Stochastic Modeling Based DGPS Estimation Algorithm

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Abstract

A novel kinematic differential GPS algorithm is presented. Specifically, the accurate relative and absolute positioning of a team, or formation, of mobile vehicles is considered. The measurement situation on hand is correctly modeled, a stochastic framework is developed, and a novel centralized estimation algorithm is rigorously derived.

1 Introduction

GPS positioning accuracy is limited by measurement errors that can be classified as either common mode or non-common mode. Common mode errors have nearly identical effects on all receivers operating in a limited geographic area, as is the case in formation flight. The non-common mode errors are dominated by receiver noise (and multipath). Conventional DGPS uses a reference station at a known ECEF position to determine corrections that other local, and presumably mobile, GPS receivers, can use to reduce the effects of GPS common mode errors.

Conventional DGPS navigation exploits the known position of a reference station and the existence of a communications channel to the moving vehicle (the "rover"), to broadcast corrections to the GPS receiver on the rover, and thus improve the latter's positioning accuracy. In its most basic form the DGPS methodology entails the application of reference station broadcast differential corrections to the user (rover) measured pseudoranges. Thus, DGPS yields a stand alone and improved user (rover) position estimate [1]. The residual error in DGPS is exclusively caused by measurement noise. Hence, there is a strong incentive to develop methodologies for mitigating the deleterious effects of measurement noise in DGPS. Obviously, an approach which relies on the averaging out of the random measurement noise is called for. Now, in Kinematic GPS (KGPS), the use of a measurement record obtained over multiple observation epochs, the stipulation of a kinematic model for the user's motion, and the "centralized" processing of the GPS pseudoranges taking into account the underlying temporal dependence of the kinematic variables, allows the mitigation of the measurement noise - induced effects. Hence, improved user position and velocity estimates are obtained. This improvement in navigation performance is obtained irrespective of whether differential corrections, as provided by DGPS, are applied to the raw pseudorange measurements.

In this paper the concept of synergetically employing DGPS and KGPS navigation in a monolythic computational algorithm is undertaken. The developed algorithm is referred to as Kinematic Differential GPS (KDGPS). The pseudoranges measured by all the team's receivers are communicated to a central processor and are operated on by a centralized (optimal) estimation algorithm, where the common errors are properly accounted for - thus obtaining improved estimation performance. Thus, a *navigation web* concept is advanced. Also, the *navigation web* development is motivated by self - calibration measurement methods, e.g., in [2], the self calibration concept was exploited for GDOP reduction in a pseudolites - based measurements scenario. In this paper, its application to GPS positioning is vigorously pursued, and the required mathematical development is presented. The application to spacecraft formation flying is considered - see, eg., Refs. [3] - [4].

2 Theory

A sampling interval of ΔT seconds is used. The duration, T, of the measurement interval cosists of $N+1$ discrete measurement epochs. Thus, the time instants when measurements are taken are $t_j = \Delta T j \;,\quad j = 0, 1, ..., N$. For the short measurement duration being stipulated, the users' kinematics are modeled as constant speed and rectlinear motion, viz., $\mathbf{x_k(t)} = \mathbf{x_{0_k}} + \mathbf{V_k t}$, where $k = 1, 2, ..., m$ and m is the number of users; $\mathbf{x_k}, \mathbf{x_{0_k}}, \mathbf{V_k} \in \mathbf{R^3}$.

The pseudorange from the k'th user to the i'th satellite at time t is modeled as

$$\begin{aligned} \rho_i^{(k)}(t) &= \sqrt{[x_k(t) - x_{s_i}(t)]^2 + [y_k(t) - y_{s_i}(t)]^2 + [z_k(t) - z_{s_i}(t)]^2} \\ &+ \tau_k + \tau_{s_i} \end{aligned} \tag{1}$$

and the measurement equation is

$$Z_i^{(k)}(t) = \rho_i^{(k)}(t) + v_i^{(k)}(t) \tag{2}$$

where τ_k is the k'th user range-equivalent clock bias. $k = 1, 2, ..., m$; τ_{s_i} is the i'th satellite range-equivalent clock bias. $i = 1, 2, ..., n$; $(x_{s_i}, y_{s_i}, z_{s_i})$ is the i'th satellite position at time t from the ephemeris data. The k'th user position at time t is $(x_k(t), y_k(t), z_k(t))$. The number of satellites in view is n and $v_i^{(k)}(t)$ is the measurement noise in the i'th channel of user k'th receiver at time t.

