Design of A Dual-stage Actuator Control System with Discrete-Time Sliding Mode for Hard Disk Drives

Seung-Hi Lee, Sang-Eun Baek, and Young-Hoon Kim
Nano Systems Lab. Samsung Advanced Institute of Technology
P.O.Box 111, Suwon 440-600, KOREA
shl, sebaek, younghoon@sait.samsung.co.kr

Abstract

This paper considers discrete-time design of a dual-stage actuator control system with sliding mode for hard disk drives. A state estimator based sliding mode controller is designed in the discrete time domain. A design methodology is also addressed to complete the design of the control system. The performance of the proposed sliding mode control algorithm is tested through simulations. Very fast seek in microactuator range is attained while not exciting the microactuator resonance. Through the results of simulations it is demonstrated that the proposed discrete-time sliding mode control system is a very prospective technique for disk drives with dual-stage actuators.

1 Introduction

Dual-stage actuator control is a prospective actuator-control technology for high track density disk drives. Dual-stage actuator (DSA) consists of a voice coil motor (VCM) for coarse positioning and a microactuator for fine positioning. As track density increases, it is necessary to increase control bandwidth and to suppress the effect of high frequency disturbances. However, considering its characteristics (a double integrator with structural resonances), it is difficult to achieve higher control bandwidth using VCM. Using a DSA control system is one approach to overcome this problem. It is also possible to achieve improved tracking accuracy and very fast track seek in microactuator range.

However, using DSA does not automatically mean control performance improvement unless good control systems are provided. Issues in DSA control include: i) to reduce destructive interference between the two actuators; ii) to reduce the effect of microactuator resonance; iii) to reduce the effect of microactuator uncertainties (e.g. nonlinearity, unmodeled dynamics, parameter change). We observed that the microactuator controller should be designed in such a way that the resonance of microactuator is not excited since, in general, it is not much lower than the Nyquist frequency of DSA control system.

Separate design of VCM and microactuator controller is a very accessible approach. In [5], a master-slave type separate design of DSA control was presented using a piezoelectrically driven microactuator. In [11], they considered multi-input-multi-output (MIMO) control design as a sequence of single-input-single-output (SISO) design assuming very little interaction between the SISO loops. More interaction between the subsystems causes more performance degrade. There has been some work demonstrating advantage of multi variable control over separate control design. In [3], a track-following controller was developed in the continuous time domain using μ-synthesis methodology. This approach most likely results in a very high order control system. In [4], a discrete-time LQG/LTR dual-stage controller was designed. The results of experiment using laser doppler vibrometer (LDV) are presented.

Considering its characteristics, the sliding mode control [1, 7, 12] is a good candidate for microactuator control. In sliding mode control, the trajectory of plant can be enforced to slide on a certain hyperplane not to excite undesirable resonance. Also robustness of a control system against parameter change can be increased. In [8], they presented design of a servomechanism with sliding mode. A discrete-time sliding mode control is designed for settling and tracking as to enforce the head to slide on an extended linear velocity profile, which is used as a hyperplane. Results of simulations and drive level experiments are presented. In [14], a sliding mode controller is designed in the continuous time domain and applied to VCM control for disk drives. They applied adaptive sliding mode control using a reference velocity profile as a sliding surface. However, there has been no work on discrete sliding mode control system design for dual-stage actuator disk drives.

This paper considers discrete-time sliding mode control system design of a dual-stage actuator for hard disk drive control system. Using a compact dual-stage actuator model [10], we design a multi-input-single-output (MISO) DSA control system with sliding mode. As an application study, a discrete-time sliding mode DSA control system is designed for a piezoelectrically actuated microactuator developed by Hutchinson Technology Institute (HTI). Finally, the performance of the proposed DSA control system is tested through simulations. The results of simulations are presented.

Throughout the paper, I denotes the identity matrix and $\rho(M)$ denotes the spectral radius of M. Also, it is assumed that all the matrices have appropriate dimensions such that matrix manipulation works.

