Optimal tracking with fixed order controllers

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Abstract

For discrete-time scalar systems, we propose an approach for designing feedback controllers of fixed order to minimize an upper bound on the peak magnitude of the tracking error to a given command input. The work makes use of linear programming to design over a class of closed-loop systems recently proposed for the rejection of non-zero initial conditions and bounded disturbances. Performance robustness in the form of a guaranteed upper bound on the peak magnitude of the tracking error under plant uncertainty is incorporated into the formulation.

1 Introduction

The problem of minimizing the l_∞ norm of the tracking error to a given command input was posed and given a solution by Dahleh and Pearson in [1]. While the techniques used, namely minimum norm duality theory and linear programming were those used in the solution they proposed for l_1-optimal control [2], the resulting closed-loop maps are rather different, since for the fixed input case [1] the optimal closed-loop response is not, in general, in l_1. Consequently, for the fixed input problem, a suboptimal solution must be used in order to obtain a response in l_1. This will allow zero steady state error and guarantee a stable system. The approach proposed in [1] involved an FIR closed-loop system whose order could be precomputed to give a solution within a desired tolerance of the true optimum. Alternative approaches which allow other than deadbeat closed-loop maps are by Moore and Bhattacharyya [3] and by Halpern et al [4]. In [3], the design is in effect that of an overparametrized pole-placement controller, with the poles selected by the designer, and the overparametrization calculated to minimize the l_∞ norm of the tracking error. In [4], a single closed-loop pole is incorporated into the interpolation constrained l_∞ minimization. These papers, [3], [4], left open the problem of how to select the closed-loop poles. Other results on obtaining deadbeat closed-loop systems are by Casavola and Mosca in [5]. The exact determination of infimal performance is considered in [6]. It is clearly highly desirable to maintain tracking performance in the presence of plant uncertainty. Results on robust performance when tracking fixed inputs are by Khammash [7] and Elia et al [8].

Recently Blanchini and Sznaier [9] have introduced an approach which they called design for equalized performance for bounded disturbance rejection. The method takes into account response to nonzero initial conditions by enforcing an l_1 condition on the coefficients of the denominator of the appropriate closed-loop transfer function. The approach has the additional desirable feature of allowing the design of fixed order controllers. Extensions of the approach from [9] have been carried out in [10], where the term super stable system was introduced to describe the class of closed-loop systems for which equalized performance is defined.

In the present paper we extend the design approach to deal with the problem of tracking commands with a view to bounding the l_∞ norm of the tracking error. Firstly, a new performance measure is proposed, which is an upper bound on the l_∞ norm of a signal and is defined only for super stable systems. Like equalized performance, the new measure can be minimized with respect to controller coefficients and can be used to design controllers of fixed order. The approach can also be used to obtain designs giving guaranteed tracking performance under plant uncertainty.

2 Preliminary Results

We begin by recalling some results from [9, 10] on a class of systems originally proposed in [9] for the prob-

[1] Partially supported by grants RFFI-99-01-00340 and INTAS IR-97-0782. Both authors supported by the Australian Research Council.

lem of rejecting bounded disturbances in a nonasymptotic setting and an appropriate performance measure γ.

2.1 Super stable systems

Consider a LTI discrete-time closed-loop system described by a scalar equation

$$(1 + a(z))x(k) = b(z)w(k) \quad (1)$$

where $w(k)$ are exogenous disturbances, $x(k)$ are outputs, z is a delay operator: $zx(k) = x(k-1)$, $a(z), b(z)$ are given polynomials with no constant terms:

$$a(z) = a_1 z + a_2 z^2 + \cdots + a_n z^n,$$
$$b(z) = b_1 z + b_2 z^2 + \cdots + b_m z^m.$$

To calculate $x(k)$ via (1) one must know the initial states $x_{-n}, x_{-n+1}, \ldots, x_{-1}$ and the values of disturbances $w_k, k \geq -m$. We denote $\|a\|_1 = \sum_1^n |a_i|, \|b\|_1 = \sum_1^m |b_i|$. We denote $\|a\|_\infty = \sup_i |a_i|$.

Definition 1. System (1) is *super stable*, if the polynomial $1 + a(z)$ is super stable, i.e. $\|a\|_1 < 1$.

We use the same notion for a transfer function with a super stable denominator.

