Stable Trajectory Tracking for Biped Robots

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Abstract The control of an underactuated biped robots in single support phase can be done choosing a certain number of output functions which are dependent on the state variables, *i.e.* on angular positions and velocities; the choice of these output functions is that better if they yield the definition of a minimal phase system. In the paper some examples are discussed. It is suggested that the underlying mathematical problem states and solves the problem in a concise manner. This goes through the application of the Pfaff-Darboux Theorem and differential geometric tools.

1 Introduction

In spite of more than 30 years research in biped walking [18], the control of such a robot remains still an open problem. The control of these systems rises a large set of new problems and questions. They require a careful analysis especially as stability is concerned. The problem of stability is of major importance for biped robots. Walking robots can have different structures and different problems of control occur depending of the contact between the feet and the ground. For a biped robot with large feet, the Control of ZMP (Zero Moment Point), initiated in [18] (see also [7]), results in efficient applications [10]. However as it is in human walk, the complete sole does not lie always on the ground, some rotation around the edge of feet occurs. To reduce the complexity of the robot in practical applications, it is also important to use a small number of actuators. In these cases, the contact with the ground is reduced to a single point. The robot becomes very simple as the robot which is considered herein, but the control strategy has to be adapted to this case. Most studies on pointwise ground contact [14, 6, 15, 8], correspond to ballistic motions of the robot. These approaches are interesting but are often limited to motions on a slope. In these contributions, study of stability often uses a Poincaré mapping for periodic gait. This tool is very useful as the hybrid nature of walking is taken into account [8, 9]. In this case the stability of walking is achieved via the complete cycle of walk (support phase and impact...) and stability has to be checked for each reference trajectory. The control developed in this paper is quite different because only single support phase is studied and a suitable choice of outputs is used to stabilize motions of the biped robot. A good tracking will be achieved for any sufficiently smooth reference trajectories. In single support phase, a planar walking robot with pointwise ground contact is composed of n links and $n-1$ motors.

For this class of robots it is not difficult to find a dynamical model, so we can always start the study from the internal representation, since Newton, Euler or Lagrange methods allow to write the equations which relate the angular accelerations of joints to the applied torques; then it is simple to obtain a nonlinear state space model, taking the angular positions and velocities as state variables. Starting from this model, different choices for the outputs are made: this choice is very important since it determines the applicability of some control strategy, whether the system is minimum phase or not. This is the case when output trajectory tracking is considered.

The purpose of the study is to obtain a minimum phase system; in particular it will be possible to track stable trajectories with internal stability by a suitable choice of outputs.

When we want to control a nonlinear system, we can steer it directly or we can design at first a feedback to linearize the I/O behavior of the model; the full linearization of the state cannot be processed on any system, and in general I/O linearization only yields partial linearization of the state dynamics. Loss of observability displays a nonlinear subsystem which may be unstable. This part of the dynamics characterises the zero dynamics [13]. In [1] the reader can find a method to design a minimum phase linearized system by an appropriate choice of the output, and a robotics example as well. The definition of the output proposed in the paper will be illustrated on two underactuated system: a two links inverted pendulum with a planar motion, and a five link planar biped. The case of two links is certainly elementary, but it features a robot with a very simple mechanical structure: it has a low number of degrees of freedom and the question of its stabilizations becomes sharper. It is shown that the underlying mathematical problem involved in the definition of the outputs is an application of the Pfaff-Darboux Theorem which is recalled in Section 2

Section 3 is devoted to the two link case – the inverted pendulum. The five link walking robot is considered in Section 4.

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2 Mathematical preliminaries

The problems arising when choosing the outputs for biped robots lead to a mathematical question which can be exposed in a completely abstract way; the question concerns the search of the minimal number of coordinates needed to express some one-form associated to the kinetic momentum of the robot (see Sections 3.1 and 4 below). This question is solved by the Pfaff-Darboux Theorem [2, 3, 4] which characterizes the number of coordinates needed, defining the rank of an one-form, as well.

