

# Indirect Control of High Frequency Power Converters for AC Generation<sup>1</sup>

Joaquín Alvarez, Domingo Cortés

CICESE, Applied Physics Division, Km 107 Carr. Tij-Ens, 22860 Ensenada, BC, Mexico

{jqalvar, dcortes}@cicese.mx

Jaime Alvarez

CINVESTAV, Electrical Eng. Department, AP 14-740, 07000 México, DF, Mexico

jalvarez@mail.cinvestav.mx

## Abstract

It is shown how a very simple procedure of time reversal can solve, in a much easier way than other techniques, an important problem arising in some control strategies of two conventional DC-to-DC switched power converters used to generate an alternate voltage signal: the boost and the buck-boost circuits. This problem is related to the non-minimum phase nature of the capacitor voltage normally used as output in these devices, which compels to control indirectly this variable by controlling the inductor current. A stable model that generates an adequate reference for this signal is proposed, which can be used in-line or in batch mode. The procedure is illustrated by numerical simulations using a sliding mode controller proposed elsewhere.

## 1 Introduction

DC-to-DC conversion is an important field of power electronics. This operation is normally implemented with high frequency switching converters. The basic and most popular devices achieving this conversion are the “buck”, “boost”, “buck-boost”, and “Cuk” types [4], [6]. The buck circuit is a step-down converter, that is, it can only yield a steady state output voltage that is a fraction of the source input voltage. On the contrary, the boost circuit always amplifies the constant voltage of the source, and the last two can reduce or amplify the source input voltage.

A popular approach to analyze and design these devices is the average model technique [5]. This method yields a linear model for the buck circuit and a bilinear structure for the other circuits. This last fact makes more difficult to develop strategies to control these devices which, on the other hand, are more interesting due to their amplification properties. In the last years, a great effort has been made to improve the robustness and general performance of these converters using

nonlinear control techniques (see [2], [7], [8], [10], and the references therein). In [2] a summary of the main nonlinear control techniques previously reported, and a comparison between several of them to control a DC-to-DC boost converter, are presented.

The simplicity of the DC-to-DC converter topologies, the small size of the passive electronic components, which have to eliminate only high frequency harmonics, and their commercial availability, among other characteristics, make these devices very attractive to be applied to DC-to-AC conversion schemes. This is not a new idea; in fact, a usual way to achieve this operation is by employing the bipolar buck converter. However, an important drawback in many applications is that this circuit is a step-down device. Other DC-to-AC conversion schemes use resonant converters or arrays of voltage sources and buck circuits. The first schemes usually include voluminous passive components, while the second ones are step-down schemes. These disadvantages may be eliminated using other DC-to-DC conversion topologies to achieve the DC-to-AC conversion; nevertheless, these approaches transform a regulation problem to a tracking control problem that is considerable more difficult to solve.

One of the main problems to solve when using these circuits to achieve DC-to-AC conversion is their non-minimum phase characteristics. This is a control property present in these systems when the capacitor voltage is taken as the system output, which is the situation of practical interest. On the other hand, the inductor current is a minimum phase output; therefore, any control technique that uses any kind of system inversion strategy must control the capacitor voltage in an indirect way. That is, it must control the capacitor voltage by controlling a minimum phase output, being the inductor current a natural choice normally used in practice. However, independently of the technique used to control the chosen minimum phase output, the problem of determining a reference for this signal that gives in steady state the desired (time-varying) capacitor voltage is not trivial.

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In [11] it is shown that the reference for the inductor current producing a periodic signal in the capacitor voltage is a bounded periodic solution of an Abel type equation whose solution can not be analytically found. The authors propose to use the harmonic balance technique to find an approximate solution. In [8] an iterative procedure to find also an approximate solution is proposed, based on a flatness property that present these systems. In the present paper we show that the Abel type equation discussed in [1] corresponds to the internal dynamics of the system when the capacitor voltage is taken as the output, which is non-minimum phase. This is not in contradiction to the fact that the solution proposed in [1] is bounded, because an unstable system may have bounded solutions; the problem is to find these trajectories. Here we show that, for the boost and the buck-boost converters, and under some assumptions holding in applications, a simple time reversal in the differential equation describing the internal dynamics permits us to find exact, bounded, periodic references for the inductor current that can be used to make the capacitor voltage track a desired, periodic signal. This procedure solves this problem in a much simpler way than other methods. Having found this adequate reference, the application of many techniques used to control typical DC-to-DC schemes used in DC-to-AC schemes is straightforward. We illustrate the proposed technique with numerical simulations of the boost and the buck-boost circuits controlled with the sliding mode technique reported in [7], [8].

