Design of Structured Controllers with Applications

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Abstract. For generalized plants with a specific structure we provide in this paper solutions of decentralized H_∞-controller design problems by convex optimization without any conservatism. We apply our framework to multi-objective control problems with time-domain and exact disturbance decoupling constraints and to the problem of designing robust controllers against uncertain stochastic disturbances.

Keywords. Structured controller design, parametric dynamic optimization, projections, multi-objective control, linear matrix inequalities.

1 Introduction

The goal of this paper is to provide solutions of control problems in a generalized plant configuration if imposing specific structural constraints on the controller. We will concentrate on controllers that are required to admit a block diagonal structure as shown in Figure 1. In this configuration both blocks G, F are either full dynamic controllers (without any additional structure) or F is assumed to be parametric (static) and then it can be confined to a set of matrices $\mathcal{F}$ which imposes structural and size constraints on this static gain. As the only restriction, the constraint set should admit an LMI representation. As a design objective we choose to minimize a performance criterion that is imposed on the channel $w \to z$, such as the H_∞-norm or the H_2-norm. In general, it is only asked that the specific performance criterion under consideration admits an LMI representation. For reasons of being specific, we concentrate on a bound on the H_∞-norm in continuous time as a performance criterion. If the controlled system admits the state-space description

$$\dot{x} = \mathcal{A}x + \mathcal{B}w, \quad z = \mathcal{C}x + \mathcal{D}w \tag{1}$$

the LMI representation of this performance specification amounts to verifying the existence of a symmetric

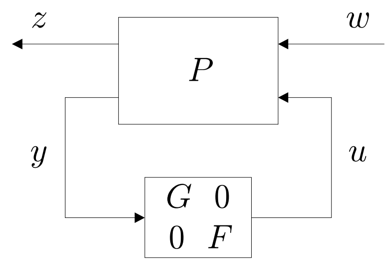

Figure 1: Decentralized Control Configuration

$\mathcal{X}$ with

$$\mathcal{X} > 0, \quad \begin{pmatrix} \mathcal{A}'\mathcal{X} + \mathcal{X}\mathcal{A} & \mathcal{X}\mathcal{B} & \mathcal{C}' \\ \mathcal{B}'\mathcal{X} & -\gamma I & \mathcal{D}' \\ \mathcal{C} & \mathcal{D} & -\gamma I \end{pmatrix} < 0. \tag{2}$$

As formulated the control problem is extremely general. In particular it comprises decentralized control problems and control by static output feedback as specific cases. It even includes general plant-controller-design scenario as described e.g. in [1] if recalling the technique of pulling uncertainties out of a control system which can as well be applied to pulling out controller blocks and to-be-designed model parameters.

Unfortunately it is impossible to provide a full solution to the most general structured controller design problem for an arbitrary open-loop generalized plant

$$\begin{pmatrix} z \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \begin{pmatrix} w \\ u_1 \\ u_2 \end{pmatrix}. \tag{3}$$

The main two new results of this paper can be summarized as follows. The structured controller design problem can be solved by convex optimization techniques under the following hypothesis on P:

- If G is dynamic and $F \in \mathcal{F}$ is parametric then P_{23}, P_{32} and P_{33} vanish.
- If both G and F are dynamic then P_{13}, P_{31} have full row, column rank on the imaginary axis including infinity, and P_{23}, P_{32} vanish.

If $P_{23} = 0$ and $P_{32} = 0$, we observe that the closed-loop transfer matrix separates as $P_{11} + P_{12}G(I -$

[1] I would like to thank Marco Dettori (TU Delft) for helpful discussions.

$P_{22}G)^{-1}P_{21} + P_{13}F(I - P_{33}F)^{-1}P_{31}$. In addition, for $P_{33} = 0$ as in the first hypothesis, the parametric controller component F is required to enter the system *affinely*. In the second hypothesis, we ask the problem of designing F (for a fixed G) to be a one-block problem in the terminology of the operator theoretic approach to controller design [2].

Although the two hypothesis put restrictions on the generalized plant, we discuss various applications that reveal the still rather striking flexibility for solving interesting control problems that are not amenable by other techniques. In particular, we will address multi-objective control problems including exact disturbance decoupling constraints and the problem of designing a robust controller against uncertain stochastic disturbances.

