Robust Output-Feedback Design Using a New Class of Nonlinear Observers[1]

Murat Arcak
murat@seidel.ece.ucsb.edu

Petar Kokotović
petar@ece.ucsb.edu

Center for Control Engineering and Computation,
University of California, Santa Barbara, CA 93106-9560

Abstract

This paper analyzes the robustness of a nonlinear observer design recently introduced by the authors. For uncertainties in the nonlinearities, bounds are given within which the observer error gradually increases with an increase in the uncertainty. For dynamic modeling errors, a robust output-feedback design is developed using ISS small-gain tools. A jet engine compressor example is used to illustrate the design.

1 Introduction

The global observer designs in the literature severely restrict classes of systems and nonlinearities. Early efforts by Thau [16], Kou *et al.* [7], Banks [2], and their recent extensions, such as Raghavan and Hedrick [13], and Rajamani [14], restrict the state dependent nonlinearities to be globally Lipschitz. An alternative is to restrict the system structure so that the nonlinearities appear as functions of the measured output. This class of systems has been characterized by Krener and Isidori [8], Bestle and Zeitz [3], and other authors. Global high-gain observers have been designed by Gauthier *et al.* [4], again under a global Lipschitz assumption.

In [1] we have presented an observer design which removes the global Lipschitz restriction from state dependent nonlinearities. This design represents the observer error system as the feedback interconnection of a linear system and a time-varying sector nonlinearity. Then, the observer gain matrices are selected to satisfy the circle criterion and, hence, to drive the observer error to zero. The class of systems for which this design is applicable is characterized with two restrictions imposed by the circle criterion. First, a linear matrix inequality (LMI) is to be feasible, which implies a positive real property for the linear part of the observer error system. The second restriction is that the nonlinearities be nondecreasing functions of linear combinations of unmeasured states. This restriction ensures that the vector time-varying nonlinearity in the observer error system satisfies the sector condition of the circle criterion.

In this paper we analyze the robustness of the new observer against modeling errors. We first study uncertainties in the nonlinearities and give bounds within which the observer error gradually increases with an increase in the modeling error. Next, we consider unmodeled dynamics, such as those studied by Praly and Jiang [12], and propose a small-gain design for observer-based control.

In Section 2 we review the observer design of [1], and analyze its robustness against inexact modeling of nonlinearities. In Section 3, we present our robust output-feedback design for unmodeled dynamics. A jet engine compressor example in Section 4 is used to illustrate the design, where the subsystem that does not meet the observer requirement is treated as unmodeled dynamics.

2 The Observer Design

For our observer design we consider the plant

$$\begin{aligned} \dot{x} &= Ax + G\gamma(Hx) + \varrho(y,u) \\ y &= Cx\,, \end{aligned} \tag{1}$$

where $x \in I\!R^n$ is the state, $y \in I\!R^p$ is the measured output, $u \in I\!R^m$ is the control input, the pair (A, C) is detectable, and, $\gamma(\cdot)$ and $\varrho(\cdot,\cdot)$ are

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locally Lipschitz. The state dependent nonlinearity $\gamma(Hx)$ is an r-dimensional vector where each entry is a function of a linear combination of the states

$$\gamma_i = \gamma_i(\sum_{j=1}^{n} H_{ij}x_j), \quad i = 1, \cdots, r\,. \tag{2}$$

Our main restriction is that each $\gamma_i(\cdot)$ be nondecreasing, that is, for all $a, b \in I\!R$, it satisfies

$$(a-b)[\gamma_i(a) - \gamma_i(b)] \geq 0\,. \tag{3}$$

If $\gamma_i(\cdot)$ is continuously differentiable, then $d\gamma_i(v)/dv \geq 0$ for all $v \in I\!R$. If, instead, $\gamma_i(\cdot)$ satisfies $d\gamma_i(v)/dv \geq g_i$, $g_i \neq 0$, we can still represent the system as in (1)-(3) by defining a new function $\tilde{\gamma}_i(v) := \gamma_i(v) - g_i v$ which satisfies $d\tilde{\gamma}_i(v)/dv \geq 0$, and absorbing $g_i v$ in the linear part of the system. When the nonlinearity $\gamma(Hx)$ also depends on y and u, we require that the nondecreasing property (3) hold for each $y \in I\!R^p$ and $u \in I\!R^m$.