## 2 DC-to-AC conversion using the boost converter

Figure 1 shows a diagram of the boost converter. A switched model of this circuit is given by

$$\begin{aligned} L \frac{dx_1}{dt} &= -ux_2 + E, \\ C \frac{dx_2}{dt} &= ux_1 - \frac{x_2}{R}, \end{aligned} \quad (1)$$

where  $x_1$  is the inductor current,  $x_2$  the capacitor voltage, and  $u \in \{0, 1\}$  defines the switch position that plays the role of the control input. The parameters  $R$ ,  $L$ ,  $C$ , and the source voltage  $E$  are constants and they are supposed to be known. An average model having the same structure than equation (1) can be obtained, where the control input  $u$  denotes the duty cycle of a one switching period, with values in the interval  $[0, 1]$ . This model is valid at high switching frequencies and away from zero inductor currents [9].

A control law can be designed using the switched model directly or for the averaged model. In the last case a pulse-width modulator (PWM) must be used to generate the switch position changes. Therefore, we can see equation (1) as a switching model or an averaged model,

having only in mind the meaning and value range taken by the control input  $u$ .

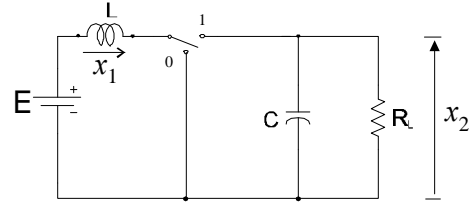


Figure 1: Diagram of the boost converter.

In a DC-to-AC conversion scheme, the control objective is to make the capacitor voltage  $x_2$  track the reference signal  $x_{2d} = A + B \sin(\omega t)$ , where  $A$  and  $B$  are positive constants. A normalized model can be obtained for this circuit via the transformation  $z_1 = x_1/(E\sqrt{C/L})$ ,  $z_2 = x_2/E$ ,  $Q = R\sqrt{C/L}$ , and a time scaling  $t = \sqrt{LC}\tau$ , yielding the model

$$\begin{aligned} \dot{z}_1 &= 1 - uz_2, \\ \dot{z}_2 &= -\frac{z_2}{Q} + uz_1, \end{aligned} \quad (2)$$

where the upper dot denotes the time derivative with respect to the new time  $\tau$ , which we will denote by  $t$  in what follows. The control objective is now to make the variable  $z_2$  follow the reference signal

$$z_{2d} = a + b \sin(\omega t), \quad (3)$$

where  $a = A/E$ ,  $b = B/E$ , and  $\omega = \sqrt{LC}$ .

### 2.1 The tracking control problem

A well known fact is that the output  $z_2$  is non-minimum phase, that is, the internal dynamics of system (2) when  $z_2$  is identical to a given reference signal  $z_{2d}$ , is given by the unstable system

$$\dot{z}_1 = 1 - \frac{f_1(t)}{z_1}, \quad (4)$$

where  $f_1(t) = z_{2d}(t)(\dot{z}_{2d}(t) + z_{2d}(t)/Q)$ . The unbounded nature of the trajectories of this system will be shown later. On the contrary, when  $z_1$  is taken as the output, the internal dynamics when this variable is identical to a given reference signal  $z_{1d}$  is given by the stable system

$$\dot{z}_2 = -\frac{z_2}{Q} + \frac{f_2(t)}{z_2}, \quad (5)$$

where  $f_2(t) = z_{1d}(t)(1 - \dot{z}_{1d}(t))$ <sup>1</sup>. Therefore, a usual approach is to indirectly control the capacitor voltage  $z_2$  by taking the inductor current  $z_1$  as the output [7]. In fact, when the desired voltage is constant, that is,

