An Almost Linear Biped [1]

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Abstract

In this paper we discuss the nonlinear control of a novel biped robot with three-degrees-of-freedom and two control inputs, whose dynamics are linear apart from the gravitational torque and the impact equations. We show that the equations of motion are locally feedback linearizable by nonlinear change of coordinates and nonlinear feedback in a region that includes all walking gates of interest. We combine the feedback linearization control with control of the resulting linear system and the nonlinear impact equations to generate a stable walking gate.

1 Introduction

In this paper we present a preliminary study of nonlinear control for the novel biped robot design first reported in [3]. That reference introduced the biped design and showed that the dynamics allow the existence of periodic limit cycles. Later work reported in [4], applied linear quadratic optimal control techniques to the linear approximation of the nonlinear dynamics in order to generate stable gaits with feedback control. In this paper we apply exact nonlinear feedback linearization control [5] to this problem. The proof of feedback linearizability, a property that does not hold for general underactuated mechanical systems [9], is a new contribution of this paper.

The structure of this biped, which we shall refer to as the inertia wheel biped, is shown in Figure 1.

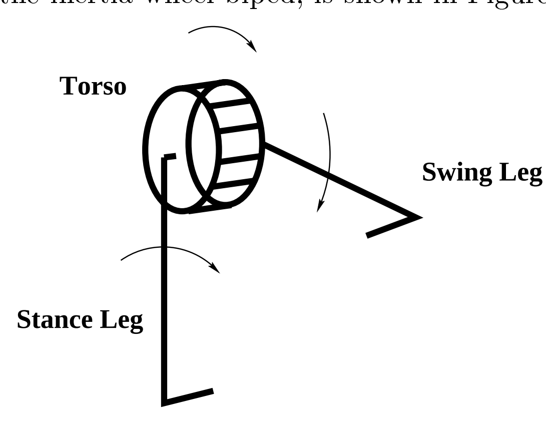

Figure 1: Inertia Wheel Biped Robot

The biped consists of two rigid legs, each of mass M_ℓ, connected through actuators (torque sources) to a central torso of mass M_b, which we assume

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possesses cylindrical symmetry. We assume that an independent torque input can be generated between each leg and the torso. The biped thus has three degrees of freedom and two control inputs.

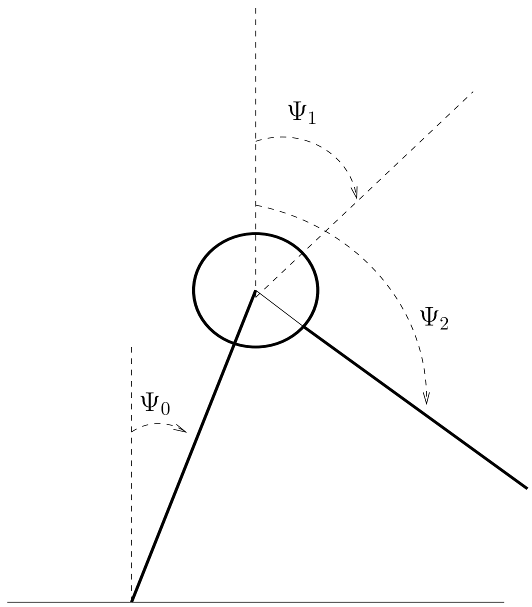

Fig 2. Coordinate Convention

Referring to Figure 2 we use absolute coordinates, Ψ_0, Ψ_1, and Ψ_2, to represent the stance leg angle, torso angle and swing leg angle, respectively, with repect to the vertical. We assume that the legs are counterbalanced so that the center of mass of all three rigid bodies is at the hip axis of rotation. Under these assumptions the dynamics equations of motion can then be computed via Lagranges equations [8] as [3]

$$\begin{aligned} J_0\ddot{\Psi}_0 &= mg\ell\sin(\Psi_0) - \tau_1 && (1)\\ J_1\ddot{\Psi}_1 &= \tau_1 - \tau_2 && (2)\\ J_2\ddot{\Psi}_2 &= \tau_2 && (3) \end{aligned}$$

where J_0, J_1, and J_2 are the rotational inertias about the hip, τ_1, τ_2 are the control input torques, and $m = M_\ell + M_b$. We see that the dynamics are linear apart from the gravitational torque, $mg\ell\sin(\Psi_0)$, acting on the stance leg. This non-anthropomorphic design has several important differences from more traditional anthropomorphic biped designs. First, counterbalancing of the two legs decouples the motion of the swing leg from the stance leg. Thus concepts, such as passive walking [6] and passivity based control designs [10], which rely on inertial and gravitational coupling to exchange energy between the swing and stance legs, are probably not feasible for this design. On the other hand, the simplified dynamics means that more traditional feedback control design methodologies are easier to apply and likely to produce good performance.