2 Parametric Dynamic Optimization

In this section, let us assume for (3) that $P_{23} = 0$, $P_{32} = 0$, $P_{33} = 0$ and without loss of generality $P_{22}(\infty) = 0$. Then we can find a minimal realization of P that admits the description

$$\begin{pmatrix} z \\ y_1 \\ y_2 \end{pmatrix} = \left[\begin{array}{ccc|ccc} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ 0 & A_{22} & A_{23} & B_{21} & B_{22} & 0 \\ 0 & 0 & A_{33} & B_{31} & 0 & 0 \\ \hline C_{11} & C_{12} & C_{13} & D_{11} & D_{12} & D_{13} \\ 0 & C_{22} & C_{23} & D_{21} & 0 & 0 \\ 0 & 0 & C_{33} & D_{31} & 0 & 0 \end{array}\right] \begin{pmatrix} w \\ u_1 \\ u_2 \end{pmatrix} \tag{4}$$

where (A_{11}, B_{13}) is controllable and (A_{33}, C_{33}) is observable. Obviously, the interconnection in Figure 1 results from a lower linear fractional transformation of

$$\left[\begin{array}{ccc|cc} A_{11} & A_{12} & A_{13}{+}B_{13}FC_{33} & B_{11}{+}B_{13}FD_{31} & B_{12} \\ 0 & A_{22} & A_{23} & B_{21} & B_{22} \\ 0 & 0 & A_{33} & B_{31} & 0 \\ \hline C_{11} & C_{12} & C_{13}{+}D_{13}FC_{33} & D_{11}{+}D_{13}FD_{31} & D_{12} \\ 0 & C_{22} & C_{23} & D_{21} & 0 \end{array}\right] =: \left[\begin{array}{c|cc} A(F) & B(F) & B \\ \hline C(F) & D(F) & D_1 \\ C & D_2 & 0 \end{array}\right] \tag{5}$$

with G. Let us now introduce the symmetric matrix variables R and S which are of the size and partitioned as the state of (4), and define

$$R_1 := \begin{pmatrix} R_{11} & 0 & 0 \\ R_{21} & I & 0 \\ R_{31} & 0 & I \end{pmatrix},\ R_2 := \begin{pmatrix} I & -R_{21}' & -R_{31}' \\ 0 & R_{22} & R_{32}' \\ 0 & R_{32} & R_{33} \end{pmatrix},$$

$$S_1 := \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ S_{31} & S_{32} & S_{33} \end{pmatrix},\ S_2 := \begin{pmatrix} S_{11} & S_{21}' & -S_{31}' \\ S_{21} & S_{22} & -S_{32}' \\ 0 & 0 & I \end{pmatrix}$$

as well as the functions

$$\begin{pmatrix} \boldsymbol{X_1}(R) & * \\ \boldsymbol{X_{12}}(R,S) & \boldsymbol{X_2}(S) \end{pmatrix} := \begin{pmatrix} R_2R_1' & * \\ S_1'R_1' & S_1'S_2 \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{A_1}(F,R) & * \\ \boldsymbol{A_{21}}(F,R,S) & \boldsymbol{A_2}(F,S) \end{pmatrix} := \\ := \begin{pmatrix} R_2A(F)R_1' & * \\ S_1'A(F)R_1' & S_1'A(F)S_2 \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{B_1}(F,R) \\ \boldsymbol{B_2}(F,S) \end{pmatrix} := \begin{pmatrix} R_2 \\ S_1' \end{pmatrix} B(F),$$

$$\begin{pmatrix} \boldsymbol{C_1}(F,R) & \boldsymbol{C_2}(F,S) \end{pmatrix} := C(F) \begin{pmatrix} R_1' & S_2 \end{pmatrix}.$$

It is straightforward to verify that all these functions are *affine* in the variables F and R, S. Now we are ready to formulate the main result of this paper.