For the plant (1), we construct an observer in the form

$$\dot{\hat{x}} = A\hat{x} + L(C\hat{x} - y) + G\,\gamma(H\hat{x} + K(C\hat{x} - y)) + \varrho(y, u)\,. \tag{4}$$

Our design task is to determine the observer matrices $K \in I\!R^{r\times p}$ and $L \in I\!R^{n\times p}$. For the observer equation to be well-defined, the uniqueness of the solutions $x(t)$ of (1) is guaranteed by restricting the control law $u = \alpha(y, \hat{x})$ to be locally Lipschitz in $(y, \hat{x})$. As will be further clarified in Section 3, we also assume that $u = \alpha(y, \hat{x})$ ensures the absence of finite escape for $x(t)$.

From (1) and (4), the dynamics of the observer error $e = x - \hat{x}$ are governed by

$$\dot{e} = (A + LC)e + G\,[\gamma(v) - \gamma(w)]\,, \tag{5}$$

where

$$v := Hx, \qquad w := H\hat{x} + K(C\hat{x} - y)\,. \tag{6}$$

We begin the observer design by representing the observer error system (5) as the feedback interconnection of a linear system and a multivariable sector nonlinearity. To this end, we view $\gamma(v) - \gamma(w)$ as a function φ of v and $z := v - w = (H + KC)e$, that is, a time-varying nonlinearity in z:

$$\varphi(t, z) := \gamma(v) - \gamma(w)\,, \tag{7}$$

where the time dependence is due to $v(t)$. Substituting (7), we rewrite the observer error system (5) as

$$\begin{aligned} \dot{e} &= (A + LC)e + G\varphi(t, z) \\ z &= (H + KC)e\,, \end{aligned} \tag{8}$$

and note from (3) that each component of $\varphi(t, z)$ satisfies

$$z_i\varphi_i(t, z_i) \geq 0\,, \quad \forall z_i \in I\!R\,. \tag{9}$$

Thanks to this sector property of each $\varphi_i(t, z_i)$, the product $\varphi(t, z)^T\Lambda z$ is nonnegative for any diagonal $\Lambda > 0$. This means that the feedback path in the observer error system depicted in Figure 1 is a multivariable sector nonlinearity. Thus, from the multivariable circle criterion, asymptotic stability is guaranteed if the linear system with input ϑ and output Λz is SPR, that is, if a matrix $P = P^T > 0$, a constant $\nu > 0$, and a diagonal matrix $\Lambda > 0$ exist such that

$$\begin{aligned} &(A + LC)^T P + P(A + LC) + \nu I \leq 0 \\ &PG + (H + KC)^T\Lambda = 0\,. \end{aligned} \tag{10}$$

In this way, the observer design for system (1) is reduced to the problem of finding observer matrices K and L such that (10) is satisfied with some $P = P^T > 0$, $\Lambda > 0$, and $\nu > 0$. Because (10) is a LMI in P, PL, Λ, ΛK and ν, we can use the efficient numerical tools available for LMI's to determine whether the problem is feasible and, if so, to compute K and L.

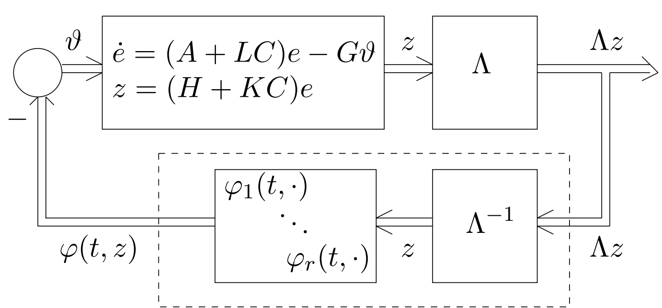

Figure 1: Observer error system.

