Simultaneous Stabilization with Near Optimal $\mathcal{H}_\infty$ Performance[1]

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Abstract

In this paper we consider the use of linear periodically time-varying (LPTV) controllers for simultaneous stabilization and disturbance rejection. Here we consider the case in which the control signal u does not appear directly in the output to be controlled (z) and the disturbance w does not appear directly in the measured output y. We prove that for every finite set of such plants for which the high-frequency gains (from u to y) satisfy a linear independence condition, we can design an LPTV controller which provides not only closed-loop stability, but also near optimal disturbance rejection in the $\mathcal{H}_\infty$ sense.

1 Introduction

In designing a control system with integrity, one has a nominal model, say P_1, and at some point a structural change, such as a loss of a sensor or actuator, may occur, yielding a new model in the finite set $\{P_2, \cdots, P_q\}$. It is clearly desirable to have a single controller which not only stabilizes every possible model in the finite set, but also provides acceptable performance.

The problem of providing simultaneous stability using a linear time-invariant (LTI) controller is quite well understood [12, 14]. In the simplest case, namely, when there are only two possible systems, easily checked necessary and sufficient conditions are available, but even in that case there are simple situations where no LTI controller exists. For example, no LTI controller will simultaneously stabilize $\{P_1, P_2\}$ with transfer functions of

$$P_1(s) = \frac{1}{s-1}, \quad P_2(s) = \frac{-1}{s-1}. \tag{1}$$

Recent work on linear time-varying control has demonstrated that simultaneous stability can be easily carried out by using a time-varying controller, e.g., see [11], [10], [9], and [1], to name but a few of the approaches. However, none of these provide a guarantee that the closed-loop performance is acceptable. Indeed, the controllers of [10], [9], and [1] are based on generalized holds (GH), and GH based controllers often have poor intersample behavior [7]. A new approach to the problem has been proposed by the first co-author in [13], where it is proven that it is possible to achieve simultaneous stability with near optimal LQR-type performance using a time-varying controller. However, in the problem formulation there is no disturbance, and it is not at all obvious how to extend the approach of [13] to deal with disturbances.

This brings us to our result. Here we measure the performance using the $\mathcal{L}_2$-induced norm or the $\mathcal{H}_\infty$ norm: for each possible model we design a (near) optimal LTI $\mathcal{H}_\infty$ controller and then we show how to design a linear periodically time-varying (LPTV) controller which not only simultaneously stabilizes each model but also provides a level of performance as close as we would like to the optimal one. The approach we adopt is as follows. We periodically probe the system and use this information together with the fact that the high frequency gains from the control input u to the measured output y satisfy a linear independence condition to discern which plant is being controlled; we then apply the appropriate feedback. The resulting controller consists of a bank of integrators to implement the dynamics of the LTI controllers being emulated together with a sampled-data output feedback controller which implements the probing as well as the choice of control signal. Our approach uses fast sampling, and the closer that one wishes to get to optimality, the smaller the sampling period needs to be. Although we impose several stringent conditions on the plant for this approach to work, we emphasize that this is the *first approach to the simultaneous stabilization problem for which near optimal disturbance rejection is guaranteed.*

The notation we use is quite standard. With $A \in \mathbf{R}^{n \times m}$, we use $(A)_i$ to denote the i^{th} column of A.

The paper is organized as follows. In the next section,

[1] This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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we set the stage for the problem, describe a procedure for constructing an LPTV controller and explain why it should work; a rigorous proof is quite involved and hence omitted for this conference version. In Section 3, we study an example with two plants P_1 and P_2 as given in (1), and show that the proposed control scheme produces satisfactory performance. Finally, Section 4 summarizes the main contribution of the paper and offers some concluding remarks.

2 Problem Formulation and Main Result

We will formulate the problem, state the assumptions, and give a detailed description of the controller used.