2 Modeling Dual-stage Actuators

Dual-stage actuators can be modeled either in multi-input-multi-output (MIMO) or in multi-input-single-output (MISO). MISO control requires only the head position which represents the combined position, while MIMO control requires additional sensors to measure the relative position error signal (RPES) between the two actuators, which is in general very difficult. Thus, in the paper, we focus on MISO control design. However, the proposed control system design method can also be applied to MIMO control design.

A MISO dual-stage actuator model [10] is described by

$$G_p = \left[\begin{array}{cc|cc} A_v & 0 & B_v & 0 \\ 0 & A_m & -B_{mv} & B_m \\ \hline C_v & C_m & 0 & 0 \end{array}\right] \qquad (1)$$

where

$$A_v = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_v = \begin{bmatrix} 0 \\ (l_v + l_m) K_v / J_v \end{bmatrix},$$

$$A_m = \begin{bmatrix} 0 & 1 \\ -k_m/J_e & -b_m/J_e \end{bmatrix}, \quad B_{mv} = B_v l_m / (l_v + l_m),$$

$$B_m = \begin{bmatrix} 0 \\ l_m K_m / J_e \end{bmatrix}, \quad C_m = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_v = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Here, x_v is the position of VCM, x_m is the relative position error between the two actuators, u_v and u_m are the controls applied to VCM and microactuator, and y is the output measurement, respectively. Also, l_v (l_m) denotes the length of VCM (microactuator), k_m and b_m denote the stiffness and damping factor of microactuator, K_v (K_m) denotes the torque constant of VCM (microactuator), J_v (J_m, J_a) denotes the inertia of VCM (microactuator, DSA), and $J_e = J_a J_m / (J_a + J_m)$. Finally, we let $x = \begin{bmatrix} x_v & \dot{x}_v & x_m & \dot{x}_m \end{bmatrix}^T$, the vector of states, and $u = \begin{bmatrix} u_v & u_m \end{bmatrix}^T$, the vector of controls.

For a sampling period of T_s, the discrete-time equivalent of the dual-stage actuator model is expressed by

$$G_p = \left[\begin{array}{c|c} \Phi_p & \Gamma_p \\ \hline C_p & 0 \end{array}\right] = \left[\begin{array}{cc|cc} \Phi_v & 0 & \Psi_v B_v & 0 \\ 0 & \Phi_m & -\Psi_m B_{mv} & \Psi_m B_m \\ \hline C_v & C_m & 0 & 0 \end{array}\right] \qquad (2)$$

where

$$\Phi_p = e^{A_p T_s}, \; \Phi_m = e^{A_m T_s},$$

$$\Psi_v = \int_0^{T_s} e^{A_v t} dt, \; \Psi_m = \int_0^{T_s} e^{A_m t} dt.$$

Also, note that

$$\Gamma_v = \Psi_v B_v, \; \Gamma_m = \Psi_m B_m.$$

It can be found easily that (Φ_p, Γ_p, C_p) is stabilizable and detectable. We let, for notational convenience, $\Gamma_{mv} = \Phi_m B_{mv}$. It is useful to note that $\Gamma_{mv} = \Gamma_m K_v J_e / (K_m J_v)$. In the case of $K_v J_m / (K_m J_v) \ll |u_m / u_v|$ in the effective operating region, the contribution of Γ_{mv} becomes negligible.

3 Design of Dual-stage Actuator Control System with Estimator Based Sliding Mode

This section is dedicated to design a DSA control system using discrete-time sliding mode based on the discrete-time model of dual-stage actuator (2). In the design of a DSA control system, cooperation between VCM and microactuator control is very important for improved control performance. Also, a DSA control system should be designed such that VCM works even in the case that microactuator fails. Considering the different characteristics of the two actuators, it is natural to use different control ideas. By introducing sliding mode in the design of dual-stage actuator control systems, such design concepts can be realized with increased robustness. The structure of the sliding mode DSA control system is shown in Figure 1.