2 Impact Dynamics

The above dynamical system describes the motion during a single step until the swing leg impacts the ground. At impact the role of the two legs reverses - the swing leg becomes the stance leg and vice versa. The complete description of the dynamics of the biped therefore includes the impact dynamics and transfer of support. We assume that the impact with the ground is perfectly inelastic and that the transfer of support from the stance to the swing leg is instantaneous. Under these assumptions there is then an instantaneous change of velocity. Since angular momentum is conserved by the impact, the velocities just after impact can be computed from the velocities just prior to impact from the conservation of angular momentum condition as [3]

$$\begin{bmatrix} \dot{\Psi}_0^+ \\ \dot{\Psi}_1^+ \\ \dot{\Psi}_2^+ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -m\ell\cos(\Psi_2 - \Psi_1)/J_0 & 0 & J_2/J_0 \end{bmatrix} \begin{bmatrix} \dot{\Psi}_0^- \\ \dot{\Psi}_1^- \\ \dot{\Psi}_2^- \end{bmatrix} \quad (4)$$

where the plus (resp. minus) signs represent the velocities immediately after (resp. before) impact. We notice another interesting feature of this design, namely, that the velocities of the torso and stance leg are continuous throughout the impact, i.e., the postimpact and preimpact velocities are the same. Only the swing leg velocity is changed abruptly by its impact with the ground. This is an important observation for later motion planning and control.

Immediately after impact the roles of the swing leg and stance leg reverse which is represented by the substitution

$$\begin{bmatrix} \Psi_0 \\ \Psi_1 \\ \Psi_2 \end{bmatrix} \longrightarrow \begin{bmatrix} \Psi_2 \\ \Psi_1 \\ \Psi_0 \end{bmatrix} \quad (5)$$

As in all such studies of systems like this, we will ignore the foot scuffing problem [6]. In practice, a mechanism (knees or moveable feet) would have to be provided for foot clearance. Here we are only concerned with the mathematical details of the controller design of the system assuming the foot scuffing problem is dealt with separately.

3 Feedback Linearization

In this section we show that the nonlinear dynamics of the inertia wheel biped during a single step are locally feedback linearizable via a diffeomorphic change of coordinates and nonlinear feedback [5]. This system, in fact, is an interesting example of a multi-input feedback linearizable system. For space reasons, we omit the detailed calculations, which are straightforward, and present only the resulting linearizing coordinate transformation and nonlinear feedback.

Defining state variables

$$\begin{aligned} x &= (x_1, x_2, x_3, x_4, x_5, x_6)^T \\ &= (\Psi_0, \dot{\Psi}_0, \Psi_1, \dot{\Psi}_1, \Psi_2, \dot{\Psi}_2)^T \end{aligned} \tag{6}$$

the state space description of the system is given by

$$\dot{x} = f(x) + Gu \tag{7}$$

with

$$f(x) = \begin{bmatrix} x_2 \\ \frac{mg\ell}{J_0} sin(x_1) \\ x_4 \\ 0 \\ x_6 \\ 0 \end{bmatrix}; G = [g_1; g_2] = \begin{bmatrix} 0 & 0 \\ \frac{1}{J_0} & 0 \\ 0 & 0 \\ \frac{1}{J_1} & -\frac{1}{J_1} \\ 0 & 0 \\ 0 & \frac{1}{J_2} \end{bmatrix} \tag{8}$$

$$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{9}$$

Let

$$\mathcal{G}_\ell = \text{span}\{\text{ad}_f^j g_i : 1 \leq i \leq 2,\ 0 \leq j \leq \ell\} \tag{10}$$

Then it is a straightforward calculation to show that $\mathcal{G}_0, \ldots, \mathcal{G}_4$ are involutive and of constant rank, and that rank $\mathcal{G}_5 = 6$ in a neighborhood of the origin, which implies that the system is locally feedback linearizable.