2.2 Equalized Performance γ

A useful performance criterion for super stable systems was termed "equalized performance level" in [9]. For a super stable system (1) with transfer function,

$$\phi(z) = \frac{b(z)}{1 + a(z)}$$

equalized performance, γ, is defined as

$$\gamma(\phi) = \frac{\|b\|_1}{1 - \|a\|_1}. \quad (2)$$

2.3 Properties of γ

The value of γ partially determines the response of a system to nonzero initial conditions.

Lemma 1. Consider (1) as a difference equation with initial conditions $|x_{-n}| \leq \alpha, |x_{-n+1}| \leq \alpha, ..., |x_{-1}| \leq \alpha$ and bounded disturbances: $|w_k| \leq 1 \quad \forall k$. Suppose that the system is super stable. Then for all $k \geq 0$

$$|x_k| \leq \gamma + q^{\frac{k}{n}+1}(\alpha - \gamma)_+, \quad (3)$$

here $q = ||a||_1, c_+ = \max\{0, c\}$.

Corollary 1. If $\alpha \leq \gamma$ (i.e. if $|x_i| \leq \gamma, i = -n, ... - 1$) then $|x_k| \leq \gamma$ for all $k \geq 0$.

This result provides motivation for the minimization of γ.

If $\phi(z)$ is a super stable transfer function, then

$$\gamma(\phi) \geq \|\phi\|_1. \quad (4)$$

If both $\phi(z)$ and $\phi(z)/(1-\alpha z)$, where $|\alpha| < 1$, are super stable transfer functions, then in general

$$\gamma(\phi) \neq \gamma(\phi \frac{1 - \alpha z}{1 - \alpha z}), \quad (5)$$

thus caution is required for cancellation of terms in numerator and denominator if one applies γ-optimization.

3 Main Results

3.1 New Tracking Performance Criterion, β

Firstly we introduce a quantity analogous to γ for the minimum amplitude tracking of a given input. Let $h(z)$ be any causal super stable rational transfer function. It can be factored as $h(z) = b(z)c(z)$ where

$$b(z) = \sum_{i=0}^{B} b_i z^i$$

and

$$c(z) = \frac{1}{1 + \sum_{i=1}^{A} a_i z^i},$$

is super stable, that is to say, $\sum_{i=1}^{A} |a_i| < 1$.

Consider the impulse response h of the transfer function $h(z)$. Now

$$\begin{aligned} \|h\|_\infty &\leq \|b\|_\infty \|c\|_1 && (6) \\ &\leq \|b\|_\infty \gamma(c) && (7) \\ &= \frac{\|b\|_\infty}{1 - \|a\|_1} := \beta(h). && (8) \end{aligned}$$

Thus for superstable $h(z)$, we see that $\beta(h)$ is an easily computed upper bound on the l_∞ norm of h. We show later that it can be minimized using linear programs.

3.2 Properties of β

It is of interest to examine the sharpness of the inequalities (6, 7). Equality is obtained in (6) when $c(z) = 1$, that is when $h(z)$ is a polynomial.

A condition for equality of (7) is obtained next. It is noted in [10] that $\gamma(h) = \|h\|_1$ for polynomial $h(z)$. Here we show that such equality is also obtained for certain all pole rational functions.

Lemma 2: Assume $c(z) = 1/(1+a(z))$ is super stable.

(a) If all $a_i \leq 0$, then $\gamma(c) = c(1) = \|c\|_1$.

(b) If all $-1^i a_i \leq 0$, then $\gamma(c) = c(-1) = \|c\|_1$.

For example if $c(z) = 1/(1 - .2z - .3z^2 - .1z^3 - .2z^8)$ or $c(z) = 1/(1 + .2z - .3z^2 + .1z^3 - .2z^8)$, then $\gamma(c) = \|c\|_1 = 5.0$.

3.3 Application to Tracking Problem

Here, we are given a plant $P(z)$ and a command $w(z)$ and our goal is to design a controller to force the output of the plant to track the command. The plant $P(z)$ has input u and output y and is described by its transfer function:

$$P(z) = \frac{b(z)}{1 + a(z)} = \frac{b_1 z + b_2 z^2 + \cdots + b_B z^B}{1 + a_1 z + \cdots + a_A z^A}, \qquad (9)$$

with $y = Pu$, while the command is the impulse response of $w(z) = w_1(z)/w_2(z), w_2(0) = 1$. Here $w(z)$ is allowed to be unstable. Our treatment of the problem will require the *tracking error* $\phi(k)$, which is given by

$$\phi = w - y,$$

to be super stable. Since $\phi(z) = T(z)w(z)$ where $T(z)$ is a stable closed-loop transfer function, it is clear that if $w(z)$ is not super stable, then at least some of its poles will need to be cancelled by zeros of $T(z)$ in order to have $\phi(z)$ super stable. In fact, even if $w(z)$ is super stable, one may choose to cancel all or part of its denominator in order to allow more freedom in the design of the denominator of the transfer function of $\phi(z)$. (Such cancellations are hidden; they do not affect stability; they only affect the response to initial conditions at the input and output of the command generator $w(z)$. We assume these initial conditions are zero valued.)

