)\| = 0$ and $\xi(t)$ is bounded, it follows from (8) and (13) that $\lim_{t\to\infty} \|\tilde{\zeta}(t)\| = 0$, with

$$
\begin{aligned}
\tilde{\zeta}_1 &= \tilde{z}_1 \\
\tilde{\zeta}_j &= \tilde{z}_j + \xi_{j-1}\tilde{\theta}, \quad 2 \le j \le n+2 \qquad (14)
\end{aligned}
$$

From (8), rewriting $\zeta = T(\theta) \begin{bmatrix} x \\ w_1 \\ w_2 \end{bmatrix}$, we define

$$
\hat{\zeta} = T(\hat{\theta}) \begin{bmatrix} \hat{x} \\ \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} . \qquad (15)
$$

Since $T(\hat{\theta})$ is nonsingular for all $\hat{\theta}$, we have

$$
\begin{bmatrix} \hat{x} \\ \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} = T^{-1}(\hat{\theta})\hat{\zeta} \qquad (16)
$$

Since $\tilde{\zeta}(t)$ and $\tilde{\theta}(t)$ tend asymptotically to zero, from (16) it follows that $\lim_{t\to\infty} \|x(t) - \hat{x}(t)\| = 0$, $\lim_{t\to\infty} |w_i(t) - \hat{w}_i(t)| = 0$, $i = 1, 2$. At this point, we define

$$
u = -\hat{w}_1 \qquad (17)
$$

The resulting dynamic output-feedback compensator (17), (16), (13), (9), (7) is of order $2n + 4$: the closed loop system dynamics are

$$
\begin{aligned}
\dot{x} &= Fx + g\tilde{w}_1 \\
y &= hx
\end{aligned}
$$

In conclusion, since $\tilde{w}_1(t)$ is bounded and goes asymptotically to zero, $\lim_{t\to\infty} y(t) = 0$ for any initial condition $x(0)$, $\hat{z}(0)$, $\hat{\theta}(0)$, $\xi(0)$.

References

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[2] R. Marino and P. Tomei, *Nonlinear Control Design - Geometric, Adaptive and Robust.* Prentice Hall, London, 1995.