Automatic Elasticity Tuning of Industrial Robot Manipulators

E. Berglund[†] and G.E. Hovland[*]

[†]ABB Industry, Marine Division,
Hasleveien 50, PO Box 6540, Rodeløkka,
N-0501 Oslo, Norway.
E-mail: einar.berglund@no.abb.com
Phone: int + 47 22872696

[*]ABB Corporate Research,
Info. Tech. and Control Systems Division,
Bergerveien 12, N-1375 Billingstad, Norway.
E-mail: geir.hovland@no.abb.com
Phone/Fax: int + 47 6684 3232/3541

Abstract

In this paper we present a new method for automatic elasticity tuning of industrial robot manipulators. The main contributions of the work are a) The parameters of a mechanical mass-spring-damper equivalent of any order are solved given only partial state information (motor encoder position and motor torque). b) The method is fully automatic with no operator input and can easily be applied in the field to update the dynamic model parameters. The ability to automatically update the elasticity parameters is particularly useful when the robot operators mount flexible tooling or equipment on the robot arms. c) The method separates friction and elasticity identification d) The method is demonstrated on an industrial ABB robot. e) In the paper we combine an important result from the vibration literature, [7], with the solution of inverse eigenvalue problems, [4]. To our knowledge, this is the first time that these methods have been combined and applied to the identification of flexible robot manipulators. The main advantage of the method compared to other identification methods is the fact that only motor encoder position and motor torque are required to identify the springs, masses and dampers of an Nth order system.

1 Introduction

Path tracking accuracy is a key issue for industrial robot manufacturers. Customer requirements are pushing the robot manufacturers towards more advanced dynamic modelling and more accurate estimation of the model parameters to meet the increasing demands on path tracking. Examples of critical path tracking applications in the automotive industry are laser cutting, plasma cutting and waterjet cutting.

Accurate modelling and identification of industrial manipulators is complicated due to the following facts: a) heavy, flexible and unmodelled tools and equipment, such as cutting heads, laserfibres, waterpipes and interface electronics, are mounted on the robot arms. b) The equipment which is mounted on the robots is customer specific. Hence, accurate estimation of model parameters must be performed at the customer site. Moreover, the estimation methods have to be practical to use in the field and preferably fully automatic with little or no operator input. c) Most industrial manipulators only provide motor measurements, such as joint torque and position, and no link position measurements. The estimation methods must be able to estimate as many model parameters as possible with only partial information about the robot's state variables.

In this paper we present a new method for automatic elasticity tuning of industrial manipulators. The main contributions of the paper are: a) The parameters of a mechanical mass-spring-damper equivalent of any order are solved given only partial state information (motor's encoder position and motor torque). b) The method is fully automatic with no operator input. c) The method separates the friction and elasticity identification which results in more accurate and reliable parameter estimates. d) The method is demonstrated on an industrial ABB robot. e) In the paper we combine an important result from the vibration literature, [7], with the solution of inverse eigenvalue problems, [4]. To our knowledge, this is the first time that these methods have been combined and applied to the identification of flexible robot manipulators.

The problem of controlling robots with elastic behaviour has received a significant amount of attention in the literature, see for example [2, 3, 6]. The identification problem, especially identification of link elasticities, has received less attention. One example of the identification of multivariable joint elasticities in the presence of coupled dynamics was presented by [5]. The work in the literature dealing with the control of elastic robots often assume that the elasticity parameters are known and these parameters are used directly in the design of the control laws. The methods and results presented in our paper will be directly beneficial to several of the existing control strategies in the literature.