To analyze the robustness of the observer against inexact modeling of nonlinearities, we suppose that instead of (1), the plant is

$$\dot{x} = Ax + G[\gamma(Hx) + \Delta(Hx)\mu(t)] + \varrho(y, u)\,, \tag{11}$$

where $\mu(t)$ is an unknown bounded disturbance. Then, using the observer (4), we get the observer error system

$$\dot{e} = (A+LC)e + G\left[\gamma(v) - \gamma(w) + \Delta(v)\mu(t)\right], \quad (12)$$

where v and w are as in (6).

We now characterize admissible nonlinearities $\Delta(\cdot)$ for which the observer (4) guarantees an ISS property from the disturbance $\mu(t)$ to the observer error $e(t)$.

Theorem 1 *Consider the plant (11) and the observer (4). Suppose $x(t)$ exists for all $t \geq 0$, and that the LMI (10) holds with a matrix $P = P^T > 0$, a constant $\nu > 0$, and a diagonal matrix $\Lambda > 0$. If, for each $i = 1, \cdots, r$, there exists a class-$\mathcal{K}$ function $\sigma_i(\cdot)$ such that, for all $a, b, \mu \in I\!R$,*

$$(a-b)[\gamma_i(a) - \gamma_i(b) + \Delta_i(a)\mu] \geq -\sigma_i(|\mu|), \quad (13)$$

then the observer error $e(t)$ satisfies, for all $t \geq 0$,

$$|e(t)| \leq \kappa |e(0)| \exp(-\beta t) + \rho\left(\sup_{0\leq\tau\leq t} |\mu(\tau)|\right), \quad (14)$$

where $\kappa = \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$, $\beta = \frac{\nu}{2\lambda_{max}(P)}$, and the ISS-gain from $\mu(t)$ to $e(t)$ is

$$\rho(\cdot) = \kappa \sqrt{\frac{2}{\nu}\sum_{i=1}^{r} \lambda_i\, \sigma_i(\cdot)}. \quad (15)$$

Proof: We use $V = e^T P e$ as an ISS-Lyapunov function, and evaluate its derivative for (12):

$$\dot{V} \leq -\nu|e|^2 - 2\sum_{i=1}^{r} \lambda_i \left(v_i - w_i\right)\left[\gamma_i(v_i) - \gamma_i(w_i) + \Delta_i(v_i)\mu\right]. \quad (16)$$

Substituting (13), we obtain

$$\dot{V} \leq -2\beta V + 2\sum_{i=1}^{r} \lambda_i\, \sigma_i(|\mu|), \quad (17)$$

from which it follows that

$$V(t) \leq V(0)\exp(-2\beta t) + \frac{1}{\beta}\left(\sum_{i=1}^{r} \lambda_i \sup_{0\leq\tau\leq t} \sigma_i(|\mu(\tau)|)\right). \quad (18)$$

Thus, (14) and (15) result from $\frac{1}{\beta\lambda_{min}(P)} = \frac{2\kappa^2}{\nu}$. □

The ISS property established by Theorem 1 shows that $e(t)$ degrades gracefully with the increase in the magnitude of the disturbance $\mu(t)$. As $\mu(t)$ vanishes, we recover exponential convergence of the observer error to zero.