To illustrate some of the ideas involved, we set $w(z) = 1/(1 - z)$, which is not superstable. We consider both one-degree-of-freedom (1-DOF) and 2-DOF configurations.

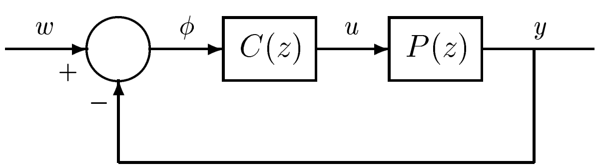

Fig. 1: 1-DOF Feedback Configuration for Tracking

3.3.1 1-DOF system: Initially we consider a 1-DOF system, which, for a step command gives a controller with integral action. Our goal is to design a controller of fixed structure:

$$C(z) = \frac{g(z)}{(1 - z)f(z)} = \frac{g_0 + g_1 z + \cdots + g_G z^G}{(1 - z)(1 + f_1 z + \cdots + f_F z^F)} \qquad (10)$$

(with prescribed orders F, G and $f(0) = 1$) which ensures super stability of the closed-loop system and minimizes the performance index $\beta(\phi)$ where ϕ is the tracking error, $\phi = w - y$. For the system in Fig. 1, the tracking error $\phi(z)$ is given by

$$\phi(z) = \frac{1}{1 - z}\left(\frac{a(1 - z)f}{(1 - z)af + bg}\right).$$

As it stands, this transfer function cannot be made super stable through selection of f, g since the denominator of w is not super stable. In order to obtain a super stable transfer function, which is required for the development in this paper, it is necessary to remove the factor $(1 - z)$ from the denominator, by cancelling with the numerator as discussed earlier. After this cancellation, the tracking error becomes

$$\phi(z) = \frac{af}{(1 - z)af + bg}$$

for which

$$\beta(\phi) = \frac{\|af\|_\infty}{1 - \|(1 - z)af + bg - 1\|_1} \qquad (11)$$

provided $(1 - z)af + bg$ is super stable.

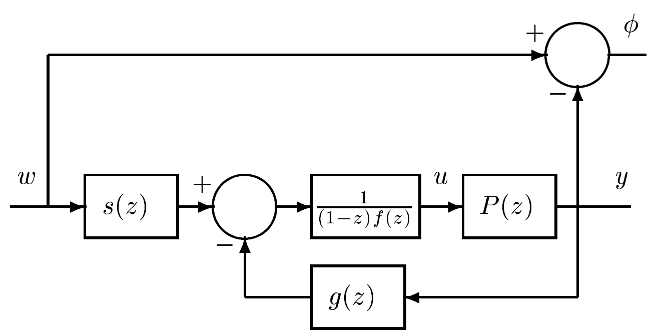

Fig. 2: 2-DOF Feedback Configuration for Tracking

3.3.2 2-DOF system: Now, it is well known that a two degree-of-freedom (2-DOF) system offers at least two advantages over a 1-DOF system when tracking given commands. Firstly, more independent design of disturbance rejecting and tracking transfer functions is possible. Secondly, for unstable plants, the restriction on achievable tracking error imposed by the unstable plant poles is removed if a 2-DOF structure is used. These issues are discussed in [11].

For our problem, it is clear only that we have a larger number of parameters over which to optimize if we use a 2-DOF structure. Consider the 2-DOF discrete-time SISO system depicted in Fig. 2. The controller now comprises a "setpoint prefilter" polynomial $s(z) = s_0 + s_1 z + \cdots + s_S z^S$ as well as polynomials g, f. Note our use of a polynomial (as opposed to a rational) setpoint filter restricts the ability to design separately the setpoint tracking transfer functions and any disturbance rejecting transfer functions. The control input u for this configuration is given by

$$(1-z)f(z)u(k) = s(z)w(k) - g(z)y(k). \qquad (12)$$

The tracking error $\phi = w - y$ is given by

$$\phi(z) = \frac{1}{1-z}\left(\frac{(1-z)af + bg - sb}{(1-z)af + bg}\right).$$

As for the 1-DOF case, it is necessary to remove the factor $(1-z)$ from the denominator. For steady state tracking of a step command, we need $g(1) = s(1)$. We then have

