A LMI APPROACH TO ROBUST OBSERVER DESIGN FOR LINEAR TIME-DELAY SYSTEMS

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Abstract

This paper is concerned with a robust observer design for linear time-delay systems via linear matrix inequality approach. The proposed method not only guarantees the stability of the proposed observer, but also reduces the effects of different unstructured uncertainties on the estimated error.

Keywords: Time-delay systems; Robust observers; Linear Matrix Inequality (LMI).

1 Introduction

Many observer schemes for linear systems with time-delays have been proposed in the literature [1]-[9].

In general, disturbances, modeling errors may destabilize the observer practically if they are not taken into account when designing the observer [10].

H_∞ observers for time-delay systems have been proposed in the literature [11, 12] with a disturbance reduced effects on the estimated error. The proposed methods are based on some modified algebraic Riccati equations.

When the system uncertainties are considered as an unknown input, observer schemes for this type of systems have been proposed by Sename [13] and Fattouh *et al.* [14] using a polynomial approach.

A parameterization of all stable observers for a time-delay system has been proposed by Yao *et al.* [15]. An explicit expression of the parameterized term has been given as a model matching problem such that the *disturbance effects* on the estimated states is minimized. However, the system is assumed to be spectrally co-canonical and there is no procedure to solve the model matching problem. The authors have given in [16] an explicit expression of the parameterized term as an optimization problem such that the effects of different system *unstructured uncertainties* on the estimated states is minimized. A suboptimal solution for this optimization problem has been proposed. Again the system is assumed to be spectrally co-canonical and the proposed procedure needs some calculations.

In this paper, a method for robust observer design is proposed using a LMI approach. The system is not assumed to be spectrally co-canonical and the observer is obtained easily by solving an LMI using one of the methods proposed in [17] for example. The paper is organized as follows. Some preliminaries are given in Section 2. Section 3 is devoted to design a robust observer for time-delay systems. Two illustrative examples are given in Section 4. The paper concludes with Section 5.

Notations:

$\mathbb{R}$ is the field of real numbers,
$\mathbb{N}$ is the set of integer numbers,
$\mathbb{F}^+ = \{a \in \mathbb{F} : 0 < a < \infty\}$, $\mathbb{F}$ denotes $\mathbb{R}$ or $\mathbb{N}$,
s denotes the Laplace variable, $z = e^{-sh}$ and $h \in \mathbb{R}^+$ fixed,
$\mathbb{R}[\bullet]$ is the ring of polynomials in $\bullet$ with coefficients in $\mathbb{R}$,
$\mathbb{R}[z][s] = \{\sum_{k=0}^{m} a_k(z)s^k : a_k(z) \in \mathbb{R}[z], m \in \mathbb{N}^+\}$,
$\mathbb{R}(s,z)$ is the field of rational functions in s, z with coefficients in $\mathbb{R}$,
$\Theta = \{p(s,z) = \frac{b(s,z)}{a(s)} \in \mathbb{R}(s,z) : b(s,z) \in \mathbb{R}[z][s], a(s) \in \mathbb{R}[s], deg_s(a(s)) > deg_s(b(s,z))$ and $p(s,z)$ is entire$\}$,
$\Theta[z]$ is the ring of polynomials in z with coefficients in Θ,
$\mathcal{M}(\bullet)$ denotes the set of matrices with elements in $\bullet$,
$\|.\|_2$ is the H_2-norm defined by: $\|X(s)\|_2 = [\frac{1}{2\pi}\int_{-\infty}^{\infty} X'(-j\omega)X(j\omega)d\omega]^{\frac{1}{2}}$,
$\|.\|_\infty$ is the H_∞-norm defined by: $\|T(s)\|_\infty = \sup_\omega \{\sigma_{max}(T(j\omega))\}$; $\sigma_{max}(T)$ denotes the maximum singular value of the matrix T, j is the imaginary number,
$\mathcal{C}[a,b]$ is the set of continuous functions $[a,b] \to \mathbb{R}^n$,
I_n denotes the $(n \times n)$ identity matrix,
X' is the transpose matrix of X.

In order to simplify the notation, X will design either $X(s,z)$ or $X(s)$ when there is no confusion.