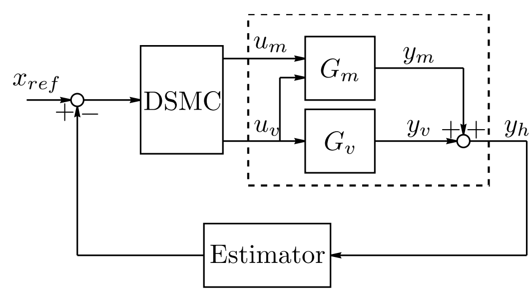

Figure 1: MISO dual-stage control system

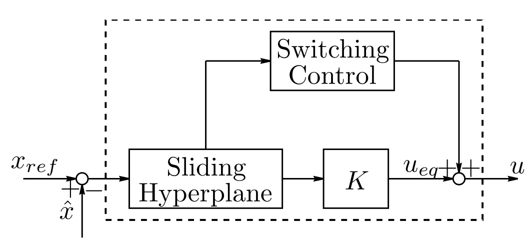

Figure 2: Discrete sliding mode controller (DSMC)

3.1 Design of State Estimator

The current estimator consists of update and prediction part expressed by

$$\begin{aligned} \hat{x}(k) &= \bar{x}(k) + L_c\left(y(k) - C_p\bar{x}(k)\right) \\ \bar{x}(k+1) &= \Phi_p\hat{x}(k) + \Gamma_p u(k) \end{aligned} \quad (3)$$

where $\bar{x}(k)$ and $\hat{x}(k)$ denote the state prediction and the state update by the output measurement $y(k)$. The gain L_c of the estimator can be determined by $\mathcal{H}_2$ design method. The current estimator gain L_c that minimizes the estimation error in the $\mathcal{H}_2$ sense with given weightings can be obtained by solving the discrete algebraic Riccati equation

$$Y - \Phi_p Y \Phi_p^T + \Phi_p Y C^T \left(V_2^T + C_p Y C_p^T\right)^{-1} C_p Y \Phi_p^T - V_1^T = 0 \quad (4)$$

where V_1 and V_2 are weightings. The estimator gain L_c is computed by

$$L_c = Y C^T \left(C_p Y C_p^T + V_2^T\right)^{-1} \quad (5)$$

where $Y > 0$ is the solution of (4). Frequency shaping functions can also be introduced to adjust the frequency characteristics of the estimator.

3.2 Design of Sliding Mode Controller

An observer based sliding dynamical sequence in the state space is defined as

$$s(k) = G\hat{x}(k) + r(k). \quad (6)$$

Here, $r(k)$ is the reference and

$$s(k) = \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix}, \; G = \begin{bmatrix} G_1(k) \\ G_2(k) \end{bmatrix}$$

where

$$G_i = \begin{bmatrix} G_{i1} & G_{i2} & \cdots & G_{in} \end{bmatrix}, \quad i = 1, 2.$$

Hereafter, we assume that G is selected such that $G\Gamma_p$ is nonsingular.

From the condition for the existence of the ideal sliding mode, $s(k+1) = s(k) = 0$, the equivalent control

$$u_{eq}(k) = -\left(G\Gamma_p\right)^{-1} G\Phi_p\hat{x}(k) \quad (7)$$

is obtained. The reduced order system in the sliding mode is

$$\hat{x}(k+1) = \left(\Phi_p - \Gamma_p\left(G\Gamma_p\right)^{-1} G\Phi_p\right)\hat{x}(k). \quad (8)$$

For notational convenience, we let

$$K_c = \left(G\Gamma_p\right)^{-1} G\Phi_p \quad (9)$$

and

$$\Phi_{eq} = \Phi - \Gamma K_c = \Phi_p - \Gamma_p\left(G\Gamma_p\right)^{-1} G\Phi_p.$$

We design G such that the equivalent system is stable, ie. the spectral radius $\rho\left(\Phi_{eq}\right) < 1$.

If the *convergent sliding mode* [1, 7, 12] is not the ideal sliding mode, then the sliding points lie inside the layer after a certain time depending upon the layer width.