The dependence of admissible nonlinearities $\Delta(\cdot)$ on $\gamma(\cdot)$ is characterized by (13). For example, if $\gamma(\cdot)$ is cubic, then $\Delta(\cdot)$ is allowed to be linear. In this case, (13) is satisfied because

$$(a-b)[a^3 - b^3 + a\mu] \geq -\frac{1}{3}\mu^2 \quad (19)$$

holds for all $a, b, \mu \in I\!R$ due to the identity $a^3 - b^3 = (a-b)(a^2+ab+b^2)$. On the other hand, (13) does not hold for cubic $\gamma(\cdot)$ and quadratic $\Delta(\cdot)$. To see this, we evaluate $(a-b)[a^3 - b^3 + a^2\mu]$ with $b = a + \frac{1}{a}$, and note that, for any fixed $\mu > 0$, the resulting function $3 + \frac{3}{a^2} + \frac{1}{a^4} - a\mu$ tends to $-\infty$ as $a \to +\infty$, thus violating (13).

3 Robust Output-Feedback Design

We now extend the results of Section 2 to achieve robustness against dynamic modeling errors. We consider the problem of output-feedback stabilization for the locally Lipschitz system

$$\begin{aligned} \dot{x} &= Ax + G[\gamma(Hx) + \Delta(Hx)\mu] + \varrho(y,u) \quad (20)\\ y &= Cx \\ \dot{\xi} &= q(\xi, h(x)) \quad (21)\\ \mu &= p(\xi, h(x)), \end{aligned}$$

where $\gamma(\cdot)$ and $\Delta(\cdot)$ are as in (13), and the ξ-subsystem (21) represents unmodeled dynamics. This formulation extends the applicability of our observer because, if there is a subsystem to which the observer is not applicable, it can be treated as unmodeled dynamics.

The unmodeled dynamics (21) are assumed to possess the following input-to-output stability (IOS), and ISS properties:

$$|\mu(t)| \leq \max\left\{\beta_\mu(|\xi(0)|, t), \rho_{\mu h}\left(\sup_{0\leq\tau\leq t} |h(x(\tau))|\right)\right\} \quad (22)$$

$$|\xi(t)| \leq \max\left\{\beta_\xi(|\xi(0)|, t), \rho_{\xi h}\left(\sup_{0\leq\tau\leq t} |h(x(\tau))|\right)\right\} \quad (23)$$

where $\beta_\mu(\cdot,\cdot)$, $\beta_\xi(\cdot,\cdot)$ are class-$\mathcal{KL}$ functions, and $\rho_{\mu h}(\cdot)$, $\rho_{\xi h}(\cdot)$ are class-$\mathcal{K}$ functions.

The ISS property of the observer error is now rewritten in the 'max' form of Teel [15], which is suitable for the small-gain analysis we will employ. Using the fact that for each $\rho(\cdot) \in \mathcal{K}_\infty$, $\forall a, b \geq 0$, $a + b \leq \max\{(I+\rho)(a), (I+\rho^{-1})(b)\}$, where $I(\cdot) = (\cdot)$ represents the identity function, we get

$$|e(t)| \leq \max\left\{\beta_e(|e(0)|,t), \rho_{e\mu}\left(\sup_{0\leq\tau\leq t}|\mu(\tau)|\right)\right\}. \quad (24)$$

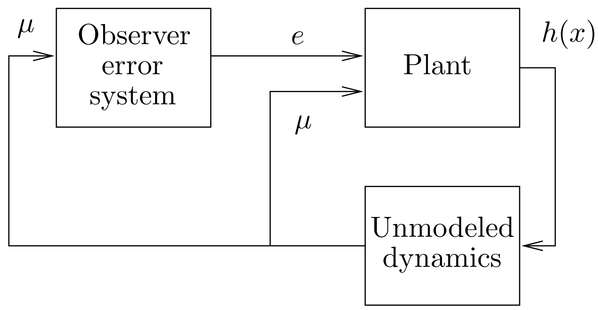

Figure 2: Closed-loop system with observer feedback.