The measurement noise at time j is modeled as

$$v_{i,j}^{(k)} = N(0, \sigma^2) \tag{3}$$

and

$$E(v_{i,j}^{(k)} v_{i',j'}^{(k')}) = 0 \;\; for \;\; i \neq i' \;, j \neq j' \;, k \neq k' \tag{4}$$

Incorporating the kinematic model for the users' positions for each time instant j, i.e., setting

$$x_{0_k} + (V_{k_x} \Delta T) j \;\; \leftarrow \;\; x_k(t) \tag{5}$$

$$x_{0_y} + (V_{k_y} \Delta T) j \;\; \leftarrow \;\; y_k(t) \tag{6}$$

$$z_{0_k} + (V_{k_z} \Delta T) j \;\; \leftarrow \;\; z_k(t) \tag{7}$$

where the subscript 0 denotes the initial position at time $t = 0$ ($j = 0$), allows eq. (1) to be written as

$$\begin{aligned} \rho_{i,j}^{(k)} &= \sqrt{[x_{0_k} + (V_{k_x} \Delta T) j - x_{s_i}(t)]^2 + [y_{0_k} + (V_{k_y} \Delta T) j - y_{s_i}(t)]^2 + [z_{0_k} + (V_{k_z} \Delta T) j - z_{s_i}(t)]^2} \\ &+ \tau_k + \tau_{s_i} \end{aligned} \tag{8}$$

Note that in eq. (8), $t = j\Delta T$, $j = 0, 1, ..., N$.

The parameter of interest is

$$\begin{aligned} \theta = \; & [x_{0_1}, y_{0_1}, z_{0_1}, V_{1_x}\Delta T, V_{1_y}\Delta T, V_{1_z}\Delta T, x_{0_2}, \\ & y_{0_2}, z_{0_2}, V_{2_x}\Delta T, V_{2_y}\Delta T, V_{2_z}\Delta T, \tau_1, \tau_2, \\ & \tau_{s_1}, \tau_{s_2}, \tau_{s_3}, \tau_{s_4}, \tau_{s_5}]^T \end{aligned} \tag{9}$$

where $(x_{0_k}, y_{0_k} z_{0_k})$ represents the k'th user initial position, $k = 1, 2, ..., m$; $(V_{k_x}, V_{k_y}, V_{k_z})$ represents the k'th user velocity; τ_k represents the k'th user range-equivalent receiver clock error, and τ_{s_i} represents the i'th satellite range-equivalent clock error, $i = 1, 2, ..., n$.

In the special case where two users ($m = 2$) and five ($n = 5$) satellites are considered, the parameter vector is shown in eq. (9) and $\theta \in R^{19}$. In the general case, the parameter

$$\theta \in R^{7m+n} \tag{10}$$

In the algorithm, the pseudoranges received are composed as follows. First, the pseudoranges received by the k'th user from n satellites at time instant j are used to form the $n \times 1$ vector

$$Z_j^{(k)} = \begin{bmatrix} \rho_{1,j}^{(k)} \\ . \\ . \\ . \\ \rho_{n,j}^{(k)} \end{bmatrix}$$

Composing the received pseudoranges over the $N+1$ time instants yields the $n(N+1) \times 1$ vector

$$Z^{(k)} = \begin{bmatrix} Z_0^{(k)} \\ . \\ . \\ . \\ Z_N^{(k)} \end{bmatrix}$$

and finally, composing for the m users yields the $mn(N+1) \times 1$ "measurement vector"

$$Z = \begin{bmatrix} Z^{(1)} \\ . \\ . \\ . \\ Z^{(m)} \end{bmatrix}$$

The pseudorange expression, $\rho(\theta)$, is composed similarly: Define

$$\begin{aligned} f_{i,j}^{(k)}(\theta) &= \sqrt{[x_{0_k} + (V_{k_x}\Delta T)j - x_{s_i}(t)]^2} \\ &\quad \overline{+[y_{0_k} + (V_{k_y}\Delta T)j - y_{s_i}(t)]^2} \\ &\quad \overline{+[z_{0_k} + (V_{k_z}\Delta T)j - z_{s_i}(t)]^2} \\ &\quad \tau_k + \tau_{s_i} \end{aligned} \tag{11}$$

and

$$\rho_{i,j}^{(k)}(\theta) = f_{i,j}^{(k)}(\theta) + \tau_k + \tau_{s_i} \tag{12}$$