Corollary 1 [12] *A sufficient condition for the existence of the discrete sliding mode is that there exists a positive integer k_o such that*

$$|s_i(k+1)| \le \alpha\,|s_i(k)|\,, \; 0 \le \alpha < 1, \; 1 \le i \le 2, \; k \ge k_o \quad (10)$$

in the region $\mathcal{N}_\epsilon = \{|s_i(k)| = |G_i\hat{x}(k)| < \epsilon, \; 1 \le i \le 2\}$.

The sufficient condition (10) is equivalent to

$$-\alpha\,|s_i(k)| \le s_i(k+1) \le \alpha\,|s_i(k)|\,, \; 0 \le \alpha < 1, \; 1 \le i \le 2.$$

Here, we let (assuming $s_i(k) \ne 0$)

$$\alpha_i(k) = \frac{s_i(k+1)}{|s_i(k)|} = \frac{G_i\Gamma_p\left(-u_{eq}(k) + u(k)\right)}{|s_i(k)|}, \; 1 \le i \le 2 \quad (11)$$

and

$$\alpha(k) = \operatorname{diag}\left[\alpha_1(k), \alpha_2(k)\right].$$

Then, clearly a sufficient condition for the existence of the sliding mode is $|\alpha(k)| < I$. From (11) the control is

$$u(k) = u_{eq}(k) + \left(G\Gamma_p\right)^{-1}\alpha(k)|s(k)|.$$

Let

$$M(k) = \operatorname{diag}\left[\operatorname{sgn}s_1(k), \operatorname{sgn}s_2(k)\right].$$

Then, the control is expressed by

$$u(k) = u_{eq}(k) + \left(G\Gamma_p\right)^{-1}\alpha(k)M(k)G\hat{x}(k). \quad (12)$$

In the above formulation we assumed $s_i(k) \ne 0$. However, notice that (12) is still valid for $s_i(k) = 0$. Here, we introduce a constant diagonal matrix W such that

$$\alpha(k) = WM(k).$$

Then

$$u(k) = u_{eq}(k) + \left(G\Gamma_p\right)^{-1} WG\hat{x}(k). \quad (13)$$

The matrix W is determined to make the discrete-time closed loop system be stable with desired behavior [8].

Theorem 1 *Suppose that L_c and G are determined such that $\rho\left(\Phi_p - \Phi_p L_c C_p\right) < 1$ and $\rho\left(\Phi_{eq}\right) < 1$. Then, there always exist some diagonal matrices W such that $\rho\left(\Phi_{eq} + \left(G\Gamma_p\right)^{-1} WG\right) < 1$, and moreover, the discrete-time closed loop system is stable. In this case, there exists a convergent discrete sliding mode with the matrix W.*

Proof: Proof is not presented here due to space limitation. ■

Corollary 2 *Suppose that* $\rho\left(\Phi_{eq} + (G\Phi_p)^{-1} WG\right) < 1$. *Then, there always exist some diagonal matrix* Ξ, $\Xi > 0$, *such that* $\rho\left(\Phi_{eq} + \Xi\Gamma_p K_c + (G\Phi_p)^{-1} WG\right) < 1$.

In the design of a dual-stage actuator control system, we should make VCM work even in the case that microactuator fails. Therefore, we use a sliding hyperplane expressed by

$$G = \begin{bmatrix} \Lambda_v & 0 \\ \Lambda_{mv} & \Lambda_m \end{bmatrix} \tag{14}$$

where

$$\Lambda_v = \begin{bmatrix} G_{11} & G_{12} \end{bmatrix},\ \Lambda_m = \begin{bmatrix} G_{23} & G_{24} \end{bmatrix}$$

and

$$\Lambda_{mv} = \begin{bmatrix} G_{21} & G_{22} \end{bmatrix}.$$

In this case, we have

$$s(k) = G\hat{x}(k) = \begin{bmatrix} \Lambda_v \hat{x}_v(k) \\ \Lambda_m \hat{x}_m(k) + \Lambda_{mv}\hat{x}_v(k) \end{bmatrix}.$$

Note that the VCM part of the sliding hyper plane is independent of the microactuator part.