Theorem 1 *Given an arbitrary $F \in \mathcal{F}$, there exists a dynamic controller G such that the realization (1) of the system (3) controlled with $u_1 = Gy_1$, $u_2 = Fy_2$ admits an $\mathcal{X}$ with (2) if and only if there exist R, S, K, L, M, N that satisfy the linear matrix inequalities*

$$\begin{pmatrix} \boldsymbol{X_1}(F,R) & \boldsymbol{X_{12}}(F,R,S) \\ \boldsymbol{X_{12}}(F,R,S)' & \boldsymbol{X_2}(F,S) \end{pmatrix} > 0, \tag{6}$$

$$\left(\begin{array}{cc|cc} sy(\boldsymbol{A_1}{+}LC) & * & \boldsymbol{B_1}{+}LD_2 & * \\ \boldsymbol{A_{21}}{+}BNC{+}K & sy(\boldsymbol{A_2}{+}B_2M) & \boldsymbol{B_2}{+}BND_2 & * \\ \hline * & * & -\gamma I & * \\ \boldsymbol{C_1}{+}D_1NC & \boldsymbol{C_2}{+}D_1M & D(F){+}D_1ND_2 & -\gamma I \end{array}\right) < 0. \tag{7}$$

For reasons of space, we used the abbreviation $\mathrm{sy}(M) = M{+}M'$ and we suppressed the blocks $*$ defined by symmetry as well as the function arguments in (7).

Sketch of Proof. The H_∞-synthesis inequalities for (5) are [3, 4]

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0,$$

$$\left(\begin{array}{cc|cc} \mathrm{sy}(XA(F){+}\tilde{L}C) & * & XB(F){+}\tilde{L}D_2 & * \\ A(F){+}BNC{+}\tilde{K} & \mathrm{sy}(A(F)Y{+}B\tilde{M}) & B(F){+}BND_2 & * \\ \hline * & * & -\gamma I & * \\ C(F){+}D_1NC & C(F)Y{+}D_1\tilde{M} & D(F){+}D_1ND_2 & -\gamma I \end{array}\right) < 0$$

in the variables X, Y, $\tilde{K}$, $\tilde{L}$, $\tilde{M}$, N. Let us partition X and Y according to the state of (4) and define, in extension of [5], the blocks of R and S through the factorizations $R_1X = R_2$ and $YS_1 = S_2$ respectively. It is easy to see that $X \to R$, $Y \to S$ are bijective transformation from the set of positive definite matrices onto the the set of R, S with

$$R_{11} > 0, \begin{pmatrix} R_{22} & R_{32}' \\ R_{32} & R_{33} \end{pmatrix} > 0, \begin{pmatrix} S_{11} & S_{21}' \\ S_{12} & S_{22} \end{pmatrix} > 0, S_{33} > 0.$$

After a congruence transformation (left-multiplying the first two rows with R_1, S_1' and right-multiplying the first tow columns with R_1', S_1), the first inequality transforms into (6) whereas the first two block columns of the second inequality are given by

$$\left(\begin{array}{cc} \mathrm{sy}(R_2A(F)R_1' + R_1\tilde{L}C_2R_1') & * \\ S_1'(A(F)+BNC_2+\tilde{K})R_1' & \mathrm{sy}(S_1'A(F)S_2 + S_1'B\tilde{M}S_1) \\ \hline (R_2B(F)+R_1\tilde{L}D_{21})' & (S_1'B(F)+S_1'BND_2)' \\ C(F)R_1' + D_{12}NCR_1' & C(F)S_2 + D_1\tilde{M}S_1 \end{array}\right).$$

Let us perform the transformation

$$K = S_1'\tilde{K}R_1',\ L = R_1\tilde{L},\ M = \tilde{M}S_1.$$

Just for simple structural reasons, we observe

$$S_1'B = B \ \text{ and } \ CR_1' = C.$$

This leads to the synthesis inequalities (7) in the variables R, S, K, L, M, N.

■

Remarks

- If $\mathcal{F}$ admits a description as the solution set of a fixed linear matrix inequality, verifying the existence of $F \in \mathcal{F}$ and R, S, K, L, M, N with (6)-(7) amounts to solving a standard LMI problem. After having found a solution, the corresponding dynamic controller component G is obtained by inverting the controller parameter transformation of [3, 4] for the parameters $\tilde{K} = (S_1^{-1})'K(R_1^{-1})'$, $\tilde{L} = R_1^{-1}L$, $\tilde{M} = MS_1^{-1}$ and N.