2.1 Problem Formulation

Our plant model is of the form

$$\begin{array}{rcl} \dot{x}(t) &=& Ax(t)+B_1w(t)+B_2u(t), x(0)=x_0, \\ z(t) &=& C_1x(t)+D_{11}w(t) \\ y(t) &=& C_2x(t), \end{array} \qquad (2)$$

with $x(t)\in\mathbf{R}^n$ the state, $w(t)\in\mathbf{R}^{m_1}$ the disturbance signal, $u(t)\in\mathbf{R}^{m_2}$ the control input, $z(t)\in\mathbf{R}^{r_1}$ the output to be controlled, and $y(t)\in\mathbf{R}^{r_2}$ the measured output; we will associate the system with the 6-tuple $(A,B_1,B_2,C_1,C_2,D_{11})$ or simply P. Notice that we have assumed that there is no weighting on the control signal in the controlled output ($D_{12}=0$) and that the disturbance does not appear directly in the measured output ($D_{21}=0$); both assumptions are crucial for our approach to work. (We notice here that the assumption $D_{21}=0$ is not restrictive considering that we intend to implement the controller in sampled-data form and typically anti-aliasing filters are used prior to sampling [4].) We assume that the plant is not known exactly: it lies in the finite set

$$\{(A^i,B_1^i,B_2^i,C_1^i,C_2^i,D_{11}^i): \quad i=1,\cdots,q\},$$

each of which has the same number of inputs and outputs but possibly different state dimensions; we associate the i^{th} model with P_i. Our standing assumptions are:

Assumption 1: Each (A^i,B_2^i) is stabilizable.

Assumption 2: Each (C_2^i,A^i) is detectable.

Assumption 3: The set of high frequency gains $\{C_2^iB_2^i : \ i=1,\cdots,q\}$ are such that there exists a vector $v\in\mathbf{R}^{m_2}$ and an integer $j\in\{1,2\cdots,q\}$ so that

$$\{(C_2^iB_2^i-C_2^jB_2^j)v: \quad i=1,\cdots,j-1,j+1,\cdots,q\}$$

are linearly independent.

While the first two assumptions are quite natural, the last one requires further explanation. Our goal is to occasionally probe the system with a large constant input for a brief period of time (h) and then look at y; in this case, the change in y is dominated by $hC_2^iB_2^iu(t)$. Hence, since we identify the system by its high frequency gain, it makes sense that there should be a constraint on these quantities. To explain this constraint further, let us examine several special cases.

Example 1 *The dimension 1 case.*

Here we assume that there are two possible plants ($q=2$), each of dimension one ($n_1=n_2=1$), with a scalar control input and a scalar measured output ($m_2=r_2=1$). We may as well assume that $C_2^1=C_2^2=1$, so Assumption 3 requires that

$$B_1^1\neq B_1^2.$$

For example, this would certainly be the case if

$$C_2^1(sI-A^1)^{-1}B_2^1=\frac{1}{s-1},\ \ C_2^2(sI-A^2)^{-1}B_2^2=\frac{-1}{s-1}.$$

Example 2 *Actuator failure case with state feedback.*

Suppose that we have state feedback:

$$C_2^i=C_2=I,\ \ i=1,\cdots,q,$$

and that we have a nominal system P_1 with the remaining systems arising from having exactly one actuator fail, so that $q=m_2+1$. That is,

$$A^i=A,\ \ B_1^i=B_1,$$

and

$$\begin{array}{rcl} B_2^1 &=& \left[\begin{array}{cccc} b_1 & b_2 & \cdots & b_{m_2}\end{array}\right], \\ B_2^2 &=& \left[\begin{array}{cccc} 0 & b_2 & \cdots & b_{m_2}\end{array}\right], \\ B_2^3 &=& \left[\begin{array}{ccccc} b_1 & 0 & b_3 & \cdots & b_{m_2}\end{array}\right], \\ & \vdots & \\ B_2^q &=& \left[\begin{array}{cccc} b_1 & \cdots & b_{m_2-1} & 0\end{array}\right]. \end{array}$$

If $\{b_1,\cdots,b_{m_2}\}$ are linearly independent, then setting $j=1$ and $v=\left[\begin{array}{cccc}1 & 1 & \cdots & 1\end{array}\right]^T$, we see that Assumption 3 is satisfied.