The controller gain K_c is expressed by

$$K_c = \begin{bmatrix} K_{cv} & 0 \\ K_{cmv} & K_{cm} \end{bmatrix} = (G\Gamma_p)^{-1} G\Phi_p$$

where

$$K_{cv} = (\Lambda_v\Gamma_v)^{-1} \Lambda_v\Phi_v, \tag{15}$$

$$K_{cm} = (\Lambda_m\Gamma_m)^{-1} \Lambda_m\Phi_m, \tag{16}$$

and

$$K_{cmv} = (\Lambda_m\Gamma_m)^{-1} \left(\Lambda_m\Gamma_{mv}K_{cv} + \Lambda_{mv}\left(\Phi_v - \Gamma_v K_{cv}\right)\right). \tag{17}$$

In the case of $K_v J_m / (K_m J_v) \ll |u_m/u_v|$ in the effective operating region, we can neglect the effect of the term Γ_{mv} as an approximation for simple algorithm computation. In this case, we have a simpler expression

$$K_{cmv} = (\Lambda_m\Gamma_m)^{-1} \Lambda_{mv}\left(\Phi_v - \Gamma_v K_{cv}\right). \tag{18}$$

The validity of such approximation is surely dependent on plant parameters as well as specific control algorithms used, and thus it should be verified later.

Once we have a properly determined sliding hyper plane G, we can compute control gains K_c from Equations (15), (16), and (17) (or (18)). Then, we have

$$u_v(k) = K_{cv}\hat{x}_v(k)$$

and

$$u_m(k) = K_{cm}\hat{x}_m(k) + K_{cmv}\hat{x}_v(k).$$

Using $\Lambda_{mv} = 0$ results in $K_{cmv} \approx 0$. However, Λ_{mv} that yields $K_{cmv} = 0$ may not exist because $\Phi_v - \Gamma_v K_{cv}$ is always singular in the discrete-time sliding mode control with a hyper plane defined by (14).

4 Application Example

The proposed discrete-time sliding mode DSA control system is applied to a hard disk drive with a dual-stage actuator with a piezoelectric microactuator developed by HTI. The disk drive has 25,000 tracks per inch (TPI) and a sampling frequency of 18 kHz. The parameters of the dual-stage actuator are listed in Table 1. The HTI piezoelectric microactuator has lightly damped resonance at 6.2 kHz.

Table 1: Parameters of the dual-stage actuator

K_v	:	0.796 N-m/amp;
K_m	:	7.7267e-5 N-m/V;
J_v	:	32.9 gm-cm^2;
J_m	:	0.329 gm-cm^2;
l_v	:	1.4 inch;
l_m	:	0.5669 inch;
k_m	:	49.927 N-m;
b_m	:	1.1392e-3 N-m-s.

4.1 Controller Design with Sliding Mode

Given a sampling frequency of 18 kHz, a scaled discrete-time model is developed for control system design. From (5) we first obtain the current state estimator gain $L_c = \begin{bmatrix} 0.34158 & 2156 & 0.64547 & -8651 \end{bmatrix}^T$ such that $\rho\left(\Phi_p - \Phi_p L_c C_p\right) = 0.59534,\ 0.59534,\ 0.17766,\ 0.17766$.

