

# Observers for the biotechnological processes with unknown kinetics. Application to wastewater treatment

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**ABSTRACT.** In this paper we present nonlinear observers for a class of biotechnological processes. These observers are an extension of the asymptotic observers (observers with unknown inputs) devoted to biotechnological systems for which some parts of the model are unknown. We take benefit of the outputs which are non linear functions of the state to design a close loop observer. The global convergence of these nonlinear observers is proven. The method is illustrated with an experimental application to a water treatment process.

**Keywords:** nonlinear observers, unknown inputs, mass balance models, bioprocesses.

## 1 Introduction and motivation

The dynamical behaviour of a continuously stirred tank bioreactor (CSTR) is often described by a general nonlinear mass-balance model of the following form [1]:

$$\dot{\xi}(t) = Kr(\xi(t)) - D(t)\xi(t) + D(t)\xi_{in}(t) - Q(\xi(t)) \quad (1)$$

In this model, the vector  $\xi = (\xi_1, \xi_2, \dots, \xi_n)^t$  is the vector of the concentrations of the biological or chemical species inside the liquid medium. The first term  $Kr(\xi)$  on the right-hand side represents the biological and biochemical conversions in the reactor (per unit of time) according to the underlying reaction network. The  $(n \times k)$  matrix  $K$  is a constant (stoichiometric-like) yield coefficient matrix. The vector  $r(\xi) = (r_1(\xi), r_2(\xi), \dots, r_k(\xi))^t$  is a vector of **reaction rates** (or conversion rates). The influent feeding

concentration is represented by  $\xi_{in}(t)$ , and  $D(t)$ , the dilution rate is the ratio of the influent flow rate and of the volume of the fermenter. For sake of brevity the time dependence will be omitted in the future. The last term  $Q(\xi)$  represents the gaseous exchange with the outside of the fermenter.

Very often, the precise formulation of the kinetics  $r(\xi)$  is not known, and therefore to build observers for these systems it is often preferable to consider  $r(\xi)$  as an unknown input and apply the linear theory of rejection of unknown inputs [2, 3]. This is the principle of the mass balance observers [1, 4]. These observers are based on mass-balance considerations, in order to eliminate the unknown kinetics  $r(\xi)$ . But since they are in open-loop, they are not very robust to errors on the estimation of the influent masses. Nevertheless these observers require the measurements of some gaseous flow rates. In this paper we take benefit of these outputs and of other measurements rather easy to perform in practice (pH, conductivity, ...) to build a closed-loop nonlinear observer, that achieves better properties of robustness and convergence rate.

## 2 Notations and definitions

In the sequel the mappings  $r(\xi)$ ,  $Q(\xi)$  are assumed to be  $C^1$  on the considered open domain  $\Omega$ . The solutions of the ordinary differential equations are well defined and unique.

We assume that a part  $\xi_1$  of the state  $\xi$  can be measured on-line. The part of the state which is not measured,

$\xi_2$ , is of dimension  $p$ . We have thus  $\xi = (\xi_1, \xi_2)^t$ . We assume moreover that the output  $y$  of the system (i.e. the set of measurements) can be split into 3 parts:

$$y = (y_1 \ y_2 \ y_3)^t \quad \text{with: } y_1 = \xi_1 \in \mathbf{R}^{n-p}, \\ y_2 = Q(\xi) \in \mathbf{R}^n, \quad y_3 = h(\xi_1, \xi_2) \in \mathbf{R}^m$$

In other words we suppose that we can measure the gaseous flow rates  $Q(\xi)$  and also a set of functions of the state  $h(\xi_1, \xi_2)$ . They can be physical variables (pH, partial pressure of a gas, etc.) related to the state  $\xi$ . As it will be illustrated in the following, in the general case there does not exist any injective relationship between  $y$  and  $\xi$ . Indeed the vector  $Q(\xi)$  is, in many realistic cases, mainly composed of zeros, and the non-zero terms can be complicated and non-injective expressions of the whole state. We have therefore to estimate the variables with an observer.