Here we consider linear causal controllers of the form $u=Ky$, with K having a finite-dimensional state-space model. We would like to design the controller K to provide closed-loop stability and to make the closed-loop map from w to z in Figure 1, denoted $\mathcal{F}(P,K)$, as small as possible in the $\mathcal{L}_2$-induced ($\mathcal{H}_\infty$) norm sense; in defining this map, we assume that there are zero initial conditions on the plant and controller. We define closed-loop stability using state models in the usual way: let x and ρ be the plant and controller states, and assume $w = 0$ in Figure 1; closed-loop stability means that for any $x(0) = x_0$ and $\rho(0) = \rho_0$,

$$\begin{bmatrix} x(t) \\ \rho(t) \end{bmatrix} \to 0 \text{ as } t \to \infty.$$

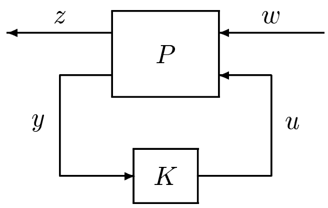

Figure 1: The plant and controller setup.

If there is no plant uncertainty, then we simply design a (near) optimal LTI controller using $\mathcal{H}_\infty$-optimal control theory. The problem is that in the face of plant uncertainty, there may not exist an LTI controller to stabilize all possible plants, let alone provide decent performance. Here the goal is to construct a linear time-varying (finite-dimensional) controller which not only provides closed-loop stability but also provides near optimal performance for each possible model.

The first step is to design a near optimal LTI controller K_i for each plant P_i. So suppose that the following LTI controller, given in state-space form, stabilizes P_i and provides acceptable closed-loop performance:

$$\begin{aligned} \dot{\rho} &= F^i\rho + G^i y, \;\; \rho(0) = \rho_0, \\ u &= H^i\rho + J^i y. \end{aligned} \tag{3}$$

Our standing assumptions are:

Assumption 4: Each (F^i, G^i) is stabilizable.

Assumption 5: Each (H^i, F^i) is detectable.

Now closed-loop stability is equivalent to

$$\begin{bmatrix} A^i + B_2^i J^i C_2^i & B_2^i H^i \\ G^i C_2^i & F^i \end{bmatrix}$$

being stable, since both the plant and controller representations are stabilizable and detectable by assumption.

We may as well assume that each controller is of the same order l, for if not we can pad out the ones of low order by some observable but uncontrollable stable modes. The corresponding level of performance that K_i provides when applied to P_i is

$$\gamma_i := \|\mathcal{F}(P_i, K_i)\|. \tag{4}$$

We would like to convert our dynamic LTI controller to a static control law. To this end, we augment our plant with a bank of l integrators:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\rho} \end{bmatrix} &= \underbrace{\begin{bmatrix} A^i & 0 \\ 0 & 0 \end{bmatrix}}_{=:\bar{A}^i} \underbrace{\begin{bmatrix} x \\ \rho \end{bmatrix}}_{=:\bar{x}} + \underbrace{\begin{bmatrix} B_1^i \\ 0 \end{bmatrix}}_{=:\bar{B}_1^i} w \\ &\quad + \underbrace{\begin{bmatrix} B_2^i & 0 \\ 0 & I \end{bmatrix}}_{=:\bar{B}_2^i} \underbrace{\begin{bmatrix} u \\ \dot{\rho} \end{bmatrix}}_{=:\bar{u}} \\ z &= \underbrace{\begin{bmatrix} C_1^i & 0 \end{bmatrix}}_{\bar{C}_1^i} \begin{bmatrix} x \\ \rho \end{bmatrix} + D_{11}^i w \\ \underbrace{\begin{bmatrix} y \\ \rho \end{bmatrix}}_{=:\bar{y}} &= \underbrace{\begin{bmatrix} C_2^i & 0 \\ 0 & I \end{bmatrix}}_{=:\bar{C}_2^i} \begin{bmatrix} x \\ \rho \end{bmatrix}. \end{aligned}$$

These equations define the augmented plant $\bar{P}_i$. Notice that the LTI controller in (3) can be rewritten as

$$\bar{u} = \underbrace{\begin{bmatrix} J_i & H_i \\ G_i & F_i \end{bmatrix}}_{=:\bar{K}_i} \bar{y}. \tag{5}$$