To prepare for a small-gain design of $u = \alpha(y, \hat{x})$, we represent the closed-loop system with the block-diagram in Figure 2. The gains of the unmodeled dynamics block and the observer error block are $\rho_{\mu h}(\cdot)$ and $\rho_{e\mu}(\cdot)$, respectively. Let $\rho_{he}(\cdot)$ and $\rho_{h\mu}(\cdot)$ be the plant gains from e and μ to $h(x)$, respectively. The task for the control law $u = \alpha(y, \hat{x})$ is to render $\rho_{he}(\cdot)$ and $\rho_{h\mu}(\cdot)$ small enough for the inner loop gain $\rho_{h\mu} \circ \rho_{\mu h}(\cdot)$, and the outer loop gain $\rho_{he} \circ \rho_{e\mu} \circ \rho_{\mu h}(\cdot)$ to satisfy, for all $s > 0$,

$$\rho_{h\mu} \circ \rho_{\mu h}(s) < s \quad (25)$$

$$\rho_{he} \circ \rho_{e\mu} \circ \rho_{\mu h}(s) < s\,. \quad (26)$$

Then, global asymptotic stability (GAS) of the closed-loop system will be guaranteed as in the *nonlinear small-gain theorem* of Teel *et al.* [6, 15].

Theorem 2 *Consider the system (20)-(21), in which $\gamma(\cdot)$ and $\Delta(\cdot)$ satisfy (13), and the ξ-subsystem satisfies (22) and (23). Suppose that the observer*

$$\dot{\hat{x}} = A\hat{x} + L(C\hat{x}-y) + G\,\gamma(H\hat{x}+K(C\hat{x}-y)) + \varrho(y,u) \quad (27)$$

is such that the LMI (10) holds with a matrix $P = P^T > 0$, a constant $\nu > 0$, and a diagonal matrix $\Lambda > 0$. If the control law $u = \alpha(y, \hat{x})$ guarantees

$$|h(x(t))| \leq \max\left\{\beta_h(|x(0)|,t)\,, \rho_{h\mu}\left(\sup_{0\leq\tau\leq t}|\mu(\tau)|\right),\ \rho_{he}\left(\sup_{0\leq\tau\leq t}|e(\tau)|\right)\right\} \quad (28)$$

$$|x(t)| \leq \max\left\{\beta_x(|x(0)|,t)\,, \rho_{x\mu}\left(\sup_{0\leq\tau\leq t}|\mu(\tau)|\right),\ \rho_{xe}\left(\sup_{0\leq\tau\leq t}|e(\tau)|\right)\right\}, \quad (29)$$

where $\rho_{h\mu}(\cdot)$ and $\rho_{he}(\cdot)$ satisfy (25) and (26), respectively, then the origin of the closed-loop system (20), (21), (27) is globally asymptotically stable.

4 Design Example

An axial compressor model, which has been the starting point for jet engine control studies, is the following single-mode approximation of a PDE model due to Moore and Greitzer [11],

$$\begin{aligned}
\dot{\phi} &= -\psi + \frac{3}{2}\phi + \frac{1}{2} - \frac{1}{2}(\phi+1)^3 - 3(\phi+1)R \\
\dot{\psi} &= \frac{1}{\beta^2}(\phi + 1 - u) \\
\dot{R} &= \sigma R(-2\phi - \phi^2 - R)\,, \quad R(0) \geq 0\,,
\end{aligned} \quad (30)$$

where ϕ and ψ are the deviations of the mass flow and the pressure rise from their set points, the control input u is the flow through the throttle, and, σ and β are positive constants. This model captures the main surge instability between the mass flow and the pressure rise. It also incorporates the nonnegative magnitude R of the first stall mode.

A state feedback GAS control law in [10, Section 2.4] was replaced by a design using ϕ and ψ in [9]. With a high-gain observer, Isidori [5, Section 12.7] obtained a semiglobal result using the measurement of ψ alone. With $y = \psi$, we will now achieve GAS for (30). Our exact observer cannot be designed because of the nonlinearities ϕR and $\phi^2 R$. However, the (ϕ, ψ)-subsystem contains the nondecreasing nonlinearity $(\phi+1)^3$, and is of the form (20) with disturbance $\mu = R$. This suggests that we treat the R-subsystem as unmodeled dynamics and apply the design of Section 3.