<sup>1</sup>The bounded nature of trajectories of system (5) will be discussed later.

when  $z_{2d} = a$ , then the steady state value of the inductor current must be  $z_{1d} = a^2/Q$ , which can be used as the desired reference for this variable. Because  $z_1$  is a minimum phase output, and a reference value for this variable that yields a desired steady state value of the practical output  $z_2$  is available, then it is now possible to use several control techniques to attain this objective. A simple and effective strategy is, for example, the control law proposed in [7] [8]. Using this technique the sliding manifold

$$\sigma = z_1 - z_{1d} = 0$$

is produced by means of the control law

$$u = 0.5(1 + \text{sgn}\sigma). \quad (6)$$

In DC-to-AC conversion the situation is much more involved because the steady state value of the reference for the capacitor voltage  $z_2$  is not constant but the periodic signal (3). In fact, when  $z_2$  attains the desired tracking signal  $z_{2d}$ , the differential equation governing the inductor current can be obtained by eliminating  $u$  from (2), yielding the unstable equation (4), which corresponds to the internal dynamics of the system. It can also be seen as the equation that characterizes a set of feasible references  $z_{1d}$  for the inductor current that produce the desired steady state signal  $z_{2d}$  for the capacitor voltage  $z_2$ . In this case this equation can be written in a more explicit form defining the differential equation that the reference for the inductor current ( $z_{1d}$ ) must satisfy, given a desired reference signal for the capacitor voltage ( $z_{2d}$ ),

$$\dot{z}_{1d} = 1 - \frac{1}{z_{1d}} \left( \dot{z}_{2d} + \frac{z_{2d}}{Q} \right) z_{2d}. \quad (7)$$

When the forcing term of this equation ( $z_{2d}$ ) is constant, the unique equilibrium point  $\bar{z}_{1d} = \bar{z}_{2d}^2/Q$  is unstable, a fact that can be concluded from the linearization of system (7) about this equilibrium.

When the forcing term of equation (7) is periodic and positive, which is the case in a practical situation, then the instability of the oscillation can be shown using the same technique described below. Note that in this case a bounded periodic solution may exist for this system, but it is unstable and unfortunately can not be found explicitly. In [8] an iterative technique based on the flatness property of the circuit is proposed; this procedure yields an approximate solution of (7), whose degree of approximation with the exact solution is improved at each iteration. Another technique that could be applied is the harmonic balance procedure proposed in [11]. This strategy provides a solution of (7) that is also approximate. Both of these techniques must be used in batch mode.

## 2.2 A generator of bounded tracking trajectories

We propose an alternate, non-iterative way to calculate an exact solution of system (7) when the forcing term is periodic, based on the next result. Let us define the signal

$$f(t) = \left[ -\dot{z}_{2d}(t) + \frac{z_{2d}(t)}{Q} \right] z_{2d}(t). \quad (8)$$

**Lemma 1** *Let  $f(t)$  (equation (8)) be a smooth and positive function such that  $f(t) \in [m, M]$  for all  $t \geq 0$ . If  $z_{1d}(0) > 0$ , then the trajectories of the time-reversed system of the internal dynamics (7) are bounded.*

**Proof:** The time-reversed system of (7) is given by

$$\dot{z}_{1d} = -1 + \frac{f(t)}{z_{1d}}. \quad (9)$$

First note that, because  $f$  is positive, the negative real line ( $z_{1d} < 0$ ) is invariant; in fact, in this case the trajectories exploit. Now suppose that at a certain, arbitrary time  $t_0 \geq 0$  we have  $0 < z_{1d}(t_0) < m$ . Therefore  $\dot{z}_{1d}(t_0) = -1 + f(t_0)/z_{1d}(t_0) > 0$  and  $z_{1d}(t)$  increases for  $t > t_0$  at least until it reaches the value  $m$ . On the other hand, if  $z_{1d}(t_0) > M$  then  $\dot{z}_{1d}(t_0) < 0$  and  $z_{1d}(t)$  decreases for  $t > t_0$  at least until it reaches the value  $M$ . Therefore, any trajectory initiating in  $z_{1d}(0) > 0$  converges and eventually enters into the interval  $[m, M]$  [3], hence it is bounded. ■

A special case arises when  $f$  is periodic, with period one (this can be obtained via a time scaling). A known result is that, if the solution of equation (9) is bounded, then it is periodic, with period one [3]. In this case this lemma can be established in a different way.