Furthermore, observe that

$$\bar{C}_2^i \bar{B}_2^i = \begin{bmatrix} C_2^i B_2^i & 0 \\ 0 & I \end{bmatrix},$$

which means that the natural counterpart of Assumption 3 holds here:

Assumption 3′: There exists a vector $\bar{v} \in \mathbf{R}^{m_2+l}$ and $j \in \{1, 2, \cdots, q\}$ so that

$$\{(\bar{C}_2^i \bar{B}_2^i - \bar{C}_2^j \bar{B}_2^j)\bar{v} : \quad i = 1, \cdots, j-1, j+1, \cdots, q\}$$

are linearly independent. (In terms of the vector v in Assumption 3, one can simply choose $\bar{v} = [v^T \quad 0]^T$.)

2.2 Control Scheme

At this point we are ready to motivate and define the proposed controller $\bar{K}$. It will be sampled-data and of the form

$$\begin{aligned} &\eta(k+1) = F(k)\eta(k) + G(k)\bar{y}(kh), \;\; \eta(0) = \eta_0, \\ &\bar{u}(kh+\tau) = H(k)\eta(k) + J(k)\bar{y}(kh), \;\; \tau \in [0, h), \end{aligned} \tag{6}$$

with the controller gains F, G, H, and J periodic of period p (some positive integer); the period of the controller is $T := ph$, and we associate this system with the 6-tuple (F, G, H, J, h, p). Note that (6) can be implemented with a sampler S_h, a zero-order hold H_h (subscript h denotes the sampling period), and an LPTV discrete-time system $\bar{K}_d$ of period p – see Figure 2.

Our controller will be of the above form with h small and $p = 2(r_2 + l) + 1$, so that the controller is periodic of period $T = ph$. First we provide a conceptual description of the controller and a high-level explanation

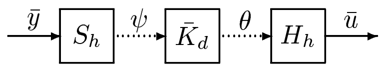

Figure 2: Implementation with a sampler and a zero-order-hold.

of why it should work. We do so in open loop first. To this end, choose $\bar{v} \in \mathbf{R}^{m_2+l}$ and $j \in \{1, 2, \cdots, q\}$ to satisfy Assumption 3′ and let $\delta \in (0.5, 1)$. Every period $[kT, kT+T)$ is divided into two phases: $[kT, kT+T-h)$ of length $2(r_2+l)h$ for probing the plant (estimation), and $[kT+T-h, kT+T)$ of length h for applying a suitable control signal.

Let us look at the first period $[0, T)$. Suppose that we set initially

$$\bar{u}(t) = h^{-\delta}\bar{v}\bar{y}_1(0), \quad t \in [0, h).$$

Solving the state equation for the i^{th} plant, we can show (see next section) that

$$\bar{x}(h) \approx \bar{x}(0) + h^{1-\delta}\bar{B}_2^i\bar{v}\bar{y}_1(0). \tag{7}$$

(Because $0.5 < \delta < 1$, the second term on the right dominates the other terms associated with $\bar{x}(0)$, $\bar{u}(t)$, and the disturbance $w(t)$.) Hence

$$\bar{y}(h) \approx \bar{y}(0) + h^{1-\delta}\bar{C}_2^i\bar{B}_2^i\bar{v}\bar{y}_1(0).$$

It follows from here that

$$h^{\delta-1}[\bar{y}(h) - \bar{y}(0)] \approx \bar{C}_2^i\bar{B}_2^i\bar{v}\bar{y}_1(0). \tag{8}$$