2 Preliminaries and problem statement

Consider the time-delay system:

$$\begin{cases} \dot{x}(t) &= \sum_{i=0}^{m} A_i x(t-ih) + B_i u(t-ih) \\ y(t) &= \sum_{i=0}^{m} C_i x(t-ih) \\ r(t) &= E x(t) \\ x(t) &= \psi(t); \qquad t \in [-mh, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^r$ is the control input vector, $y(t) \in \mathbb{R}^p$ is the measured output vector, $r(t) \in \mathbb{R}^k$ is the vector to be estimated, A_i, B_i, C_i, E, $i = 0, 1, \cdots, m$, $m \in \mathbb{N}^+$, are constant matrices with appropriate dimensions, $\psi(t) \in \mathcal{C}[-mh, 0]$ is the functional initial condition of (1), $h \in \mathbb{R}^+$ is the delay duration.

The transfer function matrix of system (1) is given by

$$G(s,z) = C(z)(sI_n - A(z))^{-1}B(z) \tag{2}$$

where $A(z) = \sum_{i=0}^{m} A_i z^i$, $B(z) = \sum_{i=0}^{m} B_i z^i$ and $C(z) = \sum_{i=0}^{m} C_i z^i$.

The system (1) is said to be spectrally co-canonical if the pair $(A(z), B(z))$ is spectrally controllable and the pair $(C(z), A(z))$ is $\mathbb{R}[z]$-observable (see [3] for the definitions).

Let $\tilde{G}(s,z) = f(G(s,z), W(s), \Delta(s,z))$ be the real plant with $W(s)$ is a fixed stable weighting transfer function matrix, $\Delta(s,z)$ is a variable stable transfer function matrix and $\|\Delta(s,z)\|_\infty \leq 1$. i.e. $W(s)$ and $\Delta(s,z)$ represent unstructured uncertainties as follows [18]:

$$\begin{cases} \tilde{G}(s,z) = G(s,z) + W(s)\Delta(s,z) \\ for\ additive\ uncertainty \\ \tilde{G}(s,z) = G(s,z)(I_r + W(s)\Delta(s,z)) \\ for\ input\ multiplicative\ uncertainty \end{cases} \tag{3}$$

Definition 1 *Given a real time-delay plant $\tilde{G}(s,z)$ and a realization (1) for the nominal system (2). $\hat{r}(t)$ is said to be a robust estimation of $r(t)$ if*

1. $\lim_{t\to+\infty}(r(t) - \hat{r}(t)) = 0$ *for* $W(s) \equiv 0$,
2. $\|r(s) - \hat{r}(s)\|_2$ *is bounded for* $W(s) \not\equiv 0$. ∎

In [16], the authors have provided a robust estimation of $r(t)$ under the assumption that the system (1) is spectrally co-canonical.

Lemma 1 *[16] Given a real time-delay plant $\tilde{G}(s,z)$. Suppose that the nominal system $G(s,z)$ has a spectrally co-canonical realization (1). A robust estimation of $r(s)$ is given by*

$$\begin{aligned} \hat{r}(s) &= \underbrace{(P(s,z)Y(s,z) - Q(s,z)\bar{N}(s,z))}_{F(s,z)} u(s) \\ &+ \underbrace{(P(s,z)X(s,z) + Q(s,z)\bar{M}(s,z))}_{H(s,z)} y(s) \end{aligned} \tag{4}$$