The sliding hyperplane is expressed by

$$s(k) = G\hat{x}(k) + r(k)$$

where $r(k) = G\Phi_p r_o(k-1)$ with

$$r_o(k) = \begin{bmatrix} 0 & 0 & \beta\left(p_r(k) - \hat{x}_1(k)\right) - \mu\hat{x}_3(k) & 0 \end{bmatrix}^T.$$

The reference $r(k)$ is introduced for dynamic adaptation of the sliding hyper plane for the microactuator to fast position error while not exciting the microactuator resonance. Figure 5 shows the resulting sliding hyper planes for VCM and microactuator. We observe that the sliding hyper plane for microactuator is dynamically adapting according to position error. Here, p_r is the position reference, β is the discount factor to adjust microactuator's response to VCM position error, μ is a factor to adjust microactuator's slew-rate. Then the equivalent control becomes

$$u_{eq}(k) = -(G\Gamma_p)^{-1} G\Phi_p\left(\hat{x}(k) + r_o(k)\right).$$

Similar results may be obtained by designing Λ_{mv} to obtain K_{cmv}. However, considering that $\Phi_v - \Gamma_v K_{cv}$ is singular and thus limits spanning of K_{cmv} by Λ_{mv}, the former approach is used.

A reference velocity profile with linear extension [9] is used to design a hyperplane in the sliding mode control of VCM. From the shape of the extended linear profile we choose $\Lambda_v = \begin{bmatrix} 4396.4 & 1 \end{bmatrix}$ to obtain $K_{cv} = \begin{bmatrix} 1394.3 & 0.3946 \end{bmatrix}$ from (15). Then, we have $\rho(\Phi_v - \Gamma_v K_{cv}) = 0.782, 0$. For microactuator control design we use Λ_{mv} and Λ_m defining the sliding hyperplane for the microactuator. Considering the fact that $K_v J_m / (K_m J_v) = 10.2 \ll |u_m/u_v|$ in the effective operating region, we use $\Lambda_{mv} = \begin{bmatrix} 0 & 0 \end{bmatrix}$ and $K_{cmv} = \begin{bmatrix} 0 & 0 \end{bmatrix}$ as an approximation for simple algorithm computation. The validity of the approximation is verified through simulations. Choosing $\Lambda_m = \begin{bmatrix} 4.5796e-9 & 1 \end{bmatrix}$ we obtain $K_{cm} = \begin{bmatrix} -1.132e6 & -20.864 \end{bmatrix}$ from (16). In this case, we obtain $\rho(\Phi_m - \Gamma_m K_{cm}) = 0.999, 0$. Also, the equivalent system yields $\rho(\Phi_p - \Gamma_p K_c) = 0.782, 0, 0.999, 0$. Here, the two nonzero eigenvalues are associated with the reduced order system for which eigenvalues can be assigned, and the last two zero eigenvalues are associated with the zero eigenvalue system.

In the selection of W, we need to choose W such that the trajectory converges to the sliding hyperplane sufficiently fast without possible control chattering. We simply choose $W = \mathrm{diag}[0.001, 0.0001]$ for sufficiently fast convergence as well as negligible chattering in the existence of simulated measurement noises. Also, we can check that the spectral radius $\rho\left(\Phi_{eq} + \Gamma_p (G\Gamma_p)^{-1} WG\right) = 0.782, 0.001, 0.0001, 0.999$.

We choose β and μ such that the time and frequency responses of closed loop system are satisfactory and $\rho\left(\Phi_{eq} + \Xi\Gamma_p K_c + (G\Phi_p)^{-1} WG\right) < 1$. There always exist some β and μ satisfying the above condition as stated in Corollary 2. For $\beta = 0.1$ and $\mu = 0.17$ we obtain satisfactory response to fast position error without microactuator resonance excitation. Also, for $\Xi = \mathrm{diag}\begin{bmatrix} 0 & 0 & \mu & 0 \end{bmatrix}$ we have $\rho\left(\Phi_{eq} + \Xi\Gamma_p K_c + (G\Phi_p)^{-1} WG\right) = 0.782, 0.491, 0.491, 0.001$.

The resulting equivalent system shows a cross-over frequency of 1.3 kHz, a phase margin of 63 degree, and a gain margin of 12 dB. We expect the resulting DSA control system to yield near minimum destructive interference (60 degree phase margin [11]). Figure 3 shows the sensitivity and complementary sensitivity plots of the equivalent system. We observe that the sensitivity has its peak value 1.5 (3.5 dB) while the complementary sensitivity has unit magnitude without a peak up to near the closed loop bandwidth of the equivalent system.