**Lemma 2** *Let  $f(t) \in [m, M]$  be a smooth, 1-periodic, positive function. Then the solution of system (9) converges asymptotically to a periodic trajectory provided that  $z_{1d}(0) = z_{1d}^0 > 0$ .*

**Proof:** The proof is in the same lines as above; the only point to be proved for asymptotically convergence is the uniqueness of the periodic solution. Let  $\varphi(t, 0, z_{1d}^0)$  be a periodic solution of (9). Then the derivative of the Poincaré map is given by (see [3])

$$\Pi'(z_{1d}^0) = \exp \left[ \int_0^1 \frac{\partial}{\partial z_{1d}} \left( -1 + \frac{f(t)}{z_{1d}} \right) \Big|_{z_{1d}=\varphi(t,0,z_{1d}^0)} dt \right],$$

that is

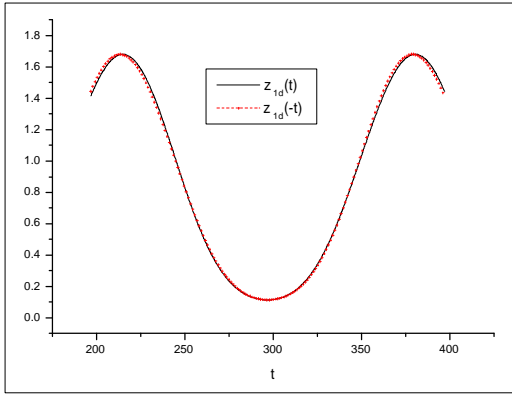
$$\Pi'(z_{1d}^0) = \exp \left[ - \int_0^1 \frac{f(t)}{[\varphi(t,0,z_{1d}^0)]^2} dt \right] < 1,$$

so  $\varphi(t, 0, z_{1d}^0)$  is the unique, asymptotically stable periodic solution of (9). ■

The usefulness of these results is that, under certain conditions, system (9) provides bounded, tracking signals for the inductor current. In fact, for this purpose it is necessary that, in steady state, the solution of system (9),  $\varphi(\tau)$ , be also a solution of system (4),  $\varphi(t)$ , where  $\tau = -t$ , that is,  $\varphi(t) = \varphi(-t)$  in an adequate time axis. It is not difficult to see that a condition for having this situation is that there must exist a certain number  $\delta$  such that the forcing delayed function  $f(t - \delta)$  be even. This is the case for the function  $f$  defined by equation (8).

### 2.3 Simulation results

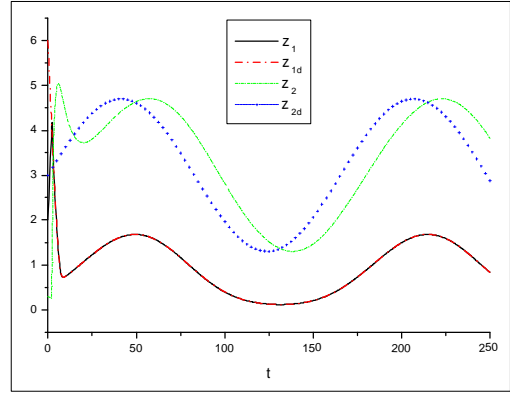
Let us consider the application of this scheme to the control of a boost circuit used to generate an AC voltage. To illustrate this application, we consider the control law (6), where  $z_{1d}$  is generated by the model (9), and the parameters were set to  $R = 48$ ,  $C = 28.2\mu F$ ,  $L = 0.36mH$ , and  $E = 50V$ , which represent practical values. The desired capacitor voltage is taken as  $x_{2d}(t) = 150 + 85 \sin(120\pi t)$ . With these values the parameters of the normalized model are given by  $Q = 13.4343$ ,  $z_{2d}(t) = 3 + 1.7 \sin(0.0379845t)$ . Figure 2 shows the symmetry of the steady state solution of the reversed-time system (9).