2 Nth-order Elasticity Model

The method presented in this paper estimates the mass, spring and damper coefficients of an Nth-order model. The model is illustrated in Figure 1. The method pre-

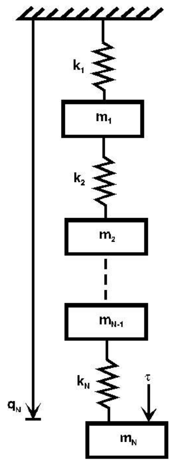

Figure 1: *Mechanical N-mass/spring/damper system. The dampers $d_1, \cdots, d_N$ are not shown in the figure, but are connected in parallel with the springs $k_1, \cdots, k_N$. The measurements available are the position q_N and the force τ acting on the last mass.*

sented here is general and can be applied to a range of systems (not only mechanical) that can be modelled by the system in Figure 1. In this paper, however, we illustrate the method on one axis of a robot manipulator. The last mass in Figure 1 corresponds to the motor inertia. The other masses correspond to the inertias of the robot links. The first spring, k_1, is set to zero, since the links are not connected to any external reference point. The last spring k_N corresponds to the transmission elasticity. The other springs $k_2, \cdots, k_{N-1}$ correspond to link and joint (for example bearings connecting the arms) elasticities. To summarise, the Nth-order mass-spring-damper equivalent in Figure 1 describes a robot axis with one transmission elasticity plus the dominating $N-2$ vibration modes of the links.

3 Automatic Tuning Method

The automatic tuning of the Nth-order elasticity model in Figure 1 is described in this section. Section 3.1 describes an approach to friction estimation. The friction parameters are estimated first and the effects from friction is removed from the measured joint torque before the elasticity parameters are estimated. The method first identifies the mass and the spring coefficients in sections 3.2-3.3. Then, after the springs and masses are known, the dampers are identified. Identifying the dampers after the other parameters results in a small identification error. It is, however, well-known that small dampers have a very little influence on the resonance frequencies. Hence, the identification errors of the springs and masses due to a small damping effect will be small.

3.1 Separation of Friction and Elasticity

For robot paths where the frequency contents is well below the first resonance mode, the dynamics (rigid-body) of the robot axis is approximated by the following equation

$$(m_1 + \cdots + m_N)\ddot{q}_N + g(q_N) + C_1 sgn(\dot{q}_N) + C_2\dot{q}_N = \tau_m \tag{1}$$

where $g(q_N)$ is the gravity torque, C_1 is the Coloumb friction and C_2 is the viscous friction. The identification of the friction coefficients above can be performed as described in, for example, [1]. The measured motor torque τ_m for an example identification movement is shown in Figure 2. The Coloumb friction parameter C_1

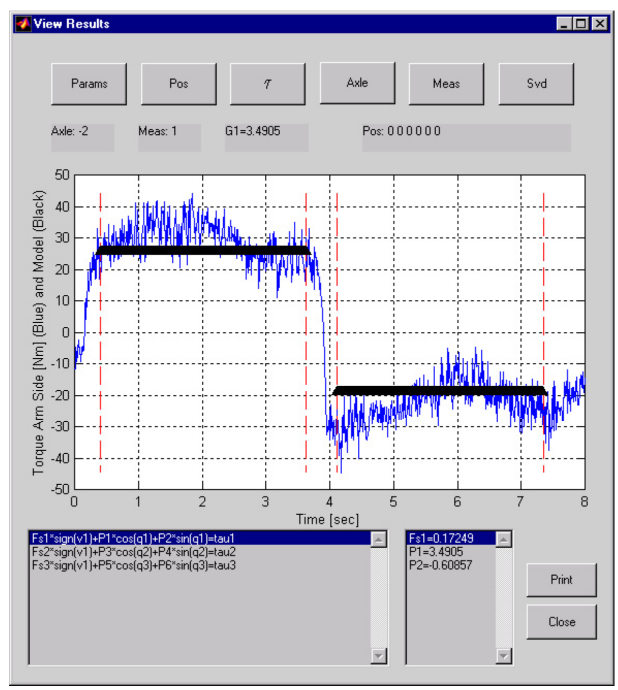

Figure 2: *Identification of Coloumb friction. The dark solid line shows the identified Coloumb friction, while the noisy curve is the measured motor torque for small movements with low velocity.*

is estimated as $C_1 = \frac{L}{2}$ where L is the discontinuous jump in Figure 2.