For $i = 1, ..., n$ we compose the vectors $f_j^{(k)}$, for $j = 0, ..., N$ we compose the vectors $f^{(k)}$, and finally, composing for the m users, $k = 1, ..., m$, yields the function $f : R^{6m} \rightarrow R^{mn(N+1)}$. The vector ρ is similarly composed, thus obtaining the function $\rho(\theta) : R^{7m+n} \rightarrow R^{mn(N+1)}$

The nonlinear GPS equations are

$$Z = \rho(\theta) + W \tag{13}$$

where the $mn(N+1)$ "equation error" W represents the composed measurement noise vector with covariance - see, e.g., eq. (4) -

$$R_1 = E(WW^T) = \sigma^2 I_{mn(N+1)} \tag{14}$$

Linearization of eq. (13) with respect to the parameter θ at the l'th iteration, about the current parameter estimate $\hat{\theta}^{(l)}$, yields the linear regression in θ

$$Z + \frac{\partial \rho}{\partial \theta} \mid_{\hat{\theta}^{(l)}} \hat{\theta}^{(l)} - \rho(\hat{\theta}^{(l)}) = \frac{\partial \rho}{\partial \theta} \mid_{\hat{\theta}^{(l)}} \theta + W \tag{15}$$

The calculation of the regressor matrix requires the composition of the partials of $\rho(\theta)$ with respect to the parameter vector θ, viz., let

$$H_{i,j}^{(k)}(\theta) = \frac{\partial \rho_{i,j}^{(k)}(\theta)}{\partial \theta} \tag{16}$$

$H_{i,j}^{(k)}$ is an $1 \times (7m+n)$ row vector. Its entries are explicitly given by

$$\begin{aligned} H_{i,j}^{(k)}(\theta) = \frac{1}{f_{i,j}^{(k)}(\theta)} [(\theta)^T E_{i,k}(j) - x_{s_{i,j}} e_{1_{i,k}}(j) \\ -y_{s_{i,j}} e_{2_{i,k}}(j) - z_{s_{i,j}} e_{3_{i,k}}(j)] + e_{i,k} \end{aligned} \tag{17}$$

where $(x_{s_{i,j}}, y_{s_{i,j}}, z_{s_{i,j}})$ is the i^{th} satellite ephemeris at time j; $e_{i,k}$, $e_{1_{i,k}}(j)$, $e_{2_{i,k}}(j)$, and $e_{3_{i,k}}(j)$ are $7m+4n$ row vectors of zeroes, with 1's, j, and j^2 located at positions indicated in their subscrips, according to

$$\begin{aligned} e_{1_{i,k}}(j) &= e_{7k-6,7k-3,7m+4i-3} \\ e_{2_{i,k}}(j) &= e_{7k-5,7k-2,7m+4i-2} \\ e_{3_{i,k}}(j) &= e_{7k-9,7k-1,7m+4i-1} \\ e_{i,k} &= e_{7k,7m+4i} \end{aligned}$$

The matrix

$$E_{i,k}(j) = e_{1_{i,k}}^T e_{1_{i,k}} + e_{2_{i,k}}^T e_{2_{i,k}} + e_{3_{i,k}}^T e_{3_{i,k}} \tag{18}$$

The composition of the regressor matrix is similar to the process employed for Z and f. First, for the n satellites, the $n \times (7m+n)$ matrix $H_j^{(k)}(\theta)$ is formed, followed by composition over the $N+1$ time epochs yielding the $n(N+1) \times (7m+n)$ matrix $H^{(k)}(\theta)$, and finally, for the m users, the $mn(N+1) \times (7m+n)$ regressor matrix

$$H(\theta) = \begin{bmatrix} H^{(1)}(\theta) \\ . \\ . \\ . \\ H^{(m)}(\theta) \end{bmatrix}$$

is obtained.