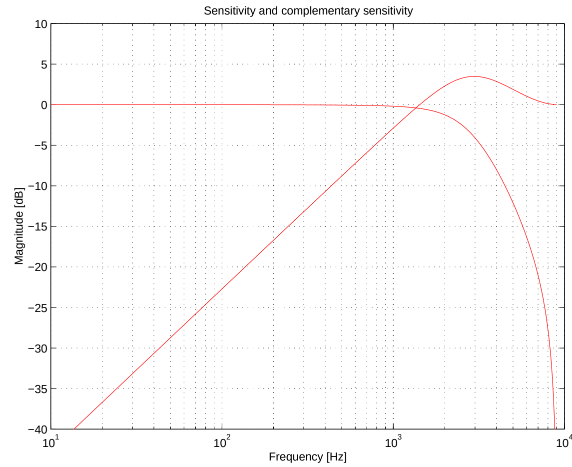

Figure 3: Sensitivity and complementary sensitivity of the equivalent system

We have completed DSA control system design with sliding mode, which is used for track follow as well as seek in microactuator range. The equivalent system is designed for good time and frequency response via the design of sliding hyper planes. W is selected such that the plant trajectories converge to the hyper planes fast enough not while exciting the microactuator resonance. The resulting discrete-time closed loop system is guaranteed to be stable according to Theorem 1 and Corollary 2.

4.2 Simulation Results

Figure 4 shows the result of single track seek in microactuator range. It is shown that the microactuator moves ahead of the VCM to reduce position error signal (PES). As a result, very fast single track seek is achieved with microactuator ontrol. The head moves into 5 % of the target track in 0.25 msec. Also, both RPES and VCM position error signal (VPES) settle down in 1 msec and are ready for another seek. Figure 5 shows phase plane of single track seek simulation. We observe that VCM is sliding down the fixed hyper plane $s_1(k)$. On the other hand, the microactuator is moving along the the microactuator hyper plane $s_2(k)$ which is also moving according to the reference $r(k)$. We observed that the behavior of VCM and microactuator can be simply adjusted by choosing appropriate sliding hyperplanes.

5 Conclusion

This paper has considered discrete-time design of a DSA control system with sliding mode for hard disk drives. A discrete-time sliding mode control algorithm

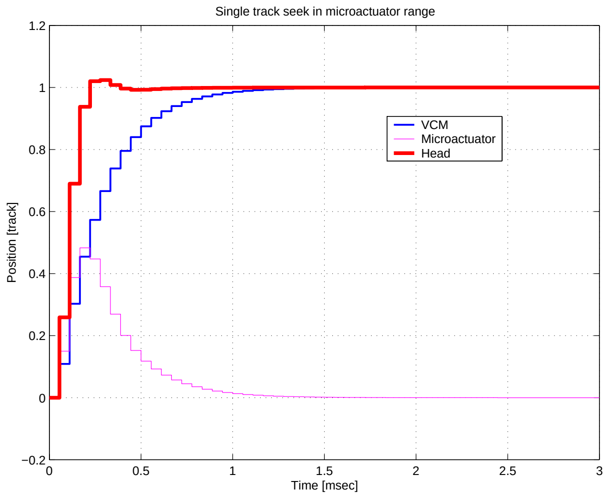

Figure 4: Time response of single track seek in microactuator range; Both VPES and RPES settle in 1 msec; Very fast seek is attained with DSA