Since $\bar{y}(0)$ and $\bar{y}(h)$ are measured, the left-hand side of (8) can be used as an estimate for the right-hand side. Of course, in carrying out this experiment we have given the state a boost – see (7), which we can largely undo by applying

$$\bar{u}(t) = -h^{-\delta}\bar{v}\bar{y}_1(0), \quad t \in [h, 2h).$$

Hence, we can obtain a good estimate of $\bar{C}_2^i\bar{B}_2^i\bar{v}\bar{y}_1(0)$ without affecting the plant very much. Of course, we do not know the value of i. However, we do know that

$$\{(\bar{C}_2^i\bar{B}_2^i - \bar{C}_2^j\bar{B}_2^j)\bar{v} : \quad i = 1, \cdots, j-1, j+1, \cdots, q\}$$

are linearly independent. We would like to apply $\bar{u} = \bar{K}_i\bar{y}$. So the part due to $\bar{y}_1(0)$ is given by $(\bar{K}_i)_1\bar{y}_1(0)$. To compute this, we define a linear map $T_1 : \mathbf{R}^{r_2+l} \to \mathbf{R}^{m_2+l}$ via

$$\begin{gathered} T_1(\bar{C}_2^i\bar{B}_2^i - \bar{C}_2^j\bar{B}_2^j)\bar{v} = (\bar{K}_i - \bar{K}_j)_1, \\ i = 1, \cdots, j-1, j+1, \cdots, q; \end{gathered} \tag{9}$$

because of the linear independence, such a map exists, and we can define it arbitrarily on the rest of $\mathbf{R}^{r_2+l}$. Note that T_1 can be pre-computed based on the given data. Hence, from (9) and (8) we get $(\bar{K}_i - \bar{K}_j)_1\bar{y}_1(0)$ to be

$$\begin{aligned} &\approx h^{\delta-1}T_1[\bar{y}(h) - \bar{y}(0)] - T_1\bar{C}_2^j\bar{B}_2^j\bar{v}\bar{y}_1(0) \\ &= h^{\delta-1}T_1\bar{y}(h) - [h^{\delta-1}T_1 + T_1\bar{C}_2^j\bar{B}_2^j\bar{v}e_1^T]\bar{y}(0), \end{aligned}$$

where e_1 is the 1^{st} basis vector in $\mathbf{R}^{r_2+l}$.

Of course, we can repeat the above for $\bar{y}_2(0)$: apply

$$\bar{u}(t) = \begin{cases} h^{-\delta}\bar{v}\bar{y}_2(0) & t \in [2h, 3h), \\ -h^{-\delta}\bar{v}\bar{y}_2(0) & t \in [3h, 4h), \end{cases}$$

estimate $\bar{C}_2^i\bar{B}_2^i\bar{v}\bar{y}_2(0)$ from $h^{\delta-1}[\bar{y}(3h) - \bar{y}(0)]$, define T_2 and so on to get $(\bar{K}_i - \bar{K}_j)_2\bar{y}_2(0)$. Similarly, we repeat for each element of $\bar{y}(0)$, defining T_k, $k = 1, \cdots, r_2+l$ in the process, thereby obtaining an approximation of

$$\begin{aligned} (\bar{K}_i - \bar{K}_j)\bar{y}(0) \;=\; &(\bar{K}_i - \bar{K}_j)_1\bar{y}_1(0) + \cdots \\ &+ (\bar{K}_i - \bar{K}_j)_{r_2+l}\bar{y}_{r_2+l}(0). \end{aligned}$$

Since j is known, we now add $\bar{K}_j\bar{y}(0)$ to this to obtain a good estimate of $\bar{K}_i\bar{y}(0)$.

At the end of the estimation phase, we are at $t = (p-1)h$; if we set

$$\bar{u}(t) \approx p\bar{K}_i\bar{y}(0), \quad t \in [(p-1)h, ph),$$

we can show

$$\begin{aligned} \bar{x}(T) \approx\; &e^{(\bar{A}^i + \bar{B}_2^i\bar{K}^i\bar{C}_2^i)T}\bar{x}(0) \\ &+ \int_0^T e^{(\bar{A}^i + \bar{B}_2^i\bar{K}^i\bar{C}_2^i)(T-\tau)}\bar{B}_1^i w(\tau)\, d\tau, \end{aligned}$$

with the approximation improving as h (and therefore T as well) goes to zero; this is exactly what we would like. Hence, the control signal that we apply is