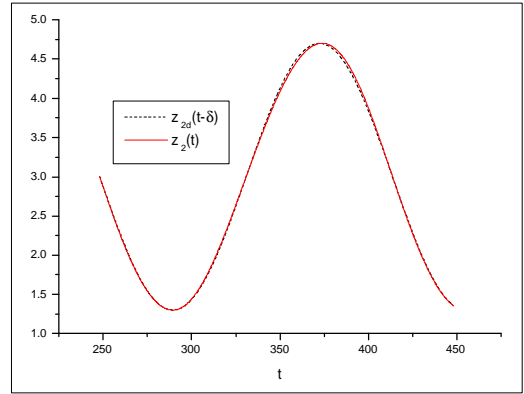
**Figure 2:** Symmetry of the steady state solution of the boost internal dynamics.

Figure 3 shows the normalized signals  $z_1$  and  $z_2$ , and their corresponding references  $z_{1d}$  and  $z_{2d}$ . The initial condition of the reference signal generator (9) was chosen far from practical values to show the speed of convergence towards the desired reference.

Note that, because the capacitor voltage  $z_2$  is indirectly controlled through the inductor current, a phase error cannot be avoided. However, this fact does not have any practical importance. Finally, figure 4 shows the controlled output capacitor voltage and its reference adequately delayed to show the perfect tracking achieved by this controller.



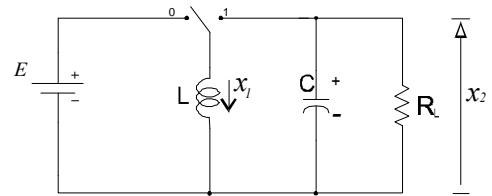
**Figure 3:** References ( $z_{id}$ ) and system states ( $z_i$ ) of the normalized boost circuit.



**Figure 4:** Steady state capacitor voltage and delayed tracking signal obtained from the normalized boost circuit.

### 3 DC-to-AC conversion with the buck-boost converter

The ideas discussed before can also be successfully applied to the buck-boost converter, shown in figure 5.



**Figure 5:** Schematic diagram of the buck-boost circuit.

The switching model, which can also be used as an average model, is given by

$$\begin{aligned} L\dot{x}_1 &= ux_2 + E(1-u), \\ C\dot{x}_2 &= -ux_1 - \frac{x_2}{R}, \end{aligned} \quad (10)$$

where the variables have the same meaning than in the boost converter. The normalized model is obtained by

defining the same variables; the only exception is  $z_2$ , that is defined as  $z_2 = x_2/E - 1$ , giving the model

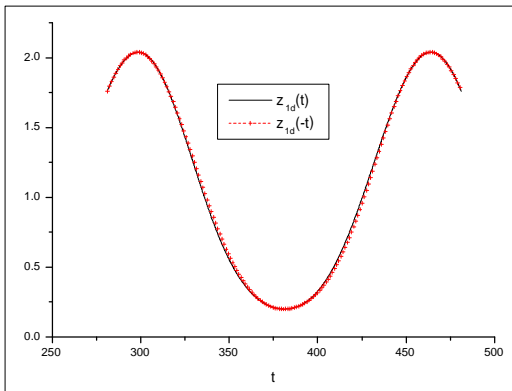
$$\begin{aligned} \dot{z}_1 &= uz_2 + 1, \\ \dot{z}_2 &= -uz_1 - \frac{z_2 + 1}{Q}. \end{aligned} \quad (11)$$