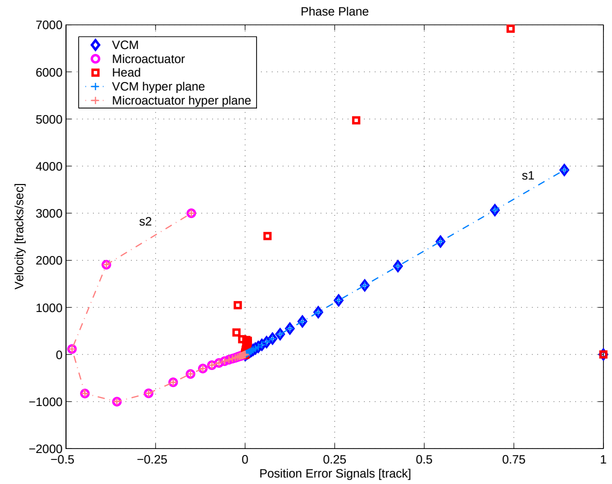

Figure 5: Phase plane of single track seek simulation: s_1 and s_2 denote sliding hyper planes of VCM and microactuator, respectively; s_1 is fixed while s_2 is dynamically adapted according to position error

was proposed for dual-stage actuator control. In addition, a design methodology was addressed to complete the design of the control system. A state estimator based sliding mode control was designed in the discrete time domain. The performance of the proposed sliding mode control algorithm was tested to show very fast seek in microactuator range while not exciting the microactuator resonance. Through the results of simulations it was demonstrated that the proposed discrete-time sliding mode control system is a very prospective technique for disk drives with dual-stage actuators.

References

[1] S. Drakunov and V. Utkin, *Sliding Mode Control in Dynamic Systems*, International Journal of Control, v. 55, 4, pp. 1029-1037, 1992.

[2] C. Edwards and S. K. Spurgeon, *Sliding Mode Control: Theory and Applications*, Taylor & Francis, 1998

[3] D. Hernandez, S.-S. Park, R. Horowitz, A. K. Packard, *Dual-Stage Track-Following Servo Design for Hard Disk Drives*, Proc. of American Control Conference, 1999.

[4] X. Hu, W. Guo, T. Huang, and B. M. Chen, *Discrete-Time LQG/LTR Dual-Stage Controller Design and Implementation for High Track Density HDDs*, Proc. of American Control Conference, 1999.

[5] S. Koganezawa, Y. Uematsu, and T. Yamada, *Dual-Stage Actuator System for Magnetic Disk Drives Using a Shear Mode Piezoelectric Microactuator*, IEEE Trans. on Magnetics, Vol. 35, No. 2, March 1999.

[6] A. J. Koshkouei and S. I. Zinober, *Discrete-Time Sliding Mode Control Design*, Proc. of IFAC, pp 481-486, 1996.

[7] C. Milosavljevic, *General Conditions for the Existence of a Quasisliding Mode on the Switching Hyperplane in Discrete Variable Structure Systems*, Automation and Remote Control 46(3), pp 307-314, 1985.

[8] S.-H. Lee, S.-E. Baek, and C. C. Chung, *Design of A Servomechanism With Sliding Mode for Disk Drives*, Proc. of IEEE Conference on Decision and Control, pp. 5253-5258, 1999.

[9] S.-H. Lee, S.-H. Chu, and C. C. Chung, *Analysis and Design of Servomechanism and Its Application to Disk Drives*, submitted to IEEE Trans. on Control Systems Technology.

[10] S.-H. Lee, Y.-H. Kim, and S.-E. Baek, *Modeling and Control of a Dual-Stage Actuator for Hard Disk Drive Servo Systems*, Proc. of American Control Conference, 2000.

[11] S. J. Schroeck and W. C. Messner, *On Controller Design For Time-Invariant Dual-Input Single-Output Systems*, Proc. of American Control Conference, 1999.

[12] W.-C. Su, S. Drakunov, and U. Ozguner, *Implementation of Variable Structure Control for Sampled-Data Systems*, chapter in the book Robust Control via Variable Structure and Lyapunov Techniques, Springer-Verlag, v. 217, pp. 87-106, 1996.

[13] V. I. Utkin, *Sliding Modes in Control and Optimization*, Springer-Verlag, 1992

[14] S. Weerasooriya, S., T. S. Low, and A. Al-Mamun, *Design of a Time Optimal Variable Structure Controller for a Disk Drive Actuator*, Proc. of American Control Conference, 1993.