$$\bar{u}(t) = \begin{cases} h^{-\delta}\bar{v}\bar{y}_1(0) & t \in [0, h) \\ -h^{-\delta}\bar{v}\bar{y}_1(0) & t \in [h, 2h) \\ h^{-\delta}\bar{v}\bar{y}_2(0) & t \in [2h, 3h) \\ -h^{-\delta}\bar{v}\bar{y}_2(0) & t \in [3h, 4h) \\ \vdots & \vdots \\ h^{-\delta}\bar{v}\bar{y}_{r_2+l}(0) & t \in [(p-3)h, (p-2)h) \\ -h^{-\delta}\bar{v}\bar{y}_{r_2+l}(0) & t \in [(p-2)h, (p-1)h) \\ \approx p\bar{K}_i\bar{y}(0) & t \in [(p-1)h, ph). \end{cases}$$

Of course, at time $T = ph$ the procedure is repeated, but now using $\bar{y}(T)$ instead of $\bar{y}(0)$.

We remark that a closed form description of the proposed estimation and control scheme can be obtained using an LPTV system of the form in (6) – see also Figure 2.

The main result of the paper is:

Theorem 1 *Subject to Assumptions 1-5, for every $\epsilon > 0$, there exists an LPTV controller K which stabilizes* *every plant in* $\{P_1, \cdots, P_q\}$ *and yields near optimal performance:*

$$\|\mathcal{F}(P_i, K)\| \leq \|\mathcal{F}(P_i, K_i)\| + \epsilon, \quad i = 1, \cdots, q.$$

The proof of the above result is constructive and is omitted due to space limitation.

3 An Illustrative Example

In this section, we apply the proposed controller to a simple case with only two plants P_1 and P_2 with transfer functions given in (1). As we mentioned earlier, it is impossible to achieve simultaneous stabilization using an LTI controller, let alone an acceptable level of performance. However, our sampled-data LPTV controller will accomplish both.

We assume the disturbance w enters at the input to the plant. The following state-space models for P_i $(i = 1, 2)$ are easily obtained:

$$\begin{aligned} \dot{x}(t) &= x(t) + B_1^i[w(t) + u(t)], \quad x(0) = 0, \\ z(t) &= y(t) = x(t), \end{aligned}$$

with $B_1^1 = -B_1^2 = 1$. It is readily verified that Assumption 3 is satisfied with $v = 1$ and $j = 1$. Static controllers are used for P_1 and P_2 respectively,

$$K_1 = -K_2 = 5,$$

giving closed-loop transfer functions (from w to z)

$$\mathcal{F}(P_1, K_1) = -\mathcal{F}(P_2, K_2) = \frac{1}{s+4}.$$

Thus for disturbances in the frequency range $0 \leq \omega < 4$ rad./s, each closed-loop system achieves about 75% attenuation.

Our proposed controller is sampled-data and we choose the sampling period $h = 10$ ms; the digital controller is second-order and LPTV with period $p = 3$. The parameter δ is selected to be 0.65 in the design. For the simulated closed-loop responses, we apply a disturbance with two frequency components:

$$w(t) = \sin t + \sin(3t).$$

In the simulations to follow, we assume the actual plant is P_1; but this fact is not known to the proposed controller, which will probe and discern the plant. Figure 3 shows the closed-loop responses for the proposed controller (solid) and controller K_1 (dash), barely distinguishable from the solid curve, in comparison with the disturbance input $w(t)$ (dot); Figure 4 shows the corresponding control signal $u(t)$ for the proposed control system; and Figure 5 gives a closer look at $u(t)$ for the

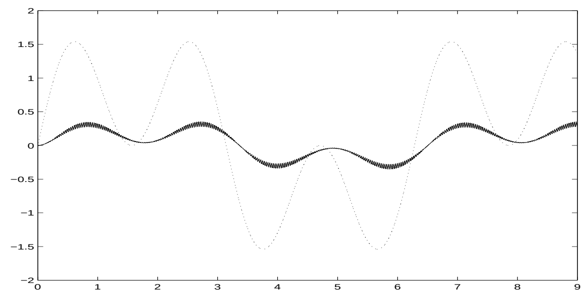

Figure 3: Closed-loop responses due to disturbance $w(t)$ (dot) for the proposed controller (solid) and controller K_1(dash).