First, we prove that $\mu = R$ satisfies the IOS property (22) with $h(x) = \phi$ as the input. With $V = R^2$ as an ISS-Lyapunov function, $R \geq 2.1|\phi|$ implies $\dot{V} \leq -0.09\sigma R^3$, because $R(t) \geq 0$ for all $t \geq 0$. This means that (22) holds with the linear gain

$$\rho_{\mu h}(\cdot) = 2.1(\cdot)\,, \tag{31}$$

and, since $\mu = \xi = R$, the ISS property (23) is also satisfied.

To design a reduced-order observer for the (ϕ, ψ)-subsystem, we let $\chi = \phi + N\psi$, and obtain

$$\begin{aligned} \dot{\chi} \;&=\; \left(\frac{3}{2} + \frac{N}{\beta^2}\right)\chi - \frac{1}{2}(\chi - N\psi + 1)^3 \\ &\quad -3(\chi - N\psi + 1)R + \bar{\varrho}(\psi, u)\,, \end{aligned} \tag{32}$$

where

$$\bar{\varrho}(\psi, u) := -\left(\frac{3}{2} + \frac{N}{\beta^2}\right)N\psi - \psi + \frac{1}{2} + \frac{N}{\beta^2}(1 - u)\,. \tag{33}$$

The resulting observer is the scalar equation

$$\begin{aligned} \dot{\hat{\chi}} \;&=\; \left(\frac{3}{2} + \frac{N}{\beta^2}\right)\hat{\chi} - \frac{1}{2}(\hat{\chi} - N\psi + 1)^3 + \bar{\varrho}(\psi, u) \\ \hat{\phi} \;&=\; \hat{\chi} - N\psi\,. \end{aligned} \tag{34}$$

For its implementation we select N such that

$$k := -\left(\frac{3}{2} + \frac{N}{\beta^2}\right) > 0\,. \tag{35}$$

To prove the ISS property (24) for the observer error $e_\phi = \phi - \hat{\phi}$, we employ the ISS-Lyapunov function $V_e = e_\phi^2$, and evaluate its derivative for

$$\dot{e}_\phi = -ke_\phi - \frac{1}{2}(a^3 - b^3 + 6aR)\,, \tag{36}$$

where $a := \chi - N\psi + 1$ and $b = \hat{\chi} - N\psi + 1$. Employing the inequality (19), and substituting $a - b = e_\phi$, we get

$$\dot{V}_e \leq -2ke_\phi^2 + 12R^2\,, \tag{37}$$

from which $|e_\phi| \geq \sqrt{\frac{6.1}{k}}|R|$ implies $\dot{V}_e \leq -0.03ke_\phi^2$, and, hence, the ISS property (24) holds with the linear gain

$$\rho_{e\mu}(\cdot) = \sqrt{\frac{6.1}{k}}(\cdot). \tag{38}$$

We are now ready to design a control law as in Theorem 2. Noting that the (ϕ, ψ)-subsystem in (30) is in strict feedback form, we apply one step of *observer backstepping*, [10]. For ψ, we design the virtual control law $\alpha_0 = c_1\hat{\phi}$. Denoting

$$\omega := \psi - c_1\hat{\phi} = \psi - c_1\phi + c_1 e_\phi, \tag{39}$$

we rewrite the $\dot{\phi}$-equation as

$$\dot{\phi} = -c_1\phi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3\phi R - \omega - 3R + c_1 e_\phi\,. \tag{40}$$