- As indicated in the proof, Theorem 1 can be viewed as a combination of what has been discussed in [3, 4] for purely dynamic controller design and in [6] for designing purely static controllers. This leads to a novel highly flexible synthesis framework with a wealth of interesting applications.

- In view of the previous remark, it is important to note that the general philosophy as outline in [3, 4] does indeed extend to this new framework. We conclude that we can handle in a straightforward fashion arbitrary performance criteria that admit LMI representations and, with certain conservatism, multi-objective cost criteria. Moreover, the systems can be either described in continuous- or discrete-time without requiring any changes in either the derivation of the synthesis inequalities or in the controller design algorithms.

3 Applications

3.1 Parametric Model-Matching

Model-matching is a popular framework for controller design. However in practice it is often hard to fix a

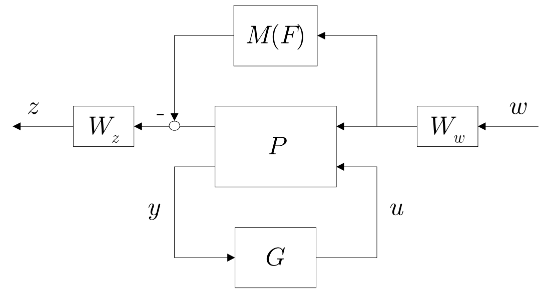

Figure 2: Parametric model-matching configuration

specific model M to be matched. In fact it is more realistic to work with a family of models $\mathcal{M}$ and to try to pick an optimal one that can be matched as closely as possible. Let us assume that the class of models to be matched is parameterized as $\mathcal{M} = \{M(F) : \ F \in \mathcal{F}\}$ where $M(F) \in RH_\infty$ depends affinely on the parameter F that is confined to the set $\mathcal{F}$ which admits an LMI representation. As a simple concrete example, one might choose $\mathcal{M}$ as the convex hull of finitely many given models M_1,...,M_m but we stress that our scenario encompasses much more interesting cases.

The corresponding control configuration (with the stable W_z and W_w as frequency dependent weights) is shown in Figure 2, and one can choose various norms for measuring the matching error. It is easily verified that the generalized plant which corresponds to the configuration in Figure 2 satisfies all the hypotheses in order to be able to apply the design algorithm described in Section 2. If measuring the matching error in the H_∞-norm, we can hence apply Theorem 1 to find both a dynamic controller and a parameter F_* leading to a choice $M(F_*)$ in the model class $\mathcal{M}$ for which the matching error is close to minimal.

Mathematically, we have provided a systematic procedure to explicitly solve minimal distance or projection problems for the set of achievable closed-loop models and rather general convex subsets (with a finite dimensional affine hull) of the space of rational proper stable matrices.

3.2 Robust Rejection of Stochastic Signals

Let us consider the discrete time system

$$\begin{pmatrix} x_{t+1} \\ z_t \\ y_t \end{pmatrix} = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_1 & D_{12} \\ C_2 & D_{21} & 0 \end{pmatrix} \begin{pmatrix} x_t \\ w_t \\ u_t \end{pmatrix}. \quad (8)$$

The disturbance w_t is a $\mathbb{R}^d$-valued wide-sense stationary zero-mean stochastic process that is uncertain in the sense that the first covariance coefficients are only known to satisfy

$$L(E(w_tw_t^T), ..., E(w_tw_{t+m}^T)) \in \mathrm{con}\{F^1, ..., F^N\}. \quad (9)$$

Here $L : \mathbb{R}^{d\times d(m+1)} \to \mathcal{H}$ is a linear mapping into some finite-dimensional real Hilbert space $\mathcal{H}$ with inner product $\langle .,. \rangle$ and $F^j \in \mathcal{H}$ are given elements. This comprises, as a specific example, upper and lower bounds on the covariance coefficients of each component of w_t. With the measurement output y_t and the control input u_t, the goal is to design an output feedback controller which internally stabilizes (in discrete-time) the controlled system and which bounds the worst-case asymptotic output variance of z_t as

$$\sup_{w_t \text{ satisfies (9)}} \lim_{t\to\infty} \frac{1}{2} E(z_t^T z_t) < \gamma.$$

This problem has been suggested and considered in [7, 8, 9, 10] and we refer to [10] for a more detailed discussion.