It is not difficult to show that  $z_2$  is a non-minimum phase output and that  $z_1$  is minimum phase [7], hence we have the same problem than in the boost circuit and must control indirectly the capacitor voltage  $z_2$  by controlling the inductor current  $z_1$ . To find an adequate tracking signal for  $z_1$  that makes the output  $z_2$  converge to a desired reference  $z_{2d}$  we put  $z_2 = z_{2d}$  and eliminate the control input from equation (11), obtaining the system

$$\dot{z}_{1d} = 1 - \frac{f_3(t)}{z_{1d}}, \quad (12)$$

where  $f_3(t) = z_{2d}(t) \{ \dot{z}_{2d}(t) + [z_{2d}(t) + 1]/Q \}$ . This equation differs from equation (4) only in the forcing term; therefore, we can use exactly the same technique proposed in the previous section to generate a suited reference for the inductor current yielding a desired steady state response for the capacitor voltage of this circuit.

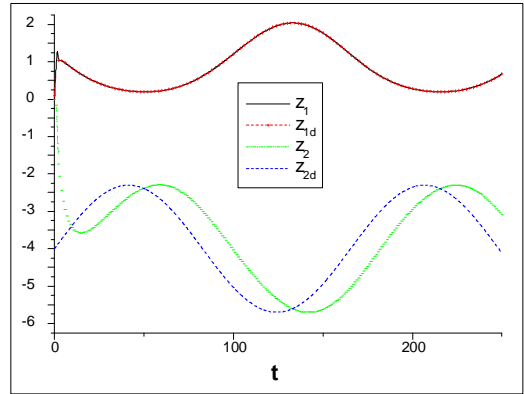
To illustrate this procedure, we show some results obtained from system (11), for a value  $Q = 13.4343$  and a reference signal  $z_{2d}(t) = -4 + 1.7 \sin(0.0379845t)$ . Figure 6 shows the symmetry of the steady state solution of system (12).



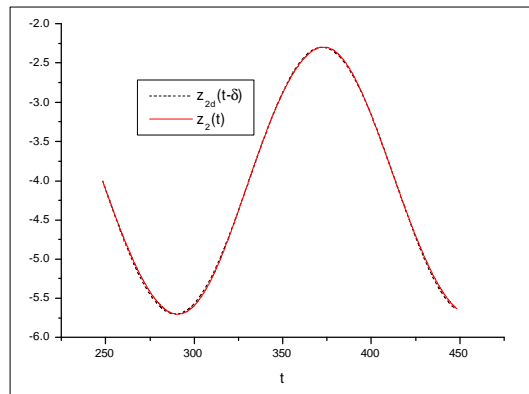
**Figure 6:** Symmetry of the steady state solution of the buck-boost internal dynamics.

As it is shown in [7], the same control law (6) yields a sliding mode along  $\sigma = z_1 - z_{1d} = 0$ . Figure 7 shows the normalized states and desired references for this circuit when this control law is applied. It can be noted a similar result than in the previous case.

Finally, figure 8 shows the controlled output capacitor voltage and the reference with an adequate time delay to show the perfect tracking.



**Figure 7:** References ( $z_{id}$ ) and states ( $z_i$ ) of the normalized buck-boost circuit.



**Figure 8:** Steady state capacitor voltage and delayed tracking signal obtained from the normalized buck-boost circuit.

## 4 Conclusions

In this paper we have discussed how the very simple procedure of time reversal can adequately solve an important problem arising in the control of two popular DC-to-DC converters used to generate sinusoidal voltages. The main problem is to find a solution of an intrinsically unstable, non-autonomous, nonlinear, first order system describing the internal dynamics of a controlled system. This problem appears in a wide class of techniques proposed to control these circuits to generate an AC output voltage, due to the non-minimum phase characteristic of this variable. An important fact that makes it possible to solve easily this problem is that the internal dynamics of the system is unidimensional, a situation that occurs at least with the two widely used circuits analyzed here, that is, the boost and the buck-boost converters. The application of the proposed procedure permits to extend the range of applicability of these devices, offering the possibility to build simple and, in consequence, less expensive circuits to solve this important problem in the field of power electronics. The same technique could be used to control (indirectly) other kind of non-minimum phase

systems with tracking tasks; it seems that a sufficient condition is that the internal dynamics have only one repeller.

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