Definition 1 *The rank r of the one-form ω is defined by*

$$(d\omega)^r \wedge \omega \neq 0, \text{ and } (d\omega)^{r+1} \wedge \omega = 0$$

Theorem 1 *(Pfaff-Darboux) If the one-form ω has constant rank r, then there locally exists a set of analytic functions $\alpha_1,...,\alpha_{r+1}$, $\lambda_1,...,\lambda_r$ so that*

$$\omega = 0$$

is equivalent to

$$d\alpha_1 + \lambda_1 d\alpha_2 + ... + \lambda_r d\alpha_{r+1} = 0$$

Equivalently, ω reads as:

$$\omega = \mu_1 d\alpha_1 + ... + \mu_{r+1} d\alpha_{r+1}$$

for some suitable functions $\mu_j, j = 1, ...r + 1$.

The celebrated Frobenius Theorem corresponds to the special case where r=0, when a single 1-form is considered. Theorem 1 is instrumental for the resolution of the problem arising when dealing with biped robots.

Elementary considerations yield that, whenever a 1-form ω can be written in n coordinates

$$\omega = \lambda_1(\alpha_1, ..., \alpha_n)d\alpha_1 + ... + \lambda_n(\alpha_1, ..., \alpha_n)d\alpha_n,$$

then $(d\omega)^r \wedge \omega$ is a $(2r + 1)$-form which is nonzero only if $2r + 1 \leq n$, *i.e.* the rank r is necessarily less than or equal to $(n - 1)/2$. In our application, it will appear in the following sections, that we are interested in rewriting ω as

$$\omega = \mu_1 dp_1 + ... + \mu_{n-1} dp_{n-1},$$

and this is always possible for the cases of interest $n \geq 2$. Practical efficiency of the computation of the new coordinates is limited only by integration problems, although some algorithms exist [12].

Now, recall some definitions from nonlinear system theory [5]. Consider a control system

$$\dot{x} = f(x) + g(x)u$$

and a 1-form $\omega \in \text{span}\{\text{dx}\}$. Its relative degree ρ is defined as

$$\rho(\omega) := \min\{k \in I\!N \mid \omega^{(k)} \notin \text{span}\{\text{dx}\}\}.$$

Define a decreasing sequence $\mathcal{H}_k$ of subspaces of span{dx} by

$$\mathcal{H}_k := \{\omega \in \text{span}\{\text{dx}\} \mid \rho(\omega) \geq \text{k}\}.$$

3 Application to a 2-link walking robot

3.1 Modelling

The first example of application of the results obtained from the theoretic point of view concerns the simplest model of walking robot. We can imagine a walking robot as a pair of legs joined together by rotational joint which is actuated. The scheme of the robot looks like a double inverted pendulum, where one of the ends of one link (one foot of the robot) is always in contact with the ground. It is obvious that the model is too simple to describe correctly all the equilibrium and control problems that can be found when dealing with walking robots, but it is anyway a first approximation, useful for further and more complex developments. The scheme of the robot is shown in Figure 1.

Two angles, α and β, describe the position of the mechanical system, while a certain number of parameters can be assigned to define the dimension of the links and their masses; to reduce the number of parameters and allow an easier study of the system, the following coefficients are introduced:

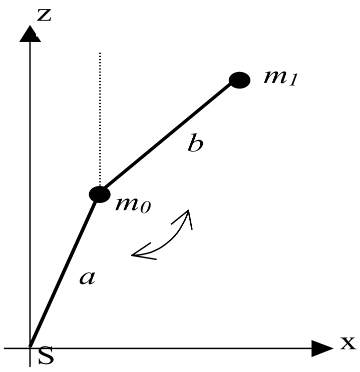

Figure 1: Scheme of two links biped robot

$$e = \frac{b}{a};\ \mu = \frac{m_1}{m_0 + m_1};\ \sigma = \frac{1}{(m_0 + m_1)a^2};\ \tilde{g} = \frac{g}{a}$$

The system is described by equations derived by Lagrange method. The system is described by an inertial matrix, a term which includes the effects of Coriolis and centrifugal forces, and finally the contribute due to the applied torque. The matrix form of the equations is the following:

$$\begin{bmatrix} 1 & \mu e\cos(\alpha-\beta) \\ \mu e\cos(\alpha-\beta) & \mu e^2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix}$$

$$+ \begin{bmatrix} \dot{\beta}^2 \\ -\dot{\alpha}^2 \end{bmatrix} \mu e\sin(\alpha-\beta) - \tilde{g} \begin{bmatrix} \sin(\alpha) \\ \mu e\sin(\beta) \end{bmatrix} = \begin{bmatrix} -\sigma \\ \sigma \end{bmatrix} \tau$$