**Example.** Along the paper we will illustrate our method with a very simple biotechnological example. We assume that a biomass of bacteria ( $X$ ) is growing in a CSTR. The micro-organisms consume the substrate  $S$  and produce a product  $P$ . As most of the time in biotechnology, the bacterial growth rate  $r(\xi)$  is a complex function of the state which is generally poorly known. In the sequel we assume that  $r(\xi)$  is unknown.

The model associated with this example is thus the following:

$$\begin{cases} \dot{X} = r(\xi) - DX \\ \dot{S} = -c_1 r(\xi) + D(S_{in} - S) \\ \dot{P} = c_2 r(\xi) - DP \end{cases} \quad (2)$$

where  $S_{in}$  is the influent substrate, and  $c_1$  and  $c_2$  are yield coefficients. This model has the form presented in (1) with:

$$K = (1, -c_1, c_2)^t, \quad \xi_{in} = (0, S_{in}, 0)^t, \quad Q(\xi) = (0, 0, 0)^t$$

We assume that the bacterial biomass and the conductivity of the solution can be measured. The conductivity is related to a positive linear combination of the ions in the solution i.e.  $S$  and  $P$ . We have therefore:

$$y_1 = X, \quad y_2 = (0, 0, 0)^t, \quad y_3 = \alpha S + \beta P$$

Our aim is to estimate  $S$  and  $P$ , without using the unknown kinetics  $r(\xi)$ .

**Remark.** In the sequel we will assume that  $\xi$  remains in a bounded domain  $\Omega_0 \subset \Omega$ , as it is generally the case for mass balance based systems [5].

## 2.1 Recall: the asymptotic observers

For details the reader may consult [1]. We separate the measured state variables  $\xi_1$  and the unmeasured state variables  $\xi_2$ . We rewrite system (1) as follows;

$$\dot{\xi}_1 = K_1 r(\xi) - D\xi_1 + D\xi_{in1} - Q_1(\xi) \quad (3)$$

$$\dot{\xi}_2 = K_2 r(\xi) - D\xi_2 + D\xi_{in2} - Q_2(\xi) \quad (4)$$

where  $(K_1, K_2), (\xi_{in1}, \xi_{in2})$  and  $(Q_1(\xi), Q_2(\xi))$  are the induced partition of  $K$ ,  $\xi_{in}$  and  $Q(\xi)$  respectively.

We state now an hypothesis on the rank of the matrix  $K_1$ :

**Hypothesis 1 (H1)** *The  $(n-p) \times k$  ( $(n-p) \geq k$ ) matrix  $K_1$  has full rank.*

From (H1), since it has full rank,  $K_1$  admits a left inverse  $P$ , let us denote  $A \stackrel{\text{def}}{=} -K_2 P$ .

We consider then the following linear change of coordinates:

$$\zeta_1 = \xi_1 \quad (5)$$

$$\zeta_2 = A\xi_1 + \xi_2 \quad (6)$$

this change of variables transforms (3) and (4) into:

$$\dot{\zeta}_1 = K_1 r(T\zeta) - D\zeta_1 + D\zeta_{in1} - Q_1(T\zeta) \quad (7)$$

$$\dot{\zeta}_2 = -D(\zeta_2 - \zeta_{in2}) - M y_2 \quad (8)$$

with

$$T \stackrel{\text{def}}{=} \begin{pmatrix} I_{n-p} & 0_{n-p,p} \\ -A & I_p \end{pmatrix}, \quad (9)$$

$$M \stackrel{\text{def}}{=} \begin{pmatrix} A & I_p \end{pmatrix} \quad \text{and} \quad \zeta_{in2} = M \xi_{in}$$

**Hypothesis 2 (H2)** *The positive scalar input  $D$  is persistently exciting i.e. there exists positive constants  $\alpha$  and  $\epsilon$  such that  $0 < \alpha \leq \int_t^{t+\epsilon} D(\tau) d\tau$*

**Lemma 1** ( see [1]) Under hypothesis (H2), the solution  $\hat{\xi}_2$  of the following open-loop observer:

$$\begin{aligned}\dot{\hat{\xi}}_2 &= -D(\hat{\xi}_2 - \zeta_{in2}) - My_2 \\ \hat{\xi}_2 &= \hat{\zeta}_2 - Ay_1\end{aligned}\quad (10)$$

converges asymptotically toward the solution  $\xi_2$  of the reduced system (4).