The robot axis dynamics after linearisation are now on the form shown in Figure 1 where the torque τ is given by

$$\tau = \tau_m - C_1 sgn(\dot{q}_N) - C_2\dot{q}_N \qquad (2)$$

3.2 Automatic Resonance Estimation

The first step in the self-tuning of the elasticity model is to identify the resonance frequencies in the transfer function $\frac{\ddot{q}_N}{\tau}(s)$, where τ is defined by eq. (2). An example transfer function is illustrated in Figure 3. The

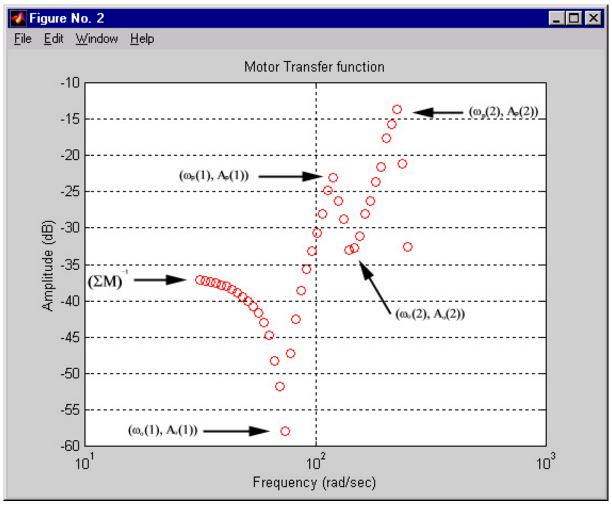

Figure 3: *Transfer function from torque τ to motor acceleration $\ddot{q}_N$. Each anti-resonance defines two complex conjugated zeros in the transfer function, while each resonance defines two complex conjugated poles. In the figure, two anti-resonance and two resonance frequencies are identified plus the low-frequency amplitude.*

anti-resonance and resonance frequency vectors are defined as

$$\omega_0 = [\omega_0(1), \cdots, \omega_0(N-1)] \qquad (3)$$
$$\omega_p = [\omega_p(1), \cdots, \omega_p(N-1)] \qquad (4)$$

The anti-resonance and resonance frequencies are easier to observe from $\frac{\ddot{q}_N}{\tau}(s)$ than from $\frac{q_N}{\tau}(s)$. To obtain $\frac{\ddot{q}_N}{\tau}(s)$ from the position measurements, every amplitude is multiplied by the corresponding frequency squared. Hence, the noise problems associated with second derivatives are avoided. For a linear N-mass system with the ladder structure, there will be $N-1$ anti-resonance and resonance frequencies, respectively. In addition to the frequency vectors ω_0 and ω_p, the corresponding amplitude vectors A_0 and A_p are recorded.

$$\mathbf{A}_0 = [A_0(1), \cdots, A_0(N-1)] \qquad (5)$$
$$\mathbf{A}_p = [A_p(1), \cdots, A_p(N-1)] \qquad (6)$$

Finally, the amplitude value A_L at low frequencies is recorded. It can be shown that

$$A_L = -20log(m_1 + \cdots + m_N) \qquad (7)$$

Alternatively, the sum of the masses can be identified at the same time as the friction model from the rigid-body analysis, see eq. (1). The frequency vectors ω_0, ω_p and the amplitudes $\mathbf{A}_0$, $\mathbf{A}_p$, A_L are automatically generated by the robot controller from the frequency response measurements and a simple search algorithm. In the next two sections we describe the method for estimating all masses, springs and dampers from these frequency vectors and the amplitudes.