The controllability indices, κ_1, κ_2, associated to this system can be computed to be

$$\kappa_i = \text{card}\{m_j \geq i : j \geq 0\} \tag{11}$$

with

$$\begin{aligned} m_0 &= \text{rank } \mathcal{G}_0 \\ m_1 &= \text{rank } \mathcal{G}_1 - \text{rank } \mathcal{G}_0 \\ m_{n-1} &= \text{rank } \mathcal{G}_{n-1} - \text{rank } \mathcal{G}_{n-2} \end{aligned}$$

Performing the indicated calculations gives $\kappa_1 = 4,\ ;\ \kappa_2 = 2$. Thus, there is a diffeomorphic coordinate transformation

$$z = \Phi(x) \tag{12}$$

and nonlinear feedback

$$u = \alpha(x) + \beta(x)v \tag{13}$$

such that

$$\begin{aligned} \dot{z}_{11} &= z_{12} && (14) \\ \dot{z}_{12} &= z_{13} && (15) \\ \dot{z}_{13} &= z_{14} && (16) \\ \dot{z}_{14} &= v_1 && (17) \\ \dot{z}_{21} &= z_{22} && (18) \\ \dot{z}_{22} &= v_2 && (19) \end{aligned}$$

One such feedback linearizing transformation can be computed as

$$\begin{aligned} z_{11} &= J_0\Psi_0 + J_1\Psi_1 + J_2\Psi_2 && (20) \\ z_{12} &= \dot{z}_{11} = J_0\dot{\Psi}_0 + J_1\dot{\Psi}_1 + J_2\dot{\Psi}_2 && (21) \\ z_{13} &= \dot{z}_{12} = J_0\ddot{\Psi}_0 + J_1\ddot{\Psi}_1 + J_2\ddot{\Psi}_2 \\ &= \sin(\Psi_0) && (22) \\ z_{14} &= \dot{z}_{13} = \cos(\Psi_0)\dot{\Psi}_0 && (23) \\ z_{21} &= \Psi_2 && (24) \\ z_{22} &= \dot{z}_{21} = \dot{\Psi}_2 && (25) \end{aligned}$$

It is easy to show that the above coordinate transformation is a diffeomorphism as long as $|\Psi_0| < \pi/2$, i.e., as long as the stance leg remains above the horizontal. Thus the feedback linearization is valid in the region of practical interest for balance and locomotion. Computing the derivatives of z_{14} and z_{22}, respectively, yields

$$\dot{z}_{14} = \cos(\Psi_0)(\frac{mg\ell}{J_0}\sin(\Psi_0) - \frac{1}{J_0}\tau_1) - \sin(\Psi_0)\dot{\Psi}_0^2 \quad (26)$$

$$\dot{z}_{22} = \tau_2 \quad (27)$$

We see that the control law

$$\tau_1 = -\frac{J_0}{\cos\Psi_0}(v_1 + \sin(\Psi_0)\dot{\Psi}_0^2 - \frac{mg\ell}{J_0}\cos(\Psi_0)\sin(\Psi_0)) \quad (28)$$

$$\tau_2 = v_2 \quad (29)$$

results in the feedback linearized system in Brunovsky form (14)-(19).

4 Outer Loop Control

In the inertia wheel biped the coupling torque between the torso and legs is that generated by the relative acceleration between the torso and legs and consequently it is important to guarantee that the torso velocity remains bounded during the locomotion. Thus, the feedback linearizability of this system is an important property as it allows arbitrary trajectory tracking of the transformed states z_1 and z_2. The desired strategy that we employ in this section is to track gait trajectories for the legs and regulate the torso position to zero.

Figure 3 shows (normalized) reference trajectories for both the swing and stance leg generated, in this case, as cubic polynomials with beginning and ending velocities equal to zero. Denote by $\Psi_0^d(t)$ and $\Psi_2^d(t)$ these desired cubic polynmial reference trajectories. Then their higher derivatives are also easily generated as polynomials. Also, let $\Psi_1^d(t)$, the reference torso position, be identically zero, which implies also that its higher derivatives are identically zero.

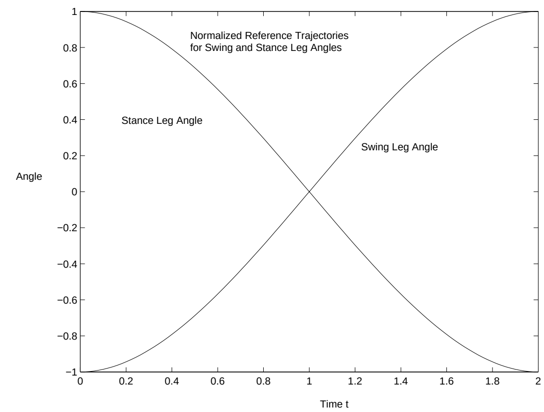

Figure 3: Normalized Reference Trajectories

We see from the impact equations (4), that tracking these desired trajectories will guarantee a continuous velocity at impact. Figure 4 shows the resulting (jumpless) gait limit cycle.