where $Q(s,z)$ is the solution of

(i) $\|H(s,z)W(s)\|_\infty \leq \gamma$ *for additive uncertainties,*

(ii) $\|F(s,z)W(s)\|_\infty \leq \gamma$ *for input multiplicative uncertainty,*

for some positive scalar γ and

$$\begin{aligned} P &= E_e(sI_e - A_o)^{-1}B_e, \quad Y = I_r - F_e(sI_e - \bar{A}_o)^{-1}B_e, \\ \bar{N} &= C_e(sI_e - \bar{A}_o)^{-1}B_e, \quad \bar{M} = I_p + C_e(sI_e - \bar{A}_o)^{-1}K_e, \\ X &= F_e(sI_e - \bar{A}_o)^{-1}K_e, \quad A_o = A_e + B_eF_e, \\ \bar{A}_o &= A_e + K_eC_e, \qquad F_e = \begin{bmatrix} F_1(s,z) & F_2(s,z) \end{bmatrix}, \\ A_e &= \begin{bmatrix} A(z) & B(z) \\ 0_{r\times n} & -I_r \end{bmatrix}, \quad B_e = \begin{bmatrix} 0_{n\times r} \\ I_r \end{bmatrix}, \\ C_e &= \begin{bmatrix} C(z) & 0_{p\times r} \end{bmatrix}, \quad E_e = \begin{bmatrix} E & 0_{k\times r} \end{bmatrix}, \\ I_e &= \begin{bmatrix} I_n & 0_{n\times r} \\ 0_{r\times n} & 0_{r\times r} \end{bmatrix}, \quad K_e = \begin{bmatrix} K(z) \\ 0_{r\times p} \end{bmatrix}. \end{aligned}$$

$F_1(s,z), F_2(s,z) \in \mathcal{M}(\Theta[z])$ and $K(z) \in \mathcal{M}(\mathbb{R}[z])$ are chosen such that

$$\begin{cases} det \begin{bmatrix} sI_n - A(z) & -B(z) \\ -F_1(s,z) & I_r - F_2(s,z) \end{bmatrix} &= \alpha(s) \\ det\,(sI_n - A(z) - K(z)C(z)) &= \beta(s) \end{cases} \tag{5}$$

$\alpha(s), \beta(s) \in \mathbb{R}[s]$ are stable polynomials. ∎

Notice that in order to get the robust estimation (4), one has, firstly, to find matrices $F_1(s,z), F_2(s,z) \in \mathcal{M}(\Theta[z])$ and $K(z) \in \mathcal{M}(\mathbb{R}[z])$ such that (5) is satisfied for some stable polynomials $\alpha(s)$ and $\beta(s)$, and, secondly, to solve the optimization problem (i) or (ii) which needs to assume that the system (1) is spectrally co-canonical and necessitates some calculations.

If system (1) is asymptotically stable, then the spectrally co-canonical assumption in not necessary and the robust estimation of $r(t)$ can be obtained as a particular case of (4) as follows.

Corollary 1 *Given a real time-delay plant $\tilde{G}(s,z)$ with a stable nominal realization (1) (not necessary spectrally co-canonical). A robust estimation of $r(s)$ is given by (4) with $Y = I_r$, $\bar{N} = G$, $X \equiv 0$, $\bar{M} = I_p$, $F_e \equiv 0$ and $K_e \equiv 0$.* ∎

It should be noted that in this case $\hat{r}(s)$ converges towards $r(s)$ with the same dynamics as system (1). If the dynamics of $r(s) - \hat{r}(s)$ has to be changed, $F_e(s,z)$

and $K_e(z)$ must be chosen different from zero and in this case one can use Lemma 1 for example to get a robust estimation.

The objective of this paper is to relax the spectrally co-canonical assumption and to provide another method in order to get a robust estimation of $r(t)$. It will be shown in the next section that this can be done by solving a linear matrix inequality.

3 Robust observer design

This section is devoted to calculate a robust estimation of $r(t)$ via linear matrix inequality approach according to Definition 1. The following Lemma will be useful for this purpose. This Lemma has been proposed by the authors in [11] and is a generalization of the result of Lee *et al.* [19] to the case of linear systems with multiple commensurate delayed states.

Lemma 2 *[11] Consider the following system:*

$$\left\{\begin{array}{lcl} \dot{\xi}(t) & = & \sum_{i=0}^{m} \mathcal{A}_i \xi(t-ih) + \mathcal{B}d(t) \\ z(t) & = & \mathcal{D}\xi(t) \\ \xi(t) & = & \phi(t); \quad t \in [-mh, 0] \end{array}\right. \tag{6}$$

where $\xi(t) \in \mathbb{R}^n$ is the state vector, $d(t) \in \mathbb{R}^r$ is the disturbance vector, $z(t) \in \mathbb{R}^p$ is the controlled output vector, $\mathcal{A}_i$, $\mathcal{B}$, $\mathcal{D}$, $i = 0, 1, \cdots, m$, are constant matrices with appropriate dimensions, $m \in \mathbb{N}^+$, $\phi(t) \in \mathcal{C}[-mh, 0]$ is the functional initial condition of (6) and $h \in \mathbb{R}^+$ is the delay duration.