$$\phi(z) = \frac{1}{1-z}\ \frac{(1-z)r}{(1-z)af + bg} \qquad (13)$$

where r is a polynomial satisfying $(1-z)r = (1-z)af + bg - sb$. It is then possible to cancel the $(1-z)$ giving

$$\phi(z) = \frac{r}{(1-z)af + bg}.$$

The performance index is

$$\beta(\phi) = \frac{\|r\|_\infty}{1 - \|(1-z)af + bg - 1\|_1} \qquad (14)$$

provided that the polynomial in the denominator is super stable.

3.4 Minimization of β

The minimization of β above can be carried out using a one-parameter family of linear programs. Controller orders F, G and, for the 2-DOF case, also S are fixed.

Theorem 1. Minimization of (11) is equivalent to a parametric linear programming problem,

$$\min_{0\le\mu<1}\ \min_{f,g} \frac{\|af\|_\infty}{1-\mu} : \qquad (15)$$

$$\|(1-z)af + bg - 1\|_1 = \mu. \qquad (16)$$

Minimization of (14) is equivalent to a parametric linear programming problem,

$$\min_{0\le\mu<1}\ \min_{f,g,s} \frac{\|r\|_\infty}{1-\mu} : \qquad (17)$$

$$\|(1-z)af + bg - 1\|_1 = \mu, \qquad (18)$$

$$(1-z)r = (1-z)af + bg - sb. \qquad (19)$$

For fixed μ, the above problems can be cast, using standard techniques, as linear programs; we omit the details. Hence their solution over $\mu \in [0,1)$ involves a one-parameter family of linear programs.

Theorem 2. If the admissible sets in (16), (18, 19) are nonempty, and the optimal values of their objective functions are denoted β^*, then the closed-loop systems with the controllers, found as the solutions of these optimization problems, are super stable with $\beta(\phi) = \beta^*$.

4 Design for Robust Performance

4.1 Robust stabilization

Here we use the same problem setup as in [10] with plant coprime factor uncertainty bounded in l_1 norm.

Consider the family of plants $\mathcal{P}$ with coprime factor uncertainty:

$$P(z) = \frac{b(z)}{a(z)} = \frac{b^0(z) + \delta b(z)}{a^0(z) + \delta a(z)}, \qquad (20)$$
$$\|\delta b\|_1 \le \varepsilon_B, \quad \|\delta a\|_1 \le \varepsilon_A.$$

Here $b(z), b^0(z), \delta b(z)$ are polynomials of degree B, with $b^0(0) = \delta b(0) = 0$; $a(z), a^0(z), \delta a(z)$ are polynomials of degree A with $a^0(0) = 1, \delta a(0) = 0$. The problem of robust stabilization with a fixed-order controller is made tractable by dealing with super stability instead of stability. Thus in a 1-DOF system we try to design a controller $C(z) = g(z)/((1-z)f(z))$ (see (10)) with fixed orders F, G which guarantees super stability of the closed-loop systems for all plants $P \in \mathcal{P}$. This means that the characteristic polynomial

$$k(z) = (1-z)a(z)f(z) + b(z)g(z), \quad k(0) = 1$$

should be super stable for all $b(z), a(z)$ in (20).

We require part of a Lemma from [10].

Lemma 3. If $p(z), q(z)$ are the polynomials of the same order m, then

$$\max_{\|q\|_1 \le r} \|pq\|_1 = r\|p\|_1.$$

The condition for robust super stability presented next is as in [10].

Theorem 3. The family $\mathcal{P}$ can be robustly super stabilized by a fixed-order controller $C(z) = g(z)/((1-z)f(z))$ if the linear inequality

$$\|(1-z)a^0 f + b^0 g - 1\|_1 + \varepsilon_B\|g\|_1 + \varepsilon_A\|(1-z)f\|_1 < 1 \qquad (21)$$

has a solution.

4.2 Robust performance

We wish to minimize β over robustly super stabilizable systems. Ideally, we would like to solve the following problem:

$$\beta^+ = \min_C \max_P \beta(\phi) \tag{22}$$

where, for a 1-DOF system,

$$\beta(\phi) = \frac{\|af\|_\infty}{1 - \|(1-z)af + bg - 1\|_1} \tag{23}$$

which is a robust equivalent of (15). Unfortunately, it is difficult to calculate $\max_P \beta(\phi)$, so instead we use an upper bound $\beta^* > \beta^+$. This is obtained by taking an upper bound on the numerator of (23) together with a lower bound on the denominator. The upper bound on the numerator uses the following Lemmas.