**Proof:** It is easy to verify that the estimation error  $e_2 \stackrel{\text{def}}{=} \hat{\xi}_2 - \xi_2 = \hat{\zeta}_2 - \zeta_2$  satisfies  $\dot{e}_2 = -De_2$ , and converges asymptotically toward  $\xi_2$ .

### 3 Closed loop mass balance observers

#### 3.1 Principle

In this section we assume that  $y_3 \in \mathbf{R}$ : the mapping  $h$  is thus defined as follows:

$$h : (\xi_1, \xi_2) \in (\mathbf{R}^{n-p} \times \mathbf{R}^p) \longrightarrow y_3 = h(\xi_1, \xi_2) \in \mathbf{R}$$

We assume that  $h$  satisfies the following hypothesis:

**Hypothesis 3 (H3)** The mapping  $h$  is monotonous with respect to  $\xi_2$ , i.e.  $\frac{Dh}{D\xi_2}$  is of fixed signs on  $\Omega$ .

**Example:** in our example  $h(\xi_1, \xi_2) = \alpha S + \beta P$ , and thus  $\frac{Dh}{D\xi_2} = [\alpha \ \beta]$ , this is of fixed signs. Let us precise that  $h$  may be nonlinear.

**Remark.** In fact, the mapping  $h$  must be saturated outside the domain  $\Omega_0$  so that the estimate  $\hat{\xi}$  remains bounded. This is not detailed here for sake of brevity.

**Proposition 1** Let  $\lambda \in \mathbf{R}^p$  be a constant unit vector ( $\|\lambda\| = 1$ ), whose signs are chosen such that  $\frac{Dh}{D\xi_2}\lambda \geq 0$ ,  $\theta$  is a (possibly time varying) positive scalar, and  $z_{in} = M\xi_{in}$ . The following system

$$\dot{\hat{z}} = -D(\hat{z} - z_{in}) - My_2 - \theta\lambda(h(y_1, \hat{z} - Ay_1) - y_3) \quad (11)$$

is an asymptotic observer of the reduced system (8).

**Proof:** the equation error is (with  $e \stackrel{\text{def}}{=} \hat{z} - z$ ):

$$\dot{e} = -De - \theta\lambda(h(y_1, \hat{z} - Ay_1) - h(y_1, z - Ay_1)) \quad (12)$$

We consider  $l_1, \dots, l_{p-1} \in \mathbf{R}^p$  an orthonormal basis of the vectorial subspace orthogonal to  $\lambda$ . If we denote  $L \stackrel{\text{def}}{=} [l_1 \dots l_{p-1}]^t$  let us consider the following LaSalle function

$$V_1(e) \stackrel{\text{def}}{=} \frac{1}{2}e^t L^t L e \quad (13)$$

then:

$$\dot{V}_1 = -De^t L^t L e = -DV_1 \quad (14)$$

The function  $V_1$  is nonnegative, it converges toward zero thanks to Hypothesis (H2). Therefore  $e$  goes towards the set  $\{e, V_1(e) = 0\}$  i.e. the kernel of  $L$ . This is precisely the vectorial subspace generated by  $\lambda$ . Note that the convergence rate toward  $\lambda$  is  $D$ .

Now, since  $e$  is bounded, we can study the system on the limit set, i.e. for  $e(t) = \alpha(t)\lambda$ . Let us rewrite the observer (12) on this limit set:

$$\lambda\dot{\alpha} = -D\alpha\lambda - \theta\lambda(h(y_1, \hat{z} - Ay_1) - h(y_1, z - Ay_1)) \quad (15)$$

this is  $\lambda$  times the same scalar expression:

$$\dot{\alpha} = -D\alpha - \theta(h(y_1, \hat{z} - Ay_1) - h(y_1, z - Ay_1)) \quad (16)$$