This matrix equation can be put into first order differential equations system, obtaining a four state space variable system: these state variables are the angular positions and velocities. The input is given by the structure of the system, while we can choose different outputs to steer the system in different ways. The choice of the output function is led by many considerations: we must prefer an output function with a quite high relative degree [5] to obtain a system with a less rich zero dynamics. $\mathcal{H}_k$ spaces [5] can be computed using computer algebra software, but these computations involve large expressions that are not always easy to be supported by computer systems ; so we can find one of the functions which can be chosen to form the basis of these spaces by using physical considerations. From mechanical considerations, it is known that the time derivative of the kinetic momentum M with respect to the ground contact point, equals the torque resulting from the external force. The derivative $\dot{M}$ of the kinetic momentum depends only on the gravity effects. Then it depends only on the system configuration (the angles). Two further time derivatives yield an explicit dependence on the torque (the input). Thus, dM belongs to $\mathcal{H}_3$, or its relative degree is 3:

$$M = (1+\mu e\cos(\alpha-\beta))\dot{\alpha} + (\mu e^2 + \mu e\cos(\alpha-\beta))\dot{\beta} =$$

$$= J_\alpha(\alpha,\beta)\dot{\alpha} + J_\beta(\alpha,\beta)\dot{\beta} \qquad (1)$$

Now we can use the theoretic results coming from Theorem 1 to see if the function M can be expressed in a different way, using a lower number of variables.

3.2 Mathematical problem in two variables

The kinetic momentum M depends on two variables α and β and their first derivatives :

$$M = J_\alpha(\alpha,\beta)\dot{\alpha} + J_\beta(\alpha,\beta)\dot{\beta}$$

The question concerns the possibility of writing the function M as a multiple of the time derivative of a certain function $p(\alpha,\beta)$:

$$M = J_p(\alpha_1,...,\alpha_n)\dot{p}(\alpha_1,...,\alpha_n) \qquad (2)$$

For this case, with only two variables, the solution always exists, even if it is not always possible to write it in explicit way, as it happens when integration problems are involved.
The following theorem can be proved:

Theorem 2 *(Frobenius) Given*

$$M = J_1(\alpha_1,...,\alpha_n)\dot{\alpha_1} + ... + J_n(\alpha_1,...,\alpha_n)\dot{\alpha_n},$$

define

$$\omega = M dt.$$

which yelds

$$\omega = J_1 d\alpha_1 + ... + J_n d\alpha_n.$$

The function M can be written as (2) if and only if:

$$d\omega \wedge \omega = 0 \qquad (3)$$

Proof:
If we assume that $d\omega \wedge \omega = 0$ we implicitly assume that the one-form ω is integrable, according to the definition of integrability and Frobenius theorem [5]. It implies that there exists locally a pair of meromorphic functions J_p and p that verify the equation

$$\omega = J_p(\alpha_1,...,\alpha_n)dp(\alpha_1,...,\alpha_n)$$

that is to say the one form ω is colinear to the exact form dp.
Finally, from the definition of the one form ω the function M becomes

$$M = \frac{\omega}{dt} = \frac{J_p(\alpha_1,...,\alpha_n)dp(\alpha_1,...,\alpha_n)}{dt} =$$

$$= J_p(\alpha_1,...,\alpha_n)\dot{p}(\alpha_1,...,\alpha_n).$$

Conversely, if we assume that we can write the function M as

$$M = J_p(\alpha_1,...,\alpha_n)\dot{p}(\alpha_1,...,\alpha_n),$$

then , from the definition of ω we can conclude that

$$\omega = M dt = J_p(\alpha_1,...,\alpha_n)dp(\alpha_1,...,\alpha_n),$$

and that means that ω is integrable.

Corollary 1 *If n=2 it is always possible to reduce the number of coordinates from 2 to 1.*

Proof:
Since in the condition (3) there is a three-form in two variables, it is always zero.