Eq. (15) thus yields the linear regression

$$Z + \frac{\partial f}{\partial \theta} |_{\hat{\theta}^{(l)}} \hat{\theta}^{(l)} - f(\hat{\theta}^{(l)}) = \frac{\partial \rho}{\partial \theta} |_{\hat{\theta}^{(l)}} \theta + W$$
$$= H(\hat{\theta}^{(l)})\theta + W \quad (19)$$

The function $\rho(\theta) : R^{7m+n} \rightarrow R^{mn(N+1)}$ is linear in the users' and satellites' clock error parameters and therefore the function $\rho(\theta)$ is replaced by the function $f(\theta)$ in the LHS of eq. (15).

2.1 Reduced Parameter Vector

As stated in eq. (10), the parameter vector contains $7m+n$ variables: 3 position components, 3 velocity components and a clock bias variable for each of the two ($m=2$) users, as well as the five ($n=5$) satellite clock errors - 19 variables in total. The main objective is to estimate the (non-reference) users' position and velocity rather than the users' and satellites' clock errors. With this in mind, the regressor's matrix structure is examined and the algorithm is modified according to the following analysis.

Define the $mn(N+1) \times (m+n)$ matrix

$$B = [b_1, b_2, ..., b_m, c_1, c_2, ..., c_n]$$

where b_k is the column of H operating on user k'th clock error variable, $k = 1, 2, ..., m$; c_i is the column of H operating on satellite i'th clock error variable, $i = 1, 2, ..., n$. Indeed, the structure of the regressor matrix $H(\hat{\theta}^{(l)})$ is

$$H = [Diag(\tilde{H}^{(k)}(\hat{\theta}^{(l)}))_{k=1}^{m} | B] \quad (20)$$

where the $n(N+1) \times 6$ matrices $\tilde{H}^{(k)}$ are the $H^{(k)}$ matrices with only the six columns $6k-5$, $6k-4$, $6k-3$, $6k-2$, $6k-1$ and $6k$, retained; e.g., in the case where $m=2$, and therefore $k = 1, 2$

$$H = \left[\begin{pmatrix} \tilde{H}^{(1)} & 0 \\ 0 & \tilde{H}^{(2} \end{pmatrix} \middle| B \right]$$

where the $\tilde{H}^{(1)}$ and $\tilde{H}^{(2)}$ matrices are explicitly given by the partials of f in $(x_{0_1}, y_{0_1}, z_{0_1}, V_{x_1}$ $\Delta T, V_{y_1}\Delta T, V_{z_1}\Delta T)$ and $(x_{0_2}, y_{0_2}, z_{0_2}, V_{x_2}\Delta T, V_{y_2}\Delta T, V_{z_2}\Delta T)$, respectively. The matrix

$$B = \left[\begin{array}{c|c} & I_n \\ & . \\ Diag(e_{n(N+1)})_{k=1}^{m} & . \\ & . \\ & I_n \end{array} \right]$$

where $e_{n(N+1)}$ is a vector of ones of size $(N+1) \times 1$ and I_n is an identity matrix of size n.

For example, for $m=2$,

$$B = \left[\begin{array}{c|c} & I_n \\ \begin{pmatrix} e_{n(N+1)} & 0 \\ 0 & e_{n(N+1)} \end{pmatrix} & . \\ & . \\ & . \\ & I_n \end{array} \right]$$

The following holds

$$\sum_{k=1}^{m} b_k = \sum_{i=1}^{n} c_i \quad (21)$$

Obviously, the matrix B (and therefore, the regressor H) is rank deficient. The rank deficiency is 1. Thus, perform the full rank factorization of B,

$$B = B_1 K \quad (22)$$

where B_1 is full rank $(m+n-1)$ and is an $mn(N+1) \times (m+n-1)$ matrix of the form

$$B_1 = [b_1, b_2, ..., b_{m-1}, \sum_{k=1}^{m} b_k, c_1, c_2, ..., c_{n-1}] \quad (23)$$

For example, in the special case where $m = 2$

$$B_1 = [b_1, b_1 + b_2, c_1, c_2, ..., c_{n-1}] \tag{24}$$

This choice of basis is responsible for inserting a column of ones in the regressor matrix H. In the parlance of linear regression, an "intercept" variable is then included. The latter has the beneficial effect of absorbing truncation errors caused by the linearization of the RHS of eq. (13). This basis choice also has the effect of yielding the estimates of clock error differences, as indicated in eq. (31) in the sequel.