We rewrite the term  $h(y_1, \hat{z} - Ay_1) - h(y_1, z - Ay_1)$  using a generalized Taylor formula:

$$h(y_1, \hat{z} - Ay_1) - h(y_1, z - Ay_1) = \phi(\hat{z}, z, y_1)e$$

with  $\phi(\hat{z}, z, y_1) = \int_0^1 \frac{Dh}{D\xi_2}(y_1, \tau\hat{z} + (1-\tau)z - Ay_1)d\tau$  then (16) becomes:

$$\dot{\alpha} = -(D + \theta\phi(\hat{z}, z, y_1)\lambda)\alpha \quad (17)$$

The choice of the signs of  $\lambda$  guarantees  $\phi(\hat{z}, z, y_1)\lambda \geq 0$ . Now let us consider:

$$V_2(\alpha) \stackrel{\text{def}}{=} \alpha^2 \quad (18)$$

it verifies:

$$\dot{V}_2 = -2(D + \theta\phi(\hat{z}, z, y_1)\lambda)V_2 \leq -2DV_2 \quad (19)$$

The Lyapunov theorem for non autonomous systems (see e.g. [6]) proves that  $V_2$  and therefore  $\alpha$  goes toward zero (we also need Hypothesis (H2)).

**Proposition 2** Suppose moreover that  $\phi\lambda$  is lower bounded by a positive real  $\eta_{min}$ :

$$0 < \eta_{min} \leq \phi(\hat{z}, z, y_1)\lambda \quad (20)$$

then the convergence rate of  $\alpha$  is greater than  $(D + \theta\eta_{min})$ . It can be adjusted using  $\theta$ .

### 3.2 Extension to larger dimensions

In the following paragraph we extend the results obtained here in the case where we use  $m$  ( $m < p$ ) on-line measurements ( $h = (h_1, \dots, h_m)^t$ ) to set the observer in closed loop. We assume nevertheless that the measurement given by  $h_1$  is a nonlinear function of the state, and that for  $i > 1$  we have  $\frac{Dh_i}{D\xi_2} = \gamma(t)k_i$  (with  $\gamma(t) \in \mathbf{R}_+^*$  and  $k_i \in \mathbf{R}^p$ ). We assume moreover that the  $k_i$  are independent (otherwise it corresponds to a problem of  $m - 1$  correcting terms). In other words,  $K_O \stackrel{\text{def}}{=} [k_2^t \dots k_m^t]$  is of full rank.

Now we consider the following observer for the reduced system (8):

$$\begin{aligned} \dot{\hat{z}} &= -D(\hat{z} - z_{in}) - M y_2 - \gamma(t)\Lambda K_O^t(\hat{z} - z) \\ &\quad - \lambda_1(h_1(y_1, \hat{z} - Ay_1) - h_1(y_1, z - Ay_1)) \end{aligned} \quad (21)$$

where  $\lambda_i \in \mathbf{R}^p$ ,  $\lambda_1$  is orthogonal to the  $k_i$ 's ( $i = 2, \dots, m$ ) and  $\Lambda \stackrel{\text{def}}{=} [\lambda_2 \dots \lambda_m]$ ;

First we consider  $l_1, \dots, l_{p-m}$  an orthonormal basis of the vectorial subspace orthogonal to  $\lambda \stackrel{\text{def}}{=} [\lambda_1 \dots \lambda_m]$ . We denote  $L = [l_1, \dots, l_{p-m}]$ .

The LaSalle function:

$$V_3 \stackrel{\text{def}}{=} \frac{1}{2} e^t L L^t e \quad (22)$$

satisfies

$$\dot{V}_3 = -D e^t L L^t e = -D V_3 \quad (23)$$

this shows that  $e$  goes toward the manifold generated by  $\lambda_1, \dots, \lambda_m$ :

$$e(t) = \lambda \epsilon(t)$$

where  $\epsilon(t) \in \mathbf{R}^m$ . Then the problem is reduced to a  $m$  dimensional problem.

We need the following technical assumption.