This theorem shows that the mathematical problem connected to the search of the outputs for a certain

system is nothing but an integration problem: the condition that the function M must verify are the integrability conditions for the associated one-form ω.
There is a further observation: the problem involves, till now, only two variable functions and one-forms, and, as it is shown in the last theorem, the problem can be converted into an integration problem. Since the condition (3) is always satisfied, the theorem leads to the conclusion that there always exists a solution (4). Theorem 2 is a special case of Theorem 1, when the rank of ω is required to be 0.

3.3 Choice of output function

We have proved that our problem has always a solution, so we can try to find the two functions p and J_p we need to express M as in (4); the result is obtained by integrating the one-form ω associated to M. We can write [17], [16]:

$$M = J_p(\alpha, \beta)\dot{p} \tag{4}$$

where

$$J_p(\alpha, \beta) = 1 + \mu e^2 + 2\mu e \cos(\alpha - \beta)$$
$$p = \frac{\alpha+\beta}{2} + \frac{(1-\mu e^2)}{\sqrt{(1+\mu e^2)^2 - 4\mu^2 e^2}} arctg\left(\sqrt{\frac{1+\mu e^2-2\mu e}{1+\mu e^2+2\mu e}} tg\left(\frac{\alpha-\beta}{2}\right)\right)$$

When writing M as a multiple of $\dot{p}$ we have found a function, p, so that $\dot{p}$ combines $\dot{\alpha}$ and $\dot{\beta}$.
From (4) and since J_p depends only on (α, β), it can be shown that p has a relative degree 3. J_p is the instantaneous moment of inertia with respect to the ground contact point, and is thus always positive. Both M and p have a relative degree 3 and no function can be found with a higher relative degree, since the $\mathcal{H}_4$ space is not integrable: any choice of output will yield a zero dynamics whose dimension is at least 1. If M is chosen as the output of the system, then output zeroing yields $M = 0$, $\dot{M} = 0$, $\ddot{M} = 0 \ldots$ Thus,

$$\dot{p} = 0.$$

This dynamics is only critically stable: the system is weakly minimum phase[11]. The same occurs when choosing p as output. A better choice is provided by $M + p$ as output for the system. The output will still have the same relative degree, 3, but when we force $M + p$ to be zero, the resulting dynamics is

$$-p = M = J_p(\alpha, \beta)\dot{p}$$

and since the coefficient J_p is positive, the zero dynamics is asymptotically stable. The solution of the problem for the 2-link case can be generalised and guide the search of the outputs for more complex examples: in the following section we will consider a five link biped robot.

4 Application to 5-link walking robots

The considered biped walks in a vertical saggital xy plane. It is composed of a trunk and two identical legs. Each leg is composed of two links articulated with a knee – see Figure 2. The knees and hips are one degree of freedom rotational joint. Introduce vector $X = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)^T$ of five generalised coordinates for describing the biped in the xy plane. All links are assumed to be massive and rigid. The inertia of the links is also taken into account. Let vector $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4)^T$ describe the torques applied at the hip and knee joints. The robot has five degrees of freedom in single support phase and there are four actuators. The dynamic model can be written as:

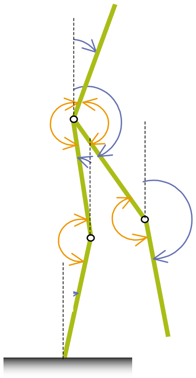

Figure 2: Scheme of five links biped robot

$$A_q(q)\ddot{q} + H_q(q, \dot{q}) = D_q\Gamma \tag{5}$$

where A_q(5 x 5) is the inertia matrix, H_q(5 x 1) is the vector of Coriolis, centrifugal and gravity forces and D_q is a (5 x 4) matrix. As it was done for the 2-link case, one can find a function with a high relative degree from mechanical facts: choose the kinetic momentum M around the ground contact point, M, and express it as

$$M = M_{\theta_1}\dot{\theta}_1 + M_{\theta_2}\dot{\theta}_2 + M_{\theta_3}\dot{\theta}_3 + M_{\theta_4}\dot{\theta}_4 + M_{\theta_5}\dot{\theta}_5 \tag{6}$$

where the five coefficients are the following:

$$\begin{array}{rcl}
M_{\theta_1} & = & a_{11} + a_{12}\cos(\theta_2 - \theta_1) + a_{13}\cos(\theta_3 - \theta_1) + \\
 & & a_{14}\cos(\theta_4 - \theta_1) + a_{15}\cos(\theta_5 - \theta_1) \\
M_{\theta_2} & = & a_{12}\cos(\theta_2 - \theta_1) + a_{22} cos + a_{23}\cos(\theta_3 - \theta_2) + \\
 & & +a_{24}\cos(\theta_4 - \theta_2) + a_{25}\cos(\theta_5 - \theta_2) \\
M_{\theta_3} & = & a_{13}\cos(\theta_3 - \theta_1) + a_{23}\cos(\theta_3 - \theta_2) + a_{33} \\
M_{\theta_4} & = & a_{14}\cos(\theta_4 - \theta_1) + a_{24}\cos(\theta_4 - \theta_2) + a_{44} + \\
 & & +a_{45}\cos(\theta_5 - \theta_4) \\
M_{\theta_5} & = & a_{15}\cos(\theta_5 - \theta_1) + a_{25}\cos(\theta_5 - \theta_2) + \\
 & & +a_{45}\cos(\theta_5 - \theta_4) + a_{55}
\end{array}$$

for some constant coefficients a_{ij}. To apply the methodology described for the two link case, we have to reduce the number of velocities involved in the kinetic momentum M, to $n-1$ which is the number of actuators. According to the theory shown in Section 2, the lowest number of variables that allow the description of the system is defined by the rank of the one-form associated to M:

$$\begin{aligned}\omega &= M_{\theta_1}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)d\theta_1+ \\ &+M_{\theta_2}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)d\theta_2+ \\ &+M_{\theta_3}(\theta_1,\theta_2,\theta_3)d\theta_3+M_{\theta_4}(\theta_1,\theta_2,\theta_4,\theta_5)d\theta_4+ \\ &+M_{\theta_5}(\theta_1,\theta_2,\theta_4,\theta_5)d\theta_5\end{aligned}$$

This 1-form is not integrable and its rank equals 2: three coordinates only are requested to express the 1-form. The problem of finding these coordinates is not simple: the theory guarantees that a solution exists, but its effective computation is a matter of integration.

In this case, as the angles θ_3 and θ_4 are the orientations of the links in two different branches of the robot (when considered as a tree with the ground foot point as the root). Then define p_1 as the result of the integration of $M_{\theta_3}(\theta_1,\theta_2,\theta_3)d\theta_3+M_{\theta_4}(\theta_1,\theta_2,\theta_4,\theta_5)d\theta_4$ where $\theta_1,\theta_2,\theta_5$ are assumed to be constant parameters. Now (6) is sought in a form involving 4 time derivatives only, as:

$$M = M_{p1}\dot{p_1}+M_{p2}\dot{p_2}+M_{p3}\dot{p_3}+M_{p4}\dot{p_4}.$$

One solution is provided by

$$\begin{aligned}M_{p1} &= 1 \\ M_{p2} &= a_{11}+a_{12}\cos(\theta_2-\theta_1)+2a_{13}\cos(\theta_3-\theta_1)+ \\ &\quad +2a_{14}\cos(\theta_4-\theta_1)+a_{15}\cos(\theta_5-\theta_1) \\ M_{p3} &= a_{12}\cos(\theta_2-\theta_1)+a_{22}\cos+2a_{23}\cos(\theta_3-\theta_2)+ \\ &\quad +2a_{24}\cos(\theta_4-\theta_2)+a_{25}\cos(\theta_5-\theta_2) \\ M_{p4} &= a_{15}\cos(\theta_5-\theta_1)+a_{25}\cos(\theta_5-\theta_2)+ \\ &\quad +2a_{45}\cos(\theta_5-\theta_4)+a_{55} \\ p_1 &= a_{13}sin(\theta_3-\theta_1)+a_{23}sin(\theta_3-\theta_2)+a_{33}\theta_3+ \\ &\quad +a_{14}sin(\theta_4-\theta_1)+a_{24}sin(\theta_4-\theta_2)+a_{44}\theta_4+ \\ &\quad +a_{45}sin(\theta_5-\theta_4) \\ p_2 &= \theta_1 \\ p_3 &= \theta_2 \\ p_4 &= \theta_5\end{aligned}$$