The substitution of (34) in (39) yields $\omega = (1 + Nc_1)\psi - c_1\hat{\chi}$, and, from (30) and (34),

$$\dot{\omega} = \frac{1 + Nc_1}{\beta^2}\phi + \frac{1}{\beta^2}(1 - u) + \Gamma(\hat{\phi}, \psi)\,, \tag{41}$$

where $\Gamma(\hat{\phi}, \psi) := c_1\psi + c_1 k\hat{\phi} + \frac{c_1}{2}(\hat{\phi} + 1)^3 - \frac{c_1}{2}$. Then, the control law

$$u = 1 + (1 + Nc_1)\hat{\phi} + \beta^2(c_2\omega + \Gamma(\hat{\phi}, \psi)) \tag{42}$$

is implementable using the signals ψ and $\hat{\phi}$, and results in

$$\dot{\omega} = -c_2\omega + \frac{1 + Nc_1}{\beta^2}e_\phi. \tag{43}$$

The remaining task is to select the design parameters c_1 and c_2 such that (28) and (29) are satisfied. For the ISS-Lyapunov function $W(\phi, \omega) := \frac{1}{2}\phi^2 + \frac{1}{2}\omega^2$, the inequalities $-\frac{3}{2}\phi^3 \leq \frac{9}{8}\phi^2 + \frac{1}{2}\phi^4$, $-\phi\omega \leq \frac{1}{2}\phi^2 + \frac{1}{2}\omega^2$, $-3\phi R \leq \frac{9}{4}\phi^2 + R^2$, $-3\phi^2 R \leq 0$ (because $R(t) \geq 0$), $c_1\phi e_\phi \leq \frac{c_1}{2}\phi^2 + \frac{c_1}{2}e_\phi^2$, and $\frac{1+Nc_1}{\beta^2}\omega e_\phi \leq \frac{(1+Nc_1)^2}{2\beta^4 c_1}\omega^2 + \frac{c_1}{2}e_\phi^2$, yield

$$\begin{aligned} \dot{W} \leq &-\left(\frac{c_1}{2} - \frac{31}{8}\right)\phi^2 \\ &-\left(c_2 - \frac{1}{2} - \frac{(1 + Nc_1)^2}{2\beta^4 c_1}\right)\omega^2 + R^2 + c_1 e_\phi^2\,. \end{aligned} \tag{44}$$

We let $c > 0$, and select c_1 and c_2 to satisfy

$$\left(\frac{c_1}{2} - \frac{31}{8}\right) > c, \quad \left(c_2 - \frac{1}{2} - \frac{(1 + Nc_1)^2}{2\beta^4 c_1}\right) > c, \tag{45}$$

so that

$$\dot{W} \leq -c(\phi^2 + \omega^2) + R^2 + (2c + \frac{31}{4})e_\phi^2\,, \tag{46}$$

from which (29) follows for $x = (\phi, \psi)$. For $h(x) = \phi$ and $\mu = R$, we now compute the gains $\rho_{h\mu}(\cdot)$ and $\rho_{he}(\cdot)$ in (28). Using the fact that for each constant $\theta > 0$, $a+b \leq \max\{(1+\theta^{-1})a, (1+\theta)b\}$ for all $a, b \geq 0$, we obtain

$$\begin{aligned} \dot{W} &\leq -c(\phi^2+\omega^2) + R^2 + (2c + \frac{31}{4})e_\phi^2 \quad (47) \\ &\leq -2cW + \quad (48) \\ &\quad \max\left\{(1+\theta^{-1})R^2,\ (1+\theta)(2c+\frac{31}{4})e_\phi^2\right\}, \end{aligned}$$

from which it follows that

$$W \geq \max\left\{\frac{(1+\theta^{-1})}{1.9c}R^2,\ \frac{(1+\theta)}{1.9c}(2c+\frac{31}{4})e_\phi^2\right\}$$

$$\Rightarrow \quad \dot{W} \leq -0.1cW\,. \quad (49)$$

Then, (28) follows because $|\phi| \leq \sqrt{2W}$, and the gains are

$$\rho_{h\mu}(\cdot) = \sqrt{\frac{(1+\theta^{-1})}{0.95c}}\,(\cdot) \quad (50)$$

$$\rho_{he}(\cdot) = \sqrt{(1+\theta)\left(\frac{2}{0.95}+\frac{31}{3.8c}\right)}\,(\cdot)\,. \quad (51)$$