**Hypothesis 4 ( $H_4$ )** *There exists  $\lambda_1$  in the kernel of  $K_O^t$  such that  $\frac{Dh_1}{D\xi_2} \lambda_1 \geq 0$ .*

**Proposition 3** *Let us take  $\lambda_1$  satisfying hypothesis ( $H_4$ ), the other  $\lambda_i$ 's ( $i > 1$ ) are chosen such that  $\frac{1}{\theta} \Lambda$  is a right inverse of  $K_O^t$  ( $\theta$  is a gain parameter). Then (21) is an asymptotic observer of (8).*

**Proof:** We define

$$V_4(e) \stackrel{\text{def}}{=} e^t K_O K_O^t e \quad (24)$$

It is easy to compute  $\dot{V}_4$ :

$$\dot{V}_4 = -2D V_4 - 2\gamma(t) e^t K_O K_O^t \Lambda K_O^t e \quad (25)$$

Since  $K_O^t$  is of full rank, it admits a right inverse: we can choose  $\Lambda$  such that  $K_O^t \Lambda = \theta I_m$ . Then equation (25) becomes:

$$\dot{V}_4 = -2(D + \theta\gamma(t))V_4 \quad (26)$$

This proves that asymptotically the error vector  $e$  belongs to the kernel of  $K_O$ . By the same argues than that given in paragraph 3.1 the limit set is contained in the vectorial subspace generated by  $\lambda_1$ :  $e(t) = \lambda_1 \alpha(t)$ .

It follows that the equation error asymptotically reduces to

$$\dot{\alpha} \lambda_1 = -(D I_p + \lambda_1(h_1(y_1, \hat{z} - Ay_1) - h_1(y_1, z - Ay_1))) \alpha \lambda_1$$

and thus, as for equation (15) we can prove that  $\alpha$  goes toward zero. It is also possible to adjust the gain  $\theta$  to increase the convergence rate of  $\alpha$ .

### 4 Illustration of the robustness properties on a simple example

We design the closed loop mass balance observer for the biotechnological process presented in (2):

$$\begin{cases} \dot{\hat{z}}_1 = D(S_{in} - \hat{z}_1) - \theta \lambda_a (\alpha \hat{S} + \beta \hat{P} - y_3) \\ \dot{\hat{z}}_2 = -D \hat{z}_2 - \theta \lambda_b (\alpha \hat{S} + \beta \hat{P} - y_3) \\ \hat{S} = \hat{z}_1 - \frac{y_1}{c_1} \\ \hat{P} = \hat{z}_2 + \frac{y_1}{c_2} \end{cases} \quad (27)$$

The convergence of this observer when  $S_{in}$  is known is a consequence of Lemma 1 if we choose  $\lambda_a > 0$  (same sign as  $\alpha$ ) and  $\lambda_b > 0$  (same sign as  $\beta$ ).

Actually, the influent substrate  $S_{in}$  is badly known, and we must use an estimate  $\hat{S}_{in}$ . We must replace the first equation of (27) with:

$$\dot{\hat{z}}_1 = D(\hat{S}_{in} - \hat{z}_1) - \theta(\alpha \hat{S} + \beta \hat{P} - y_3)$$

We choose here for sake of clarity  $\lambda = (1 \ 0)^t$ .

Now let us show the robustness properties when the estimate of  $S_{in}$  is wrong. If  $S^*$  and  $P^*$  denote the value of  $S$  and  $P$  at equilibrium, we call  $\hat{S}^*$  and  $\hat{P}^*$  the steady state value for the close loop observer. If the observer is designed in open loop ( $\theta = 0$ ) using  $\hat{S}_{in}$ , a straightforward calculation gives us:

$$\hat{S}^* = S^* + \hat{S}_{in} - S_{in} \quad (28)$$

The prediction error is therefore the error on  $S_{in}$ . With the closed loop observer, the steady state is:

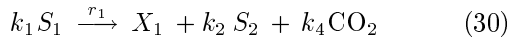
$$\hat{S}^* = S^* + \frac{D}{\theta\alpha + D}(\hat{S}_{in} - S_{in}) \quad (29)$$

If the gain  $\theta$  is high, then it is easy to see that  $\hat{S}^* \simeq S^*$ : the bias is reduced by the closed loop observer.