Let us recall the following analysis result that has been derived in [10] on the basis of a simple Lagrange dualization argument. Determine the adjoint mapping $L^* : \mathcal{H} \to \mathbb{R}^{d\times d(m+1)}$ and introduce the family of discrete-time stable transfer matrices

$$M(F,z) := \sum_{k=0}^{m} L_k^*(F) \frac{1}{z^k}$$

that depends linearly on $F \in \mathcal{H}$. Under a mild regularity condition, the controlled system whose transfer matrix is denoted as T satisfies the desired specification iff there exists some $F \in \mathcal{H}$ with

$$\langle F^j, F \rangle < \gamma \ \text{ for all } \ j = 1,\ldots,N$$

that renders

$$\begin{pmatrix} \frac{1}{2}I & T(z) \\ 0 & M(F,z) \end{pmatrix}$$

strictly positive real. This amounts to solving the parametric model-matching problem for the configuration in Figure 3 and for the strict positive real specification. Since the latter admits an LMI representation, our general design technique from Section 2 applies to solve this robust control problem by output feedback without any conservatism. In contrast to our specialized previous approach [10] that was based on variable elimination, the present technique even allows to incorporate this criterion in the multi-objective controller design paradigm as described in [3, 4].

3.3 Multi-Objective Control

In this section we intend to briefly sketch the consequences of our parametric dynamic optimization procedure for certain multi-objective control problems.

Let us consider a discrete-time generalized plant with two performance channels $w_1 \to z_1$ and $w_2 \to z_2$. With the Youla parameterization, the set of all closed-loop transfer matrices that result from internally stabilizing

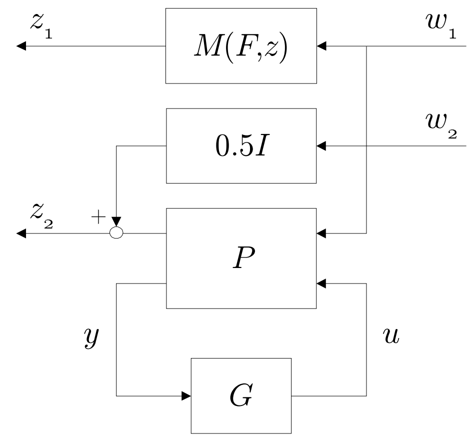

Figure 3: Robust controller design as parametric model-matching

controllers can be described as

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \left[\begin{pmatrix} T_{11} & * \\ * & T_{22} \end{pmatrix} + \begin{pmatrix} T_{13} \\ T_{23} \end{pmatrix} Q \begin{pmatrix} T_{31} & T_{32} \end{pmatrix} \right] \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{10}$$

where Q is the free stable Youla parameter.

Suppose we intend to impose finite-horizon time-domain bounds on the output sequence z_1 if w_1 is a given fixed disturbance input signal. It is not difficult to see [11] that such bounds amount, for some N, to linear inequality constraints on the first N coefficients in a Laurent expansion of Q around ∞. This suggests to represent all Youla parameters as

$$Q(z) = Q_0 + \frac{1}{z} Q_1 + \cdots + \frac{1}{z^N} Q_N + \frac{1}{z^{N+1}} G(z)$$

where G is real rational proper and stable. Then the desired time-domain constraints only affect the coefficients $Q_0, ..., Q_N$ and they typically admit an LMI representation. In addition to such time-domain constraints, one generally attempts to achieve certain closed-loop frequency response shapes that can be translated, with suitable weights, into minimizing the H_∞-norm of the channel $w_2 \to z_2$. The overall problem then results in minimizing

$$\|T_{22} + T_{23}[Q_0 + \frac{1}{z}Q_1 + \cdots + \frac{1}{z^N}Q_N]T_{32} + \\ + [\frac{1}{z^{N+1}}T_{23}]GT_{32}\|_\infty$$

over all $Q_0, ..., Q_N$ satisfying certain LMI constraints and over all proper and stable G. In the most interesting paper [11] this problem has been solved for systems where T_{23}, T_{32} have full column, row rank on the unit circle. The solution is based on the operator theoretic approach of the eighties in which four block problems are reduced by all-pass embedding techniques to one block Nehari problems. As the main disadvantage, this problem has been only addressed for the discrete-time