Using (31), (38), (50) and (51), the inner and outer loop small-gain conditions, (25) and (26) are, respectively,

$$2.1\sqrt{\frac{(1+\theta^{-1})}{0.95c}} < 1\,, \quad (52)$$

$$2.1\sqrt{\frac{6.1}{k}}\sqrt{(1+\theta)\left(\frac{2}{0.95}+\frac{31}{3.8c}\right)} < 1\,. \quad (53)$$

Selecting $c > 0$ and $k > 0$ sufficiently large ensures that (52) and (53) hold. Additional freedom for the selection of c and k is obtained from $\theta > 0$, which allocates the inner and outer loop gains.

5 Conclusion

We have analyzed the robustness of the nonlinear observer introduced in [1]. This robustness property is due to the use of the circle criterion, which rendered the linear part of the observer error system positive real. This indicates the possibility of improving robustness of observers by requiring certain structural properties for the observer error system, such as positive realness. The use of our observer in conjunction with small-gain tools has led to an output-feedback design procedure, illustrated on the jet engine compressor example. Such combined use of observer and controller design tools is a promising research direction for nonlinear output-feedback control.

References

[1] M. Arcak and P.V. Kokotović. Nonlinear observers: A circle criterion design. In *Proceedings of the 38th IEEE Conference on Decision and Control*, pages 4872–4876, Phoenix, AZ, 1999.

[2] S.P. Banks. A note on non-linear observers. *International Journal of Control*, 34:185–190, 1981.

[3] D. Bestle and M. Zeitz. Canonical form observer design for non-linear time-variable systems. *International Journal of Control*, 38:419–431, 1983.

[4] J.P. Gauthier, H. Hammouri, and S. Othman. A simple observer for nonlinear systems, applications to bioreactors. *IEEE Transactions on Automatic Control*, 37:875–880, 1992.

[5] A. Isidori. *Nonlinear Control Systems II*. Springer-Verlag, London, 1999.

[6] Z.-P. Jiang, A.R. Teel, and L. Praly. Small-gain theorem for ISS systems and applications. *Mathematics of Control, Signals, and Systems*, 7:95–120, 1994.

[7] S.R. Kou, D.L. Elliott, and T.J. Tarn. Exponential observers for nonlinear dynamic systems. *Information and Control*, 29:204–216, 1975.

[8] A.J. Krener and A. Isidori. Linearization by output injection and nonlinear observers. *Systems and Control Letters*, 3:47–52, 1983.

[9] M. Krstić, D. Fontaine, P. Kokotović, and J. Paduano. Useful nonlinearities and global bifurcation control of jet engine stall and surge. *IEEE Transactions on Automatic Control*, 43:1739–1745, 1998.

[10] M. Krstić, I. Kanellakopoulos, and P. Kokotović. *Nonlinear and Adaptive Control Design*. John Wiley & Sons, Inc., New York, 1995.

[11] F.K. Moore and E.M. Greitzer. A theory of post-stall transients in axial compression systems -Part I: Development of equations. *Journal of Turbomachinery*, 108:68–76, 1986.

[12] L. Praly and Z.-P. Jiang. Stabilization by output-feedback for systems with ISS inverse dynamics. *Systems and Control Letters*, 21:19–33, 1993.

[13] S. Raghavan and J.K. Hedrick. Observer design for a class of nonlinear systems. *International Journal of Control*, 59:515–528, 1994.

[14] R. Rajamani. Observers for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, 43:397–401, 1998.

[15] A.R. Teel. A nonlinear small gain theorem for the analysis of control systems with saturation. *IEEE Transactions on Automatic Control*, 41(9):1256–1271, 1996.

[16] F.E. Thau. Observing the state of non-linear dynamic systems. *International Journal of Control*, 17:471–479, 1973.