Given a positive scalar γ, if there exist two symmetric positive definite matrices R and P such that

$$\mathcal{A}_0'P + P\mathcal{A}_0 + P\left(\sum_{i=1}^{m} \mathcal{A}_i R^{-1} \mathcal{A}_i'\right)P + mR + \frac{1}{\gamma^2}\mathcal{D}'\mathcal{D} + P\mathcal{B}\mathcal{B}'P < 0 \tag{7}$$

then the system (6) is asymptotically stable and the inequality $\|\mathcal{D}(sI_n - \sum_{i=0}^{m} \mathcal{A}_i e^{-sih})^{-1}\mathcal{B}\|_\infty \leq \gamma$ holds. ∎

The main result of this paper is the following.

Theorem 1 *Consider a real time-delay plant $\tilde{G}(s,z) = f(G(s,z), W(s), \Delta(s,z))$ where f is given by (3). Suppose that $G(s,z)$ has a realization (1). Given a positive scalar γ, if there exist a matrix $X \in \mathbb{R}^{n\times r}$ and two symmetric positive definite matrices P and R such that*

$$\begin{bmatrix} M & P\bar{A} & X\bar{C} & P\bar{B}D_1 & XD_2 \\ \bar{A}'P & -\bar{R} & 0 & 0 & 0 \\ \bar{C}'X' & 0 & -\bar{R} & 0 & 0 \\ D_1'\bar{B}'P & 0 & 0 & -I_r & 0 \\ D_2'X' & 0 & 0 & 0 & -I_r \end{bmatrix} < 0 \tag{8}$$

where

$$\left\{\begin{array}{lcl} M & = & A_0'P + PA_0 + mR + \frac{1}{\gamma^2}E'E \\ & & -XC_0 - C_0'X' \\ \bar{A} & = & [\ A_1, A_2, \cdots, A_m\ ] \\ \bar{C} & = & [\ C_1, C_2, \cdots, C_m\ ] \\ \bar{R} & = & \frac{1}{2}\, diag\underbrace{\{R, R, \cdots, R\}}_{m\ \ times} \\ D_1 & = & \|W(s)\|_\infty \text{ and } D_2 \equiv 0 \text{ for the input multiplicative uncertainty} \\ D_1 & \equiv & 0 \text{ and } D_2 = \|W(s)\|_\infty \text{ for the additive uncertainty} \end{array}\right. \tag{9}$$

then the following system

$$\begin{array}{rcl} \hat{r}(s) & = & E(sI_n - A(z) + LC(z))^{-1}\bar{B}\bar{u}(s) \\ & + & E(sI_n - A(z) + LC(z))^{-1}Ly(s) \end{array} \tag{10}$$

where

$$\left\{\begin{array}{lcl} \bar{B} & = & [\ B_0, B_1, \cdots, B_m\ ] \\ \bar{u}(t) & = & [u'(t),\ u'(t-h),\ \cdots,\ u'(t-mh)]' \end{array}\right. \tag{11}$$

generates a robust estimation of $r(s)$ with $L = P^{-1}X$.

Proof: In order to prove this theorem, the system (1) will be firstly transformed into another asymptotically stable system. So the Corollary 1 can be applied on the new system in order to get a robust estimation of $r(t)$. Finally, it will be proven that this estimation is a robust one for the initial system.
Suppose there exist $X \in \mathbb{R}^{n\times r}$, $P = P' > 0$ and $R = R' > 0$ such that (8) is satisfied for some $\gamma \in \mathbb{R}^+$.
1- system transformation: the system (1) can be rewritten as:

$$\dot{x}(t) = \sum_{i=0}^{m}(A_i - LC_i)x(t) + [\ \bar{B} \quad L\ ]\begin{bmatrix} \bar{u}(t) \\ y(t) \end{bmatrix} \tag{12}$$

where $\bar{B}$ and $\bar{u}(t)$ are given by (9).
Since (8) is satisfied then, using Lemma 2 and the Schur complement[1], system (12) is asymptotically stable.
2- Getting a robust estimation from (12): Since system (12) is asymptotically stable, then by Corollary 1 a robust estimation of $r(t)$ is given by:

$$\hat{r}(s) = [\ P_1(s,z) \quad P_2(s,z)\ ]\begin{bmatrix} \bar{u}(s) \\ y(s) \end{bmatrix} \tag{13}$$

for $Q \equiv 0$, where

$$\begin{array}{rcl} P_1(s,z) & = & E(sI_n - A(z) + LC(z))^{-1}\bar{B} \\ P_2(s,z) & = & E(sI_n - A(z) + LC(z))^{-1}L \end{array}$$