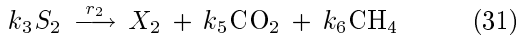
## 5 Application to an anaerobic wastewater treatment process

We consider an anaerobic digestion process used to treat wastewater. It consists mainly in two bacterial populations in a fermenter:  $X_1$  the acidogenic bacteria and  $X_2$  the methanogenic bacteria. These populations intervene in the two following biological reactions:

*Acidogenesis* (with reaction rate  $r_1$ ):



*Methanogenesis* (with reaction rate  $r_2$ ):



Where  $S_1$  and  $S_2$  represent the organic substrate, and the volatile fatty acids (VFA). If we take into account some chemical equilibria, we must also include the total alkalinity ( $Z$ ) and the total inorganic carbon ( $C$ ) in the model. For more details on the model see [7].

The model of this process has the general form given by eq. (1), with

$$\xi^t = (X_1, X_2, Z, S_1, S_2, C), \quad r(\xi)^t = (r_1(\xi), r_2(\xi))$$

$$K = \begin{pmatrix} 1 & 0 & 0 & -k_1 & k_2 & k_4 \\ 0 & 1 & 0 & 0 & -k_3 & k_5 \end{pmatrix}^t$$

$$F^t = (0, 0, Z_{in}, S_{1in}, S_{2in}, C_{in}), \quad Q^t = (0, 0, 0, 0, 0, Q_{CO_2})$$

The terms  $Z_{in}, S_{1in}, S_{2in}, C_{in}$  are the influent concentrations of  $Z, S_1, S_2, C$ , respectively. The term  $Q_{CO_2}$  is the molar flow rate of  $CO_2$  per volume of medium. In this example we are interested by the estimation of  $S_1$  and  $S_2$ . The following outputs are available :

$$y_1 = 0, \quad y_2 = (Q_{CO_2}, Q_{CH_4})^t,$$

$$y_3 = (Q_{CO_2}, Q_{CH_4}, \text{pH}, P_c)^t$$

where  $P_c$  is the  $CO_2$  partial pressure. In a previous paper [8] we have shown how to design a mass balance based observer for this process: the equations are of the general form (8), with:

$$M = \begin{pmatrix} 0 & 0 & 0 & -1 \\ a_0 & 0 & -a_1 a_3 & a_2 a_3 + a_4 \end{pmatrix}^t$$

Note that the transformation to obtain this form slightly differs from this presented in equation (9).

The predictions of the observer are then given by the following estimates:

$$\hat{Z} = \hat{z}_2, \quad \hat{S}_1 = \hat{z}_3 - a_1 \frac{\hat{z}_2 - f(\text{pH})\hat{z}_4}{1 + a_2 f(\text{pH})}$$

$$\hat{S}_2 = \frac{\hat{z}_2 - f(\text{pH})\hat{z}_4}{1 + a_2 f(\text{pH})}, \quad \hat{C} = \frac{a_2 \hat{z}_2 + \hat{z}_4}{1 + a_2 f(\text{pH})}, \quad \hat{X}_2 = \hat{z}_1$$

with  $f(\text{pH}) = \frac{1}{1 + 10^{pK - \text{pH}}}$ . Moreover, the relationships between the state and the outputs are (we consider a region where  $S_2$  does not inhibit  $r_2$ ):

$$Q_{CH_4} = h_1(z) = a_7 \frac{z_1(z_2 - f(\text{pH})z_4)}{z_2 + a_8 + f(\text{pH})(a_2 a_8 - z_4)}$$

$$Q_{CO_2}(z) = h_2(z) = a_5 \left( \frac{1 - f(\text{pH})}{1 + a_2 f(\text{pH})} (a_2 z_2 + z_4) - a_6 P_c \right)$$

Now we will present the closed loop observer for this system, and show how it improves the estimations of the variables  $S_1$  and  $S_2$ . With the notations of 3.2 we have:

$$\gamma(t) = a_5 \frac{1 - f(\text{pH})}{1 + a_2 f(\text{pH})} > 0 \text{ and } K_O = k_2^t = (0 \ a_2 \ 0 \ 1)^t$$

Moreover  $\text{sign}\left(\frac{Dh_1}{Dz}\right) = (+ \ + \ 0 \ -)$ . The vector  $\lambda_2 = (\star \ s \ \star \ (\theta - a_2 s))^t$  satisfies  $K_O^t \lambda_2 = \theta$ . We take  $\lambda_1 = (0 \ 1 \ \star \ -a_2)^t$  which satisfies  $K_O^t \lambda_1 = 0$  and  $\frac{Dh_1}{Dz} \lambda_1 \geq 0$  ((H1) is verified). From Proposition 3, the closed loop observer (21) converges.