The function M belongs to $\mathcal{H}_3$ while p_1, p_2, p_3 and p_4 belong to $\mathcal{H}_2$. If we choose M, p_1, p_2, p_3 or M, p_2, p_3, p_4 ... as outputs for the system we would obtain one transmission zero only. For instance, when M, p_2, p_3, p_4 are chosen as outputs, then output zeroing yields

$$\left\{\begin{array}{c} M_{p1}\dot{p_1}+M_{p2}\dot{p_2}+M_{p3}\dot{p_3}+M_{p4}\dot{p_4}=0 \\ p_2=0 \\ p_3=0 \\ p_4=0 \end{array}\right.$$

so we have the relation

$$M_{p1}\dot{p_1}=0$$

This zero dynamics is representative of a weakly minimum phase system. Now, choose $M+p_1, p_2, p_3, p_4$ as output functions, then output zeroing yields

$$\begin{array}{c} M_{p1}\dot{p_1}+M_{p2}\dot{p_2}+M_{p3}\dot{p_3}+M_{p4}\dot{p_4}+p_1=0 \\ p_2=0 \\ p_3=0 \\ p_4=0 \end{array}$$

and it gives rise to the following zero dynamics:

$$M_{p1}\dot{p_1}+p_1=0$$

Since M_{p1} equals 1, the stability of state trajectories is guaranteed. The zero dynamics has dimension 1 since the structure at infinity of the system is $2,2,2,3$[5].

An output can be chosen in this way and simulations will display the behavior of the system. Stability of the state variables is guaranteed while the output is forced to zero. In our simulation we can define trajectories for the outputs which corresponds to certain trajectories of the state variables: for instance we are interested to steer the system to the equilibrium configuration where

$$\theta_1=\theta_2=\theta_3=0 \quad \theta_4=\theta_5=\pi$$

and the velocities are zero, which means that the robot stands up on one leg. The control law is designed based on trajectory tracking theory to bring the system to the requested reference configuration from an initial configuration close to it:

$$\begin{array}{lll} \tilde{\theta_1}=0.05; & \tilde{\theta_2}=0.03; & \tilde{\theta_3}=-0.05; \\ \tilde{\theta_4}=\pi-0.05; & \tilde{\theta_5}=\pi-0.03. & \end{array}$$

and zero velocities.

Once we know the trajectories of the state variable, we can compute the trajectories for the outputs we have just chosen, to determine the vector of desired output trajectories y_d. Standard trajectory tracking with feedback linearization of the error, all poles being placed to -3, yields the following relation

$$y^{(3)}={y_d}^{(3)}+9(\ddot{y_d}-\ddot{y})+27(\dot{y_d}-\dot{y})+27(y_d-y).$$

The control law produces the errors between the desired and real behavior of the positions as displayed in Figure 1.

5 Conclusion

The proposed strategy can be used for different underactuated planar biped robots which have n freedom degrees but a lower number of input signals $n-1$ to steer their behavior. The derivative of the kinetic momentum around the ground point contact is only function of the configuration variable and the kinetic momentum belongs to $\mathcal{H}_3$. Applying the Pfaff-Darboux theorem, the kinetic momentum can be written as :

$$M=\sum_{i=1}^{n-1} M_{p_i}\dot{p_i}$$

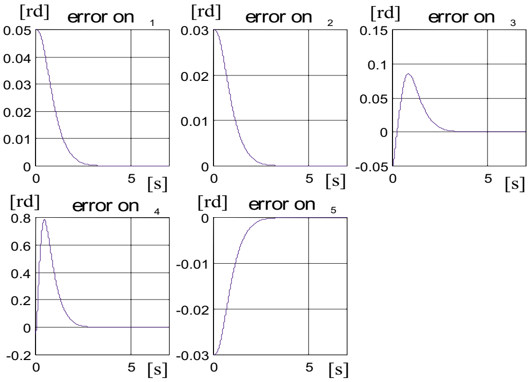

Figure 3: Position errors.

where M_{p_i} are function of the configuration of the robot.