H_∞-cost, it only applies to systems that satisfy the above mentioned regularity hypothesis, and it leads to rather intricate formulas and a complicated controller construction. We observe that our approach covers the solution of these parametric design problems and avoids the listed disadvantages. In particular, for arbitrary systems in continuous- or discrete-time it applies to any criterion that admits an LMI representation.

Let us finally remark that the possibility to solve parametric H_∞-problems has been the basis of a series of papers to solve multi-objective control problems that involve one H_∞-constraint [12, 13, 14]. Our design technique does not only allow a direct and numerically efficient solution of these multi-objective control problems in continuous-time, but it is possible to handle constraints different from H_∞-norm bounds as long as they admit an LMI representation. For reasons of space it is impossible to discuss the striking variety of problem solutions in greater detail.

3.4 H_∞-Control with Disturbance Decoupling

In addition to loop-shaping, practical MIMO control problems might require the elimination of interaction between certain input and output signals. Requirements on the shape of the closed-loop system can be translated into an H_∞-specification, whereas decoupling amounts to requiring that the corresponding closed-loop transfer matrix vanishes. To be specific let us assume that we have chosen a configuration as in Section 3.3 where $w_1 \to z_1$ has to vanish and the H_∞-norm of $w_2 \to z_2$ has to be rendered smaller than 1. A particular problem of this type, disturbance decoupling with controllers that render the system robustly stable against co-prime factor uncertainty, has been suggested and solved for only a rather restricted class of plants with totally different techniques in [15].

Let us briefly sketch how this multi-objective control problem can be reduced to a structured controller design problem. Indeed, the constraint

$$T_{11} + T_{13} Q T_{31} = 0$$

just means that the Youla parameter has to be contained in some linear sub-manifold of RH_∞. Using the Smith canonical form it is pretty straightforward to test whether this manifold is nonempty (solvability of the equation) and to parameterize the whole solution set. If Q_0 is a specific solution, the set of all solutions is given by

$$Q = Q_0 + U_1 Q_1 V_1 + U_2 Q_2 V_2$$

with easily constructed fixed $U_j, V_j \in RH_\infty$ and with two parameters Q_1 and Q_2 that vary freely in RH_∞. The loop-shaping constraint on the second channel then amounts to

$$\|[T_{22} + T_{23} Q_0 T_{32}] + [T_{23} U_1] Q_1 [V_1 T_{32}] + \\ + [T_{23} U_2] Q_2 [V_2 T_{32}]\|_\infty < 1. \quad (11)$$

We hence arrive at an H_∞ problem with a structured controller for which, to the best of our knowledge, no general solution is available.

Warning. In line with well-known alternating projection algorithms in Hilbert spaces one might speculate that one can minimize the norm of the left-hand side of (11) by coordinate search as follows: Given Q_1^ν, Q_2^ν, optimize over Q_1 for fixed $Q_2 = Q_2^\nu$ to obtain $Q_1^{\nu+1}$, and then optimize over Q_2 for fixed $Q_1 = Q_1^{\nu+1}$ to arrive at $Q_2^{\nu+1}$. Each step this procedure only requires to solve a standard H_∞ control problem and is, hence, easily implemented. However, simple counterexamples show that the achievable norm level is in general larger than the value that can be obtained by common optimization over Q_1, Q_2. Hence, despite convexity, these coordinate descent type algorithms are not converging.