Solving eq. (23) for K yields the blocked $(m + n - 1) \times (m + n)$ matrix

$$K = \begin{bmatrix} \tilde{I}_n & 0_{m\times(n-1)} & b \\ 0_{(n-1)\times m} & I_{n-1} & -e_{n-1} \end{bmatrix}$$

where

$$\tilde{I}_n = \begin{bmatrix} I_{m-1} & -e_{m-1} \\ 0_{1\times(m-1)} & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0_{(m-1)\times 1} \\ 1 \end{bmatrix}$$

and 0 is a zeroes matrix, and e_n is a vector of ones of length n.

Now, partition the regressor

$$H = [\tilde{H}|B] \tag{25}$$

where $\tilde{H} = Diag(\tilde{H}^{(k)})_{k=1}^m$ is an $mn(N+1) \times 6m$ matrix consisting of the columns of H operating on the users' position and velocity parameters only. Also define the reduced, full rank, matrix

$$H_1 = [\tilde{H}|B_1] \tag{26}$$

Next, perform the full rank factorization of the regressor H $(= [\tilde{H}|B])$

$$H = H_1 K_1 \ , \tag{27}$$

i.e., the following equation is solved for the $(7m + n - 1) \times (7m + n)$ matrix K_1:

$$[\tilde{H}|B_1 K] = [\tilde{H}|B_1] K_1 \tag{28}$$

We calculate:

$$K_1 = \begin{bmatrix} I_{6m} & 0_{6m\times(m+n)} \\ 0_{(m+n-1)\times(6m)} & K_{(m+n-1)\times(m+n)} \end{bmatrix}$$

Finally, the reduced parameter vector is defined

$$\theta_1 = K_1 \theta \tag{29}$$

The reduced parameter vector $\theta_1 \in R^{7m+n-1}$ consists of the users' position and velocity parameters as well as linear combinations of the user and satellite clock errors. For the specific scenario examined in which we have two mobile users and five satellites visible, this yields the 18×1 vector

$$\begin{aligned} \theta_1 \;=\; & [x_{0_1}, y_{0_1}, z_{0_1}, V_{x_1}\Delta T, V_{y_1}\Delta T, V_{z_1}\Delta T, x_{0_2}, y_{0_2}, \\ & z_{0_2}, V_{x_2}\Delta T, V_{y_2}\Delta T, V_{z_2}\Delta T, \tau_1 - \tau_2, \\ & \tau_2 + \tau_{s_5}, \tau_{s_1} - \tau_{s_5}, \tau_{s_2} - \tau_{s_5}, \\ & \tau_{s_3} - \tau_{s_5}, \tau_{s_4} - \tau_{s_5}]^T \end{aligned} \tag{30}$$

Since

$$H\theta = H_1 K_1 \theta = H_1 \theta_1 \tag{31}$$

eq. (19) is written with the reduced parameter θ_1:

$$Z + H_2(\hat{\theta}_2^{(l)})\hat{\theta}_2^{(l)} - f(\hat{\theta}_2^{(l)}) = H_1(\hat{\theta}_2^{(l)})\theta_1 + W \tag{32}$$

where, in addition, the further reduced, 12×1, parameter vector is used

$$\begin{aligned} \theta_2 \;=\; & [x_{0_1}, y_{0_1}, z_{0_1}, V_{x_1}\Delta T, V_{y_1}\Delta T, V_{z_1}\Delta T, x_{0_2}, \\ & y_{0_2}, z_{0_2}, V_{x_2}\Delta T, V_{y_2}\Delta T, V_{z_2}\Delta T]^T \end{aligned} \tag{33}$$

Thus, the further reduced parameter vector θ_2 is stripped of the clock error parameters of θ_1, and θ_2 (and not θ_1) is used on the LHS of eq. (33) because the function f is not dependent on the time parameters. Accordingly, the matrix H_2 is composed of the first $6m$ columns of H associated with the parameters featuring in θ_2 (positions and velocities, no clock errors). Thus, $H_2 = Diag(\tilde{H}^{(k)})_{k=1}^m$.