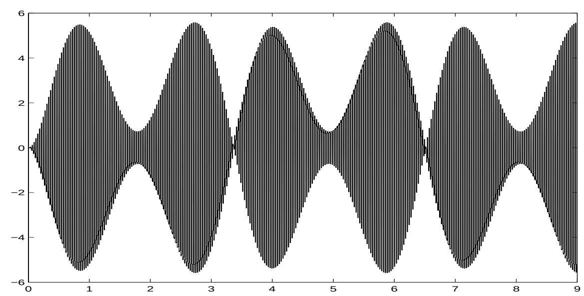

Figure 4: The control signal $u(t)$ for the proposed controller.

initial half second. We see that the performance of the proposed controller is very close to that of the actual LTI one (K_1), even for a relatively low sampling rate and a relatively small control effort.

Finally, in Figure 6 we simulate a hypothetical situation where there is an abrupt plant change from P_1 to P_2 at $t = 6$ s. In this case, the controller K_1 no longer stabilizes P_2 and so the output takes off starting from $t = 6$. However, the proposed control system is well-behaved except an abrupt change in the output response; this is because at the time of switching, the initial state of P_2 was assumed to be zero, instead of a matching state condition which would keep the transition smooth.

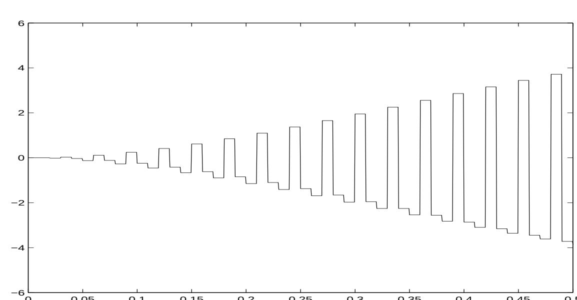

Figure 5: The control signal $u(t)$ for $0 \leq t \leq 0.5$.

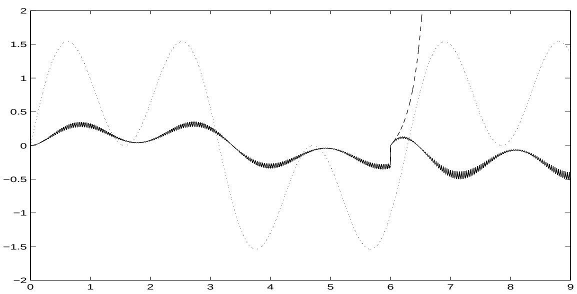

Figure 6: Closed-loop responses due to disturbance $w(t)$ (dot) for the proposed controller (solid) and controller K_1(dash) when P_1 is switched to P_2 at $t = 6$.

4 Summary and Concluding Remarks

In this paper we consider the problem of designing a controller for a finite set of models. It is well known that in many cases there is no LTI controller which simultaneously stabilizes every admissible model. While it is also well known that simultaneous stability can be provided by linear time-varying controllers, the performance provided by such control laws is unknown and from the design approaches adopted, we expect it to be poor. An exception is the recent paper of the first co-author [13], where it is proved that simultaneous stabilization with near optimal LQR-type performance is achievable. However, it is not clear how to extend that approach to deal with disturbances, so here we propose a new methodology to design a linear time-varying controller to not only simultaneously stabilize every admissible model, but also to provide near optimal disturbance rejection. Although we impose several stringent conditions on the plant for this approach to work, we emphasize that this is the *first approach to the simultaneous stabilization problem which guarantees near optimal disturbance rejection.*

In the paper, the disturbance rejection performance is measured using the $\mathcal{L}_2$-induced norm; this connects well with the popular LTI controller design paradigm based on $\mathcal{H}_\infty$ optimization. We remark that under the same assumptions, it can be shown that the proposed control scheme also provides near optimal $\mathcal{L}_1$ disturbance rejection measured using the $\mathcal{L}_\infty$-induced norm; in this case, the parameter δ is just required to be in the interval $(0, 1)$.

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