3.3 Inverse Eigenvalue Problem for Mass-Spring System

In this section we make use of an important result described in [7]. Consider the system illustrated in Figure 1. If we neglect the dampers the frequency response $\mathcal{F}(\omega)$ of the position of mass m_N to the excitation $\tau = \sin\omega t$ (on mass m_N) will have the following form

$$\mathcal{F}(\omega) = \frac{\Pi_{i=1}^{N-1}(\omega^2 - \hat{\omega}_i^2)}{M_T \Pi_{i=1}^{N}(\omega^2 - \tilde{\omega}_i^2)} \qquad (8)$$

where $\hat{\omega}_i$ and $\tilde{\omega}_i$ are the anti-resonance and resonance frequencies, respectively. M_T is a constant depending on the system's total mass, $\hat{\omega}_i$ and $\tilde{\omega}_i$. Note that the extra resonance frequency corresponds to the double integrator from motor torque to motor position.

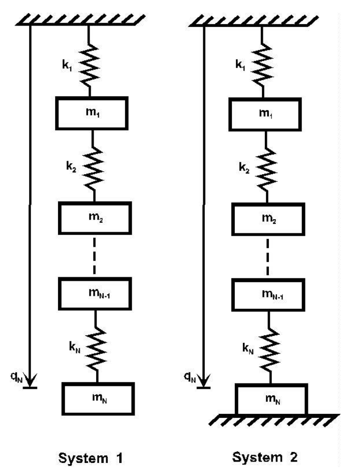

Figure 4: *Two mechanical N-mass/spring systems.*

Now consider the two systems illustrated in Figure 4. We define the eigenvalues of system 1 as $\tilde{\lambda}_i$ and the eigenvalues of system 2 as $\hat{\lambda}_i$. By using the results from [7], we have the following important lemma:

Lemma 1 *The eigenvalues of system 1 in Figure 4 are given by the squared resonance frequencies of $\mathcal{F}(\omega)$, ie. $\tilde{\lambda}_i = \tilde{\omega}_i^2$, where $i = 1, \cdots, N$. The eigenvalues of system 2 in Figure 4 are given by the squared anti-resonance frequencies of $\mathcal{F}(\omega)$, ie. $\hat{\lambda}_i = \hat{\omega}_i^2$, where $i = 1, \cdots, N-1$.* □

Given $\tilde{\lambda}_i$ and $\hat{\lambda}_i$ by the method of [7], we apply an inverse eigenvalue method for mass-spring systems described in the book [4]. The steps in the solution are:
1. Find the Nth eigenvector $u_N^{(i)}$, see 3.3.1.
2. Apply a method (Lanczos algorithm) to find a Jacobian matrix $\mathbf{B}$, see 3.3.2.
3. Use the matrix $\mathbf{B}$ to compute all springs k_i and masses m_i, see 3.3.3.

The computations required in the three steps above are described in the following subsections. For a more detailed description of the methods, see Chapter 4 in [4].

3.3.1 Computation of Nth Eigenvector: The components of the Nth eigenvector $\mathbf{u}_N$ are given by

$$[u_N^{(i)}]^2 = \Pi_{j=1}^{N-1}(\lambda_j^0 - \lambda_i)/\Pi_{j=1}^{N}{}'(\lambda_j - \lambda_i) \qquad (9)$$

where $'$ indicates that the term $j = i$ is omitted.

3.3.2 Lanczos Algorithm: Define the matrix $\mathbf{B}$ as follows.

$$\mathbf{B} = \begin{bmatrix} a_1 & -b_1 & & & \\ -b_1 & a_2 & -b_2 & & \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & a_{N-1} & -b_{N-1} \\ & & & -b_{N-1} & a_N \end{bmatrix} \qquad (10)$$

and the $N \times N$ matrix $\Lambda = diag(\lambda_i)$. The coefficients $a_1, \cdots, a_N$ and $b_1, \cdots, b_N$ are computed as follows.

$$\begin{aligned} a_N &= \mathbf{u}_N^T \Lambda \mathbf{u}_N & (11) \\ \mathbf{d}_{N-1} &= a_N \mathbf{u}_N - \Lambda \mathbf{u}_N & (12) \\ b_{N-1} &= \|\mathbf{d}\|_2 & (13) \end{aligned}$$

where the $\mathbf{d}_i$ vectors are introduced for convenience. For $i = N-1, \cdots, 2$, we have

$$\begin{aligned} \mathbf{u}_i &= \mathbf{d}_i / b_i & (14) \\ a_i &= \mathbf{u}_i^T \Lambda \mathbf{u}_i & (15) \\ \mathbf{d}_{i-1} &= a_i \mathbf{u}_i - b_i \mathbf{u}_{i+1} - \Lambda \mathbf{u}_i & (16) \\ b_{i-1} &= \|\mathbf{d}_{i-1}\|_2 & (17) \end{aligned}$$