The problem of finding the real functions M_{p_i} and p_i is not generally obvious. In the particular case, where the robot has a tree structure, for example when the robot has a body and two legs, as the variables in two branches are not mixed in the kinetic momentum, an explicit solution can be found as in the case of the 5 links robot. When the robot is a simple open structure in which the links are in a series structure, the effective computation of the function M_{p_i} and p_i remains an open issue for further research.

If there exist one function M_{p_i} which is positive and bounded in all the working space, then appropriate output function will be $M+p_i$ and p_j with $j = 1...n-1, j \neq i$. A non linear control law with dynamic feedback define to track this n-1 outputs will result in a stable behavior of all the robot.

The results in the paper give hints for the control of general underactuated systems, one has a first integral of the systems equations which does not depend on the control explicitly.

References

[1] D'Andrea B., L. Praly, "*About finite nonlinear zeros for decoupable systems*", Systems & Control Letters, vol.10, 1988, pp 103-109.

[2] Bryant R.L., S.S. Chern, R.B. Gardner,H.L. Goldshmidt, P.A. Griffiths, "*Exterior Differential Systems*", Springer Verlag 1991.

[3] E. Cartan, "*Les systèmes différentiels extérieurs et leurs applications géométriques*", Hermann, Paris 1971.

[4] E. Cartan, "*Les systèmes de Pfaff à cinq variables et les équations aux dérivées partielles du second ordre*" (Ann. Ec. Normale, t. 27, 1910, p. 109-192), Oeuvres Complètes, Editions du CNRS, 1984.

[5] Conte G., C.H. Moog, A.M. Perdon, "*Nonlinear Control Systems: an Algebraic setting*", Lecture Notes on Control and Information Science, Springer Verlag, vol.242, 1999.

[6] M.Garcia, A. Chatterjee, A. Ruina, "Speed, efficiency and Stability of Small-Slope 2D passive Dynamics Bipedal walking", IEEE International Conference on Robotics and Automation, Leuven, Belgium, 1998.

[7] A. Goswami," Foot Rotation Indicator (FRI) point: A New Gait Planning Tool to Evaluate Postural Stability of Biped Robots", IEEE International Conference on Robotics and Automation, Detroit, Michigan, 1999.

[8] A. Goswami, B. Espiau, A. Keramane, "Limit cycles and their stability in a passive bipedal gait", IEEE Int. Conference on Robotics and Automation, Leuven, Belgium, 1998.

[9] J.W. Grizzle, G. Abba, F. Plestan, "Proving asymptotic stability of a walking cycle for a five dof biped robot model", CLAWAR 99, 2nd International Conference on Climbing and Walking Robots, Portsmouth, Sept. 99.

[10] K. Hirai, M. Hirose, Y. Haikava, T. Takenaka,"The Development of Honda Humanoid Robot",IEEE Int. Conference on Robotics and Automation, Leuven, Belgium, 1998.

[11] Hauser J., S. Sastry, G. Meyer, "*Nonlinear Control Design for Slightly Non-minimum Phase Systems: Application to V/STOL Aircraft*", Automatica, 1992, vol.28, no. 4, pp.665-679.

[12] H. Hermes, *Involutive subdistributions and canonical forms for distributions and control systems*,Theory and Applications of Nonlinear Control Systems, C.I. Byrnes and A. Lindquist(editor), pp. 123-135., 1986.

[13] A. Isidori, "*Nonlinear Control Systems*", Springer Verlag, 1994.

[14] T. Mc Geer "Passive dynamic walking", *Int. Journal of Robotics Research*, 9-62-82, 1990.

[15] A.C. Smith, M.D. Berkermeier, "The motion of a finite-width rimless wheel in 3D", IEEE International Conference on Robotics and Automation, Leuven, Belgium, 1998.

[16] R. Stojic, C. Chevallereau, "On the stability of biped with point foot-ground contact", IEEE Int. Conference on Robotics and Automation, San Fransisco, 2000.

[17] R. Stojic, O. Timcenko, "On control of a class of feedback non linearizable mechanical system", WAC'98 World Automation Congress, Ancorage,Alaska,1998.

[18] M. Vukobratovic and D. Juricic,"Contribution to the synthesis of biped gait", *IEEE Trans on Biomedical Engineering*, BME-16:1-6,1969.