In the next step, it will be shown that $\|P_1(s,z)W(s)\|_\infty \leq \gamma$ for the input multiplicative

[1] the Schur complement is defined as follows [17]: the matrix $\begin{bmatrix} Q & S \\ S' & R \end{bmatrix}$ is negative definite if and only if the matrices R and $Q + SR^{-1}S'$ are negative definite.

uncertainty and $\|P_2(s,z)W(s)\|_\infty \leq \gamma$ for the additive uncertainty.

3- Robustness issue: Using the Schur complement, the LMI (8) is equivalent to:

$$A_0'P + PA_0 - XC_0 - C0'X' + mR + \frac{1}{\gamma^2}E'E$$
$$+2P\{\sum_{i=1}^{m}(A_iR^{-1}A_i' + XC_iR^{-1}C_i'X')\}P$$
$$+P\bar{B}D_1D_1'\bar{B}P + XD_2D_2'X' < 0 \quad (14)$$

For $L = P^{-1}X$, inequality (14) equals to:

$$(A_0 - LC_0)'P + P(A_0 - LC_0) + mR + \frac{1}{\gamma^2}E'E$$
$$+2P\{\sum_{i=1}^{m}(A_iR^{-1}A_i' + LC_iR^{-1}C_i'L')\}P$$
$$+P\bar{B}D_1D_1'\bar{B}P + PLD_2D_2'L'P < 0 \quad (15)$$

Using the following inequality:

$$-UR^{-1}U' - VR^{-1}V' \leq UR^{-1}V' + VR^{-1}U'$$

where U and V are any two matrices with suitable dimensions, one can write:

$$-A_iR^{-1}A_i' - LC_iR^{-1}C_i'L' \leq$$
$$LC_iR^{-1}A_i' + A_iR^{-1}C_i'L' \quad (16)$$

for $i = 1, 2, \cdots, m$.

By (16) equation (15) leads to the following inequality:

$$(A_0 - LC_0)'P + P(A_0 - LC_0)$$
$$+P\{\sum_{i=1}^{m}(A_i - LC_i)R^{-1}(A_i - LC_i)'\}P$$
$$+mR + \frac{1}{\gamma^2}E'E + PYY'P < 0 \quad (17)$$

where $Y = [\bar{B}D_1 \;\; LD_2]$.

Since (17) is satisfied then by Lemma 2 system (12) is stable and $\|P_1(s,z)D_1\|_\infty \leq \gamma$ and $\|P_2(s,z)D_2\|_\infty \leq \gamma$. Let $D_1 = \|W(s)\|_\infty$, $D_2 \equiv 0$ for input multiplicative uncertainty and $D_1 \equiv 0$, $D_2 = \|W(s)\|_\infty$ for additive uncertainty, then system (10) generates a robust estimation of $r(s)$. ■

Remark 1 *In LMI (8)-(9), γ represents a qualitative measure of system uncertainty effects on the estimated error. Hence, it will be of interest to minimize γ subject to the LMI (8)-(9).* ■

4 Illustrative examples

In this section the example proposed by [16] for a time-delay system with additive uncertainty is firstly considered. Then a second one is considered which represents an unknown input-delay system.

Example 1: Consider the unstable time-delay system: $\tilde{G}(s,z) = \{G(s,z) + \Delta(s,z)W(s) : \|\Delta\|_\infty \leq 1\}$ with

$$G(s,z) = \frac{1}{s^2 + s - z}, \qquad W(s) = \frac{0.1s^2}{s^2 + s + 1}$$

and $G(s,z)$ has a realization:

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} 0 & z \\ 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{cases}$$

Fig. 1 shows a Nyquist diagram of the nominal model $G(j\omega, e^{-j\omega h})$ with disk centered at $G(j\omega, e^{-j\omega h})$ with radius $W(j\omega)$ at each frequency.