For $i = 1$, we finally have

$$\begin{aligned} \mathbf{u}_1 &= \mathbf{d}_1 / b_1 & (18) \\ \mathbf{d}_0 &= -b_1 \mathbf{u}_2 - \Lambda \mathbf{u}_1 & (19) \\ a_1 &= \|\mathbf{d}_0\|_2 & (20) \end{aligned}$$

3.3.3 Spring and Mass Coefficients: The spring and mass coefficients are found from the $\mathbf{B}$ matrix as follows.

$$\begin{aligned} \hat{m}_N &= 1 & (21) \\ \hat{m}_i &= (a_{i+1}/b_i)^2 * \hat{m}_{i+1}, i = N-1, \cdots, 2 & (22) \\ \hat{m}_1 &= (b_1/a_1)^2 \hat{m}_2 & (23) \end{aligned}$$

Let $V = 1/(A_L \sum_{i=1}^{N} \hat{m}_i)$. The masses are then finally given by $m_i = V \hat{m}_i$, where $i = 1, \cdots, N$.

To find the spring coefficients, we first define a diagonal matrix $\mathbf{M} = diag(m_i)$, a matrix $\mathbf{C} = \mathbf{M}^{1/2}\mathbf{B}\mathbf{M}^{1/2}$ and a matrix $\mathbf{E}$ as follows.

$$\mathbf{E} = \begin{bmatrix} 1 & -1 & & & \\ 0 & 1 & -1 & & \\ \cdot & \cdot & \cdot & 1 & -1 \\ & & & 0 & 1 \end{bmatrix} \qquad (24)$$

The spring coefficients are finally given as the diagonal elements of the following matrix $\mathbf{K}$.

$$\mathbf{K} = \mathbf{E}^{-1}\mathbf{C}\mathbf{E}^{-T} \qquad (25)$$

3.4 Non-Linear Optimization of Damper Coefficients

The damper coefficients are found by using non-linear optimisation. In this section we define the optimisation function $F(\mathbf{x})$, where $\mathbf{x}$ is a vector of the damper coefficients, $\mathbf{x} = [d_1, \cdots, d_N]$.

Let $\mathbf{M} = diag(m_1, \cdots, m_N)$ and $\mathbf{K}$ and $\mathbf{D}$ be $N \times N$ matrices on the following form.

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & \cdots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 & \cdots & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & -k_N & k_N \end{bmatrix} \qquad (26)$$

$\mathbf{D}$ has the same structure as $\mathbf{K}$ except that all k_i's are replaced by d_i's. We now define the matrix $\mathbf{A}$ and the vector $\mathbf{B}$ as follows.

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} & (27) \\ \mathbf{B} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_N^{-1} \end{bmatrix} & (28) \end{aligned}$$

where $\mathbf{M}_N^{-1}$ is the Nth coloumn of $\mathbf{M}^{-1}$. Let $\mathbf{C}$ be a $1 \times 2N$ vector given by $\mathbf{C} = [0, \cdots, 0, 1]$. We now have the transfer function $\mathcal{H}(s) = \frac{\ddot{q}_N}{\tau}(s)$ given by

$$\mathcal{H}(s) = s\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \qquad (29)$$

The non-linear optimisation function $F(x)$ is now finally given by

$$F(x) = \begin{bmatrix} |\mathcal{H}(j\omega_0(1))| - A_0(1) \\ \vdots \\ |\mathcal{H}(j\omega_0(N-1))| - A_0(N-1) \\ |\mathcal{H}(j\omega_p(1))| - A_p(1) \\ \vdots \\ |\mathcal{H}(j\omega_p(N-1))| - A_p(N-1) \end{bmatrix} \qquad (30)$$

The nonlinear least-squares problem is solved using the Gauss-Newton method, similar to the methods used in the "Optimisation Toolbox" in Matlab.