3 Main Result

The linear regression featuring in the ILS algorithm is augmented to include the prior information on user 2 (position and velocity). The prior information is provided in the form

$$x_{0_2} = N(\bar{x}_{0_2}, \sigma^2_{x_{0_2}}),\ y_{0_2} = N(\bar{y}_{0_2}, \sigma^2_{y_{0_2}})$$
$$z_{0_2} = N(\bar{z}_{0_2}, \sigma^2_{z_{0_2}}),\ V_{x_2} = N(\bar{V}_{x_2}, \sigma^2_{V_{x_2}})$$
$$V_{y_2} = N(\bar{V}_{y_2}, \sigma^2_{V_{y_2}}),\ V_{z_2} = N(\bar{V}_{z_2}, \sigma^2_{V_{z_2}})$$

Using very large σ parameters is tantamount to the stipulation that no prior information on user 2^{th} initial state is available.

The linear regression is now augmented as follows.

$$Z := \begin{pmatrix} Z \\ - \\ Z_1 \end{pmatrix} \ where\ Z_1 = \begin{pmatrix} \bar{x}_{0_2} \\ \bar{y}_{0_2} \\ \bar{z}_{0_2} \\ \bar{V}_{x_2} \\ \bar{V}_{y_2} \\ \bar{V}_{z_2} \end{pmatrix}$$

In addition, the regressor H_1 is augmented

$$H_1 := \begin{bmatrix} H_1 \\ - \\ M \end{bmatrix}$$

where the $6 \times (7m + n - 1)$ selector matrix is, e.g., in the case where $m = 2$,

$$M = [0_{6\times 6} \mid I_6 \mid 0_{6\times(m+n-1)}] \tag{34}$$

and when appended to H_1 picks out the $x_{0_2}, y_{0_2}, z_{0_2}, V_{x_2}\Delta T, V_{y_2}\Delta T, V_{z_2}\Delta T$ elements of the parameter vector θ_1. Moreover,

$$H_2 := \begin{bmatrix} H_2 \\ - \\ 0_{6\times 6m} \end{bmatrix},\quad f := \begin{pmatrix} f \\ - \\ 0_{6\times 1} \end{pmatrix}$$

and also W is accordingly augmented. Additionally, a weighting matrix R is included, to correctly incorporate the confidence level in the "reference " receiver's (user 2) prior information on position, velocity, and possibly, range equivalent clock error: $R = Diag(R_1, R_2)$ where $R_1 = \sigma^2 I_{mn(N+1)}$ is determined by the measurement noise variance σ, and the diagonal matrix R_2 contains the prior information data, viz., the standard deviations of the reference station's initial position (and velocity)

$$R_2 = Diag((\sigma^2_{x_{0_2}}, \sigma^2_{y_{0_2}}, \sigma^2_{z_{0_2}}, \sigma^2_{V_{x_2}}, \sigma^2_{V_{y_2}}, \sigma^2_{V_{z_2}})) \tag{35}$$

This finally yields the enhanced ILS algorithm

$$\begin{aligned} \hat{\theta}_1^{(l+1)} &= [H_1^T(\hat{\theta}_2^{(l)})R^{-1}H_1(\hat{\theta}_2^{(l)})]^{-1}H_1^T(\hat{\theta}_2^{(l)})R^{-1}[Z \\ &\quad + H_2(\hat{\theta}_2^{(l)})\hat{\theta}_2^{(l)} - f(\hat{\theta}_2^{(l)})] \end{aligned} \tag{36}$$

and $l = 0, 1, ..., L$.

4 Conclusion

In this paper a *navigation web* - based concept is advanced and a novel algorithm for KDGPS data processing is presented. Specifically, the accurate positioning of a team, or formation, of mobile vehicles is considered. The measurement situation on hand is correctly modeled, a stochastic framework is developed, and a novel centralized estimation algorithm is rigorously derived.

References

1. Parkinson, B. W., and J. J. Spilker: "Global Positioning System: Theory and Applications", Vols. 1 and 2, AIAA Press, 1996.

2. Pachter, M., and J. B. McKay:"Geometry Optimization of a GPS - Based Navigation Reference System", *Navigation*: Journal of the Institute of Navigation, Vol. 44, No 4, Winter 1997-1998, pp. 457-470.

3. Corazzini, T., A. Robertson, J. C. Adams, A. Hassibi and J. P. How: "GPS Sensing for Spacecraft Formation Flying", Stanford University, 1997.

4. Corazzini, T., and J. How: "Onboard GPS Signal Augmentation for Formation Flying", Stanford University, 1998.