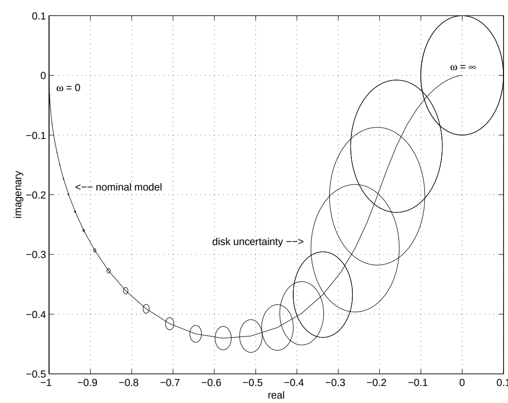

Figure 1: Nyquist diagram of $\tilde{G}(s,z)$.

In order to construct a robust observer for $r(t) = [1 \;\; 0]x(t)$ the proposed method is applied.

For $\gamma = 0.5$, $R = I_2$, $D_1 = 0$ and $D_2 = 0.1$ (additive uncertainty), the LMI (8) has a solution

$$P = \begin{bmatrix} 2.3808 & -8.4256 \\ -8.4256 & 32.7516 \end{bmatrix}, \quad X = \begin{bmatrix} -0.4454 \\ 98.8717 \end{bmatrix}$$

The observer gain matrix equals to: $L = [117.187 \;\; 33.1661]'$ and the robust estimation of $r(s)$ in this case is given by:

$$\hat{r}(s) = \frac{s + 34.1606}{s^2 + 34.1606s + 117.1658 - z}u(s)$$
$$+ \frac{117.6471(s+1) + 33.2941z}{s^2 + 34.3059s + 117.6471 - z}y(s)$$

The estimated error is shown in Fig. 2 for two values of $\Delta(s,z)$. For the simulation an initial value at time $t = -1sec.$ is used to generate an initial value function on $t \in [-1, 0]$.

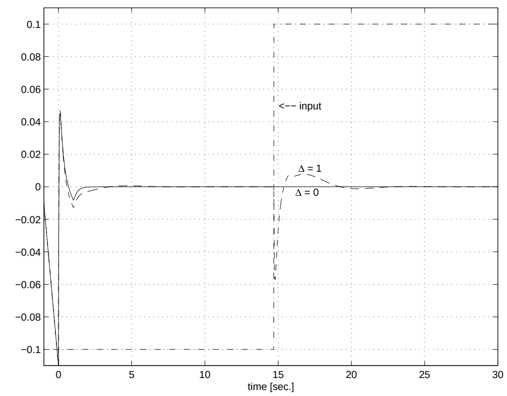

Figure 2: estimated error for $(-)\ \Delta = 0$ and $(- -)\ \Delta = 1$.

Fig. 3 shows a comparison between the estimated errors for $\Delta = 1$ obtained using the proposed method and the one obtained using the method proposed in [16]. Clearly, the proposed method gives better result concerning the convergence of the estimated error.

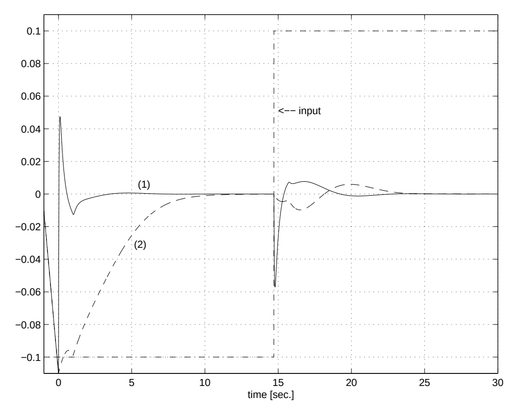

Figure 3: estimated error: (1) proposed method (2) [16]'s method.

Fig. 4 shows the singular values of $P_2(j\omega, e^{-j\omega})W(j\omega)$ versus the frequency. This plot represents the effects of the system uncertainty on the real estimation.