4 Experiments

In this section we present experimental results from an ABB industrial robot. Figure 5 shows the frequency response from the torque in eq. (2) to the motor acceleration $\ddot{q}_3$ for a 3rd order mechanical equivalent. Note that the gravity torque has no influence on the resonance frequencies and the amplitudes, due to the linearisation. Hence, the method presented here works for systems subject to gravity as well as systems with no gravity. From Figure 5 the vectors ω_0, ω_p, $\mathbf{A}_0$ and $\mathbf{A}_p$ plus the amplitude A_L are automatically generated as follows.

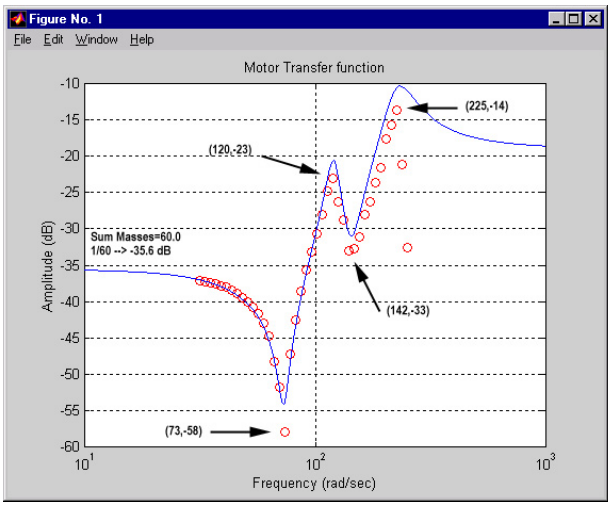

Figure 5: *Experimental results. The dotted curve shows the frequency response measurements from the manipulator. The solid curve shows the frequency response of the automatically generated 3rd order mass-spring-damper equivalent.*

$$\omega_0 = [73, 142] \quad (31)$$
$$\omega_p = [120, 225] \quad (32)$$
$$\mathbf{A}_0 = [-58, -33] \quad (33)$$
$$\mathbf{A}_p = [-23, -14] \quad (34)$$
$$A_L = -35.6dB = \frac{1}{60.3} \quad (35)$$

The corresponding eigenvalues are given by

$$\Lambda_0 = [73^2, 142^2] \quad (36)$$
$$\Lambda = [0, 120^2, 225^2] \quad (37)$$

Note that Λ has an eigenvalue at zero. This eigenvalue corresponds to the double integrator in the frequency response from motor torque to motor position. In Figure 5 we have measured the frequency response from torque to motor acceleration, where the double integrator disappears.

The computations in sections 3.3.1-3.3.3 generate the following values.

$$\mathbf{u}_3 = [0.38, 0.32, 0.87]^T \quad (38)$$
$$a_3 = 39352 \quad (39)$$
$$\mathbf{d}_2 = [15177, 7957, -9622]^T \quad (40)$$
$$b_2 = 19653 \quad (41)$$
$$\mathbf{u}_2 = [0.77, 0.40, -0.49]^T \quad (42)$$
$$a_2 = 14495 \quad (43)$$
$$\mathbf{d}_1 = [3649, -6184, 642]^T \quad (44)$$
$$b_1 = 7209 \quad (45)$$
$$\mathbf{u}_1 = [0.51, -0.86, 0.09]^T \quad (46)$$
$$\mathbf{d}_0 = [5567, -9434, 980]^T \quad (47)$$
$$a_1 = 10998 \quad (48)$$

The estimated masses and springs are

$$m_1 = 15.4 \quad (49)$$
$$m_2 = 35.8 \quad (50)$$
$$m_3 = 8.8 \quad (51)$$
$$k_1 = 0 \quad (52)$$
$$k_2 = 169000 \quad (53)$$
$$k_3 = 350000 \quad (54)$$