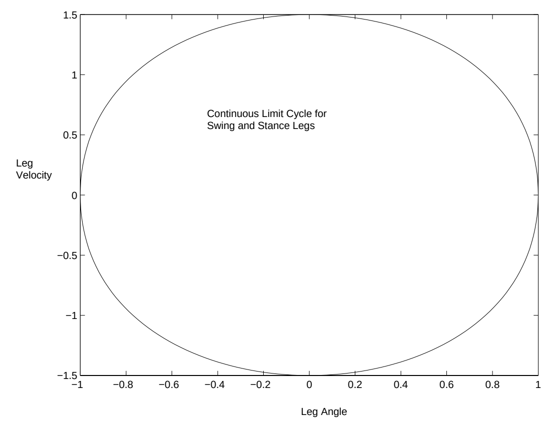

Figure 4: Limit Cycles for Leg Motion

We next define

$$z_{11}^d = J_0\Psi_0^d + J_1\Psi_1^d + J_2\Psi_2^d \quad (30)$$

$$z_{12}^d = J_0\dot{\Psi}_0^d + J_1\dot{\Psi}_1^d + J_2\dot{\Psi}_2^d \quad (31)$$

$$z_{13}^d = \sin(\Psi_0^d) \quad (32)$$

$$z_{14}^d = \cos(\Psi_0^d)\dot{\Psi}_0^d \quad (33)$$

$$z_{21}^d = \Psi_2^d \quad (34)$$

$$z_{22}^d = \dot{\Psi}_2^d \quad (35)$$

and let the outer loop control terms v_1 and v_2 be given by

$$v_1 = \dot{z}_{14}^d + k_{14}(z_{14}^d - z_{14}) + k_{13}(z_{13}^d - z_{13}) = +k_{12}(z_{12}^d - z_{12}) + k_{11}(z_{11}^d - z_{11}) \quad (36)$$

$$v_2 = \dot{z}_{22}^d + k_{22}(z_{22}^d - z_{22}) + k_{21}(z_{21}^d - z_{21}) \quad (37)$$

where the gains k_{ij} are to be designed to achieve exponential tracking of the given reference trajectories.

5 Some Extensions

Since impacts occur at finite time intervals, an alternative to the tracking controller of the previous section is to consider controllers that achieve finite time stabilization. Such controllers have been considered in bipedal locomotion in [2]. One such controller, given in [1] provides finite time stabilization of a double integrator. We can thus use this idea to guarantee finite time convergence of the swing leg to a desired angle prior to impact. This will reduce the control design of the torso/stance leg to a single input, four dimensional system, which in fact is identical to the inertia wheel pendulum considered in [7]. Following the notation of [2, 1], a finite time stabilizing controller for the subsystem (18)-(19) is given by

$$v_2 = \frac{1}{\epsilon^2}\psi_\alpha(z_{21}, \epsilon z_{22}) \tag{38}$$

where

$$\begin{aligned}\psi(x_1, x_2) :=& -\text{sign}(x_2)|x_2|^\alpha \\ &-\text{sign}(\phi_\alpha(x_1, x_2))|\phi_\alpha(x_1, x_2)|^{\frac{\alpha}{2-\alpha}}\end{aligned} \tag{39}$$

with

$$\phi_\alpha(x_1, x_2) := x_1 + \frac{1}{2-\alpha}\text{sign}(x_2)|x_2|^{\frac{\alpha}{2-\alpha}} \tag{40}$$

The parameter ϵ is used to adjust the settling time of the swing leg. With the swing leg convergent to a reference angle (with zero velocity) in finite time, we see that the input v_2 and hence the torque τ_2 reach zero in finite time. During the remainder of the step, then the dynamics of the biped reduce to the single input system

$$\dot{\bar{z}}_{11} = \bar{z}_{12} \tag{41}$$
$$\dot{\bar{z}}_{12} = \bar{z}_{13} \tag{42}$$
$$\dot{\bar{z}}_{13} = \bar{z}_{14} \tag{43}$$
$$\dot{\bar{z}}_{14} = v_1 \tag{44}$$

where the overbar denotes the value of the respective variable with $\Psi_2 = 0 = \dot{\Psi}_2$. We can thus control this single input linear system, which governs the motion of the stance leg and torso so that it reaches a desired configuration just prior to impact of the swing leg with the ground. Details are omitted here.

6 Conclusions

We have presented a feedback linearization scheme for a novel three degree of freedom biped with a symmetric torso and counterbalanced legs and provided some preliminary ideas for tracking and stabilization control. Further research issues for this system include the investigation into energy optimal control, which is of practical interest in bipedal locomotion of robots that may be required to carry their own energy sources, and also an investigation of robust/adaptive control laws to provide good disturbance rejection properties.

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