Example 2: Consider the time-delay system:

$$\tilde{G}(s) = \frac{e^{-sh}}{s^2}$$

where the time delay is assumed to be constant but unknown and lies in the interval $0 \leq h \leq 2$.
Let $h = 1$ sec. be the nominal value of the time delay, the term $e^{-s(h-1)}$ can be treated as an input multiplicative uncertainty of the nominal system $G(s,z) = \frac{z}{s^2}$, $z = e^{-s}$, by embedding $\tilde{G}(s)$ in the family

$$\{G(s,z)\,(1 + W(s)\Delta(s,z)) : \|\Delta\|_\infty \leq 1\}$$

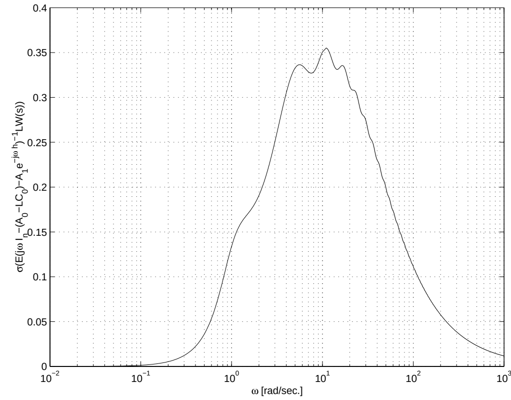

Figure 4: effects of the uncertainty on the estimated error.

with $W(s) = \frac{21s}{10s+1}$.
A minimal realization of $G(s,z)$ is given by:

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} z \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{cases}$$

In order to construct a robust observer for $r(t) = [1\ \ 0]x(t)$ the proposed method is applied.
For $\gamma = 0.3$, $R = I_2$, $D_1 = 2.1$ and $D_2 = 0$, the LMI (8) has a solution

$$P = \begin{bmatrix} 1.4012 & -15.4007 \\ -15.4007 & 744.7611 \end{bmatrix}, \quad X = \begin{bmatrix} 652.0250 \\ 751.9777 \end{bmatrix}$$

The observer gain matrix equals to: $L = [616.5645\ \ 13.7594]'$ and the robust estimation of $r(s)$ in this case is given by:

$$\begin{aligned} \hat{r}(s) &= \frac{(s + 13.7594)z}{s^2 + 13.7594s + 616.5637} u(s) \\ &+ \frac{616.5637s}{s^2 + 13.7594s + 616.5637} y(s) \end{aligned}$$

The estimated error is shown in Fig. 5 for $\Delta = 1$ and different values of $h \in [0, 2]$ and for a step input.

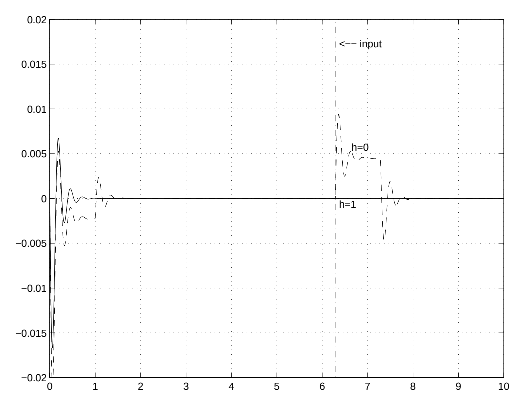

Figure 5: estimated error for $\Delta = 1$ and $(-)\ h = 1$, $(- -)\ h = 0$.

Fig. 6 shows the singular values of $P_1(j\omega, e^{-j\omega})W(j\omega)$ versus the frequency. This plot represents the effects of the system uncertainty on the real estimation.

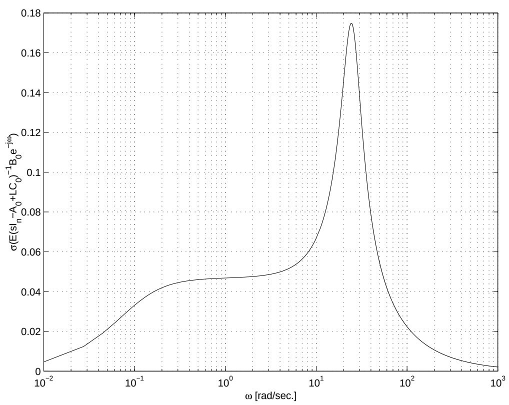

Figure 6: effects of the uncertainty on the estimated error.

5 Conclusion

In this paper, a method for a robust observer design for linear time-delay systems has been developed. The design method not only stabilizes the observer but also reduces the effects of system unstructured uncertainty on the estimated error. A sufficient condition for the existence of such an observer is given in term of linear matrix inequality to be solved.

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