The dampers are found using the method in section 3.4 and the values are

$$d_2 = 185 \quad (55)$$
$$d_3 = 456 \quad (56)$$

Note that the damper coefficients are small compared to the spring coefficients. The dampers have a very little effect on the values of the resonance frequencies. Hence, the errors in the identified spring and mass coefficients are small. The frequency response of the 3rd order mechanical equivalent with the parameters given in eqs. (49)-(56) is shown as a solid line in Figure 5. We see from the frequency response that the estimated 3rd order model is a very good approximation to the robot axis dynamics.

5 Conclusions

The following conclusions can be made. 1) If we lock the position of a mass, the eigenvalues of the locked system are given by the zeroes in the transfer function of the unlocked system. The transfer function is generated from the unlocked mass from torque and position measurements. 2) From [4] page 40 we have: *If* *a linear constraint is applied to a mechanical system, each natural frequency increases, but does not exceed the next natural frequency of the original system.* In our paper, the linear constraint on system 2 in Figure 4 makes the new natural frequencies equal to the anti-resonance frequencies of the original system. Hence, between two resonance frequencies, there will always be an anti-resonance frequency. 3) If we assume a ladder structure, as in Figure 1, we can identify a unique system with the given anti-resonance and resonance frequencies. The identification procedure requires the eigenvalues of the locked and the unlocked system, and is described in [4]. 4) If the physical system is not a ladder structure, but has $N-1$ anti-resonance frequencies and N resonance frequencies, there still exists a model with ladder structure which matches the transfer function.

In this paper we have presented and demonstrated a new method for the automatic estimation of the flexible dynamics of a robot axis. The method first identifies friction coefficients by moving the robot axis at low speeds without excitation of the resonance frequencies. Second, the mass and spring coefficients of an Nth-order mechanical equivalent are automatically identified from the robot axis anti-resonance and resonance frequencies. Third, the damper coefficients are identified by using a nonlinear optimisation method. The methods have been applied to the identification of the flexible dynamics of an industrial ABB robot. The frequency response of the identified model parameters shows very good correspondence with the measured frequency response.

We have combined an important result from the vibration literature with the solution of inverse eigenvalue problems. To our knowledge, this approach to the identification of flexible robot dynamics is new. The method has significant advantages compared to other identification algorithms. The main advantage is the fact that only motor encoder position and motor torque are required to identify the springs, masses and dampers of an Nth order system.

References

[1] B. Armstrong-Hélouvry, *Control of Machines with Friction*, Kluwer Academic Publishers, 1991.

[2] De Luca A., "Trajectory Control of Flexible Manipulators", *Control Problems in Robotics and Automation*, pp. 83-104, Editors: B. Siciliano and K.P. Valavanis, Springer, 1998.

[3] S.S. Ge, et.al., "Controller Design for a Single-Link Flexible Smart Materials Robot with Experimental Tests", *Proceedings of the 37th IEEE International Conference on Decision and Control (CDC'98)*, Tampa, Florida, pp. 421-426, 1998.

[4] G.M.L. Gladwell, *Inverse Problems in Vibration*, Dordrecht, Martinus Nijhoff, 1986.

[5] G.E. Hovland, E. Berglund and O.J. Sørdalen, "Identification of Joint Elasticity of Industrial Robots", *Proceedings of the 6th International Symposium on Experimental Robotics*, Sydney, March 26-28 1999.

[6] J.H. Oh and J.S. Lee, "Backstepping Control Design of Flexible Joint Manipulator Using Only Position Measurements", *Proceedings of the 37th IEEE International Conference on Decision and Control (CDC'98)*, Tampa, Florida, pp. 931-936, 1998.

[7] Y.M. Ram and S. Elhay, "The Theory of a Multi-Degree-of-Freedom Dynamic Absorber", *Journal of Sound and Vibration*, Vol. 195, No. 4, pp. 607-615, 1996.