4 Structured Dynamic Controller Design

For (3) we assume in this section that $P_{23} = 0$, $P_{32} = 0$ and that P_{13}, P_{31} have full row, column rank on the imaginary axis and at infinity. The goal is to render the H_∞-norm of $w \to z$ in Figure 1 smaller than one by suitably choosing a structured controller with dynamic blocks G, F that stabilize (3). Without loss of generality (Youla) this problem translates into one of finding $Q, \hat{Q} \in RH_\infty$ such that

$$\|T_0 + T_1 Q T_2 + T_3 \hat{Q} T_4\|_\infty < 1. \quad (12)$$

Due to the hypotheses on P_{13}, P_{31}, we can even assume that T_3 and T_4 are *square* and *invertible* on the imaginary axis and at infinity. Let us assume for the moment that Q is fixed. Characterizing the existence of $\hat{Q}$ satisfying (12) is called a two-sided Nudelman problem whose solution is found e.g. in [2, Theorem 18.5.5]. In order to recall this solution, we need to determine a left-null pair (A_ζ, B_+) of T_3 and a right-null pair (C_-, A_π) of T_4. (This just means that (A_ζ, B_+), (C_-, A_π) are controllable, observable, that $\sigma(A_\zeta) \subset \mathbb{C}^+$, $\sigma(A_\pi) \subset \mathbb{C}^+$, and that there exist C, B such that $T_3^{-1}(s) - C(sI - A_\zeta)^{-1} B_+$, $T_4^{-1}(s) - C_-(sI - A_\pi)^{-1} B$ are stable.) After a slight adaptation of [2, Theorem 18.5.5] (working with linear matrix inequalities instead of Lyapunov equations in view of the discussion to follow), there exists a stable $\hat{Q}$ with (12) if and only if there exist S_1, S_2, S with

$$\begin{pmatrix} S_1 & S^* \\ S & S_2 \end{pmatrix} > 0, \quad (13)$$

$$S A_\pi - A_\zeta S = B_+ C(Q) + B(Q) C_-, \quad (14)$$

$$\begin{pmatrix} A_\pi^* S_1 + S_1 A_\pi - C_-^* C_- & C(Q)^* \\ C(Q) & -I \end{pmatrix} < 0, \qquad (15)$$

$$\begin{pmatrix} A_\zeta S_2 + S_2 A_\zeta^* - B_+ B_+^* & B(Q) \\ B(Q)^* & -I \end{pmatrix} < 0 \qquad (16)$$

where

$$B(Q) := -\mathrm{Res}_+ (sI - A_\zeta)^{-1} B_+ [T_0 + T_1 Q T_2],$$
$$C(Q) := \mathrm{Res}_+ [T_0 + T_1 Q T_2] C_- (sI - A_\pi)^{-1}.$$

Here we have used the abbreviation $\mathrm{Res}_+ T := \sum_{z_0 \in \mathbb{C},\, \mathrm{Re}(z_0) > 0} \mathrm{Res}_{z=z_0} T(z)$.

Testing the existence of a stable Q and $\widehat{Q}$ with (12) hence amounts to testing the existence of a stable Q and S, S_1, S_2 that satisfy (13)-(16). Since the operator $T \to \mathrm{Res}_+ T$ is finite dimensional, this turns out to be a finite dimensional LMI feasibility problem. It is hence possible to numerically verify the existence of a solution to our structured H_∞-problem.

It is possible - and omitted in this conference paper for reasons of space - to derive a compact and explicit state-space parameterization of the ranges of $B(Q)$ and $C(Q)$ if Q varies in RH_∞. This analysis leads to the following important structural insights: The McMillan degree of the Youla parameter Q and, hence, also that of the corresponding controller component, can be explicitly bounded a priori. Moreover, it is possible to arbitrarily choose the (stable) poles of the Youla parameter and it is only required to optimize over the residues of Q. Since we did not include any specific hypothesis on the ranks of T_1, T_2, we believe it to be rather surprising that the poles of Q can be freely chosen. This aspect is crucial in reducing the structured H_∞ problem to a finite-dimensional convex feasibility problem, and it should provide the essential step in trying to find a direct LMI solution for more general systems without any rank constraints on T_3, T_4.

5 Conclusions

In this paper we have reduced specific decentralized controller design problems to convex optimization problems without any conservatism. Various applications to questions in multi-objective and robust controller design reveal the scope of applicability of these new synthesis algorithms. Finally we would like to stress that all our techniques admit natural extensions to multiple controller blocks.

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