The observer has been applied on a fermenter located

at the LBE (INRA Narbonne, France). The values of the model parameters are indicated in [8]. For the observer tuning we have chosen the following values:  $\lambda_1 = (0 \ 1.5 \ 0 \ -0.65)^t$ ,  $\lambda_2 = (0 \ 0.1 \ -0.012 \ 0.057)^t$  and  $\theta = 0.1$ .

Globally, the performances of the closed loop observer are better than the classical asymptotic observer (see Figure 1). The estimation results for  $S_2$  show bad performances for the open loop asymptotic observer (negative concentration). However, we notice that these estimations are considerably improved by the closed loop asymptotic observer which is more robust to parameter uncertainties. The estimation of the COD (the chemical oxygen demand resulting from  $S_1$  and  $S_2$ ) may be affected by the high variability of the input  $S_{1in}$  and by the changes in the composition of  $S_1$  along the experiment.

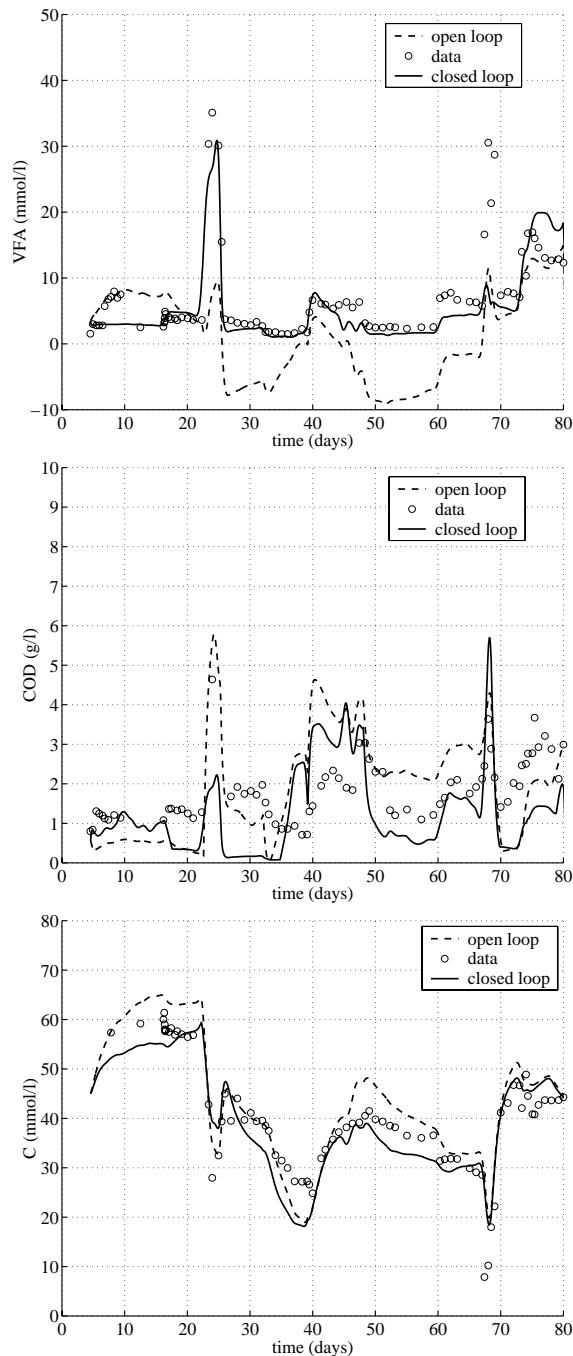
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**Figure 1:** Comparison between the open loop and the closed loop observer ( experimental data)