Lemma 4. If $p(z), q(z)$ are the polynomials of the same order m, then

$$\max_{\|q\|_1 \le r} \|pq\|_\infty = r\|p\|_\infty.$$

Lemma 5. Given fixed polynomials $f(z), a^0(z)$ with uncertain $a(z) = a^0(z) + \delta a(z)$, then

$$\max_{\|\delta a\|_1 \le \varepsilon_A} \|af\|_\infty \le \|a^0 f\|_\infty + \varepsilon_A \|f\|_\infty.$$

We then have the following robust performance result for 1-DOF systems.

Theorem 4. Suppose the parametric linear programming problem

$$\beta^* = \min_{0 \le \mu < 1} \min_{f,g} \frac{\|a^0 f\|_\infty + \varepsilon_A \|f\|_\infty}{1-\mu} :$$
$$\|a^0(1-z)f + b^0 g - 1\|_1 + \varepsilon_B \|g\|_1 + \varepsilon_A \|(1-z)f\|_1 = \mu,$$

has a solution f^*, g^*. Then the controller $C^* = g^*/((1-z)f^*)$ makes all closed-loop systems with plants $P \in \mathcal{P}$ robustly super stable with robust tracking performance less or equal β^*.

The corresponding robust performance result for 2-DOF systems is as follows.

Theorem 5. Suppose the parametric linear programming problem

$$\beta^* = \min_{0 \le \mu < 1} \min_{f,g} \frac{\|a^0 f + b^0 v\|_\infty + \varepsilon_A \|f\|_\infty + \varepsilon_B \|v\|_\infty}{1-\mu} :$$
$$v(1-z) = g - s,$$
$$\|a^0(1-z)f + b^0 g - 1\|_1 + \varepsilon_B \|g\|_1 + \varepsilon_A \|(1-z)f\|_1 = \mu,$$

has a solution f^*, g^*, s^*. Then the control

$$u(k) = \frac{s^* w(k) - g^* y(k)}{(1-z)f^*}$$

makes all closed-loop systems with plants $P \in \mathcal{P}$ robustly super stable with robust tracking performance less or equal β^*.

5 Examples

Consider the example from [5] of designing a 1-DOF controller for plant,

$$P(z) = \frac{-10z(z-0.5)}{(1-10z)(1-0.5z)}$$

tracking a step.

Following the procedure in Theorem 4, we design 1-DOF controllers to minimize $\beta(\phi)$ for some controller orders and plant uncertainty levels as shown below

	F	2	3	4	5	6
	G	2	3	4	5	6
$\varepsilon_B = \varepsilon_A = 0$	β^*	40.0	21.6	16.9	15.0	14.2
	ν_0	40.0	21.6	16.9	15.0	14.2
$\varepsilon_B = \varepsilon_A = 0.01$	β^*	48.9	25.9	20.0	17.9	16.9
	ν_0	40.0	21.6	16.9	15.0	14.2
$\varepsilon_B = \varepsilon_A = 0.05$	β^*	431	93.0	67.6	50.1	44.4
	ν_0	40.0	26.1	24.6	16.8	15.8

The table shows the values of β^*, which is the minimized value of β, our upper bound on $\|\phi\|_\infty$; and ν_0 which we introduce here to be the value of $\|\phi\|_\infty$ achieved with the nominal plant. The values of both of these quantities can be compared with the lowest possible value of $\|\phi\|_\infty$ achievable with the nominal plant and given command and 1-DOF controller structure, namely 13.5 from [5].

We show the results for controller orders $G = 3, F = 3$ in detail. The optimal controller minimizing β obtained using Theorem 1 or Theorem 4 with $\varepsilon_B = \varepsilon_A = 0$ is

$$\frac{g}{(1-z)f} = \frac{2.672 - 1.448z - 2.896z^2 + 1.472z^3}{(1-z)(1-1.86z-2.94z^2)},$$

(notice $f_3 = 0$) giving a tracking error,

$$\phi(z) = 1 - 12.3615z + 21.6017z^2 + 21.6017z^3 - 14.7186z^4,$$

so that $\beta^* = \nu_0 = 21.6$.

We now incorporate plant uncertainty. We set $\varepsilon_B = \varepsilon_A = 0.01$ and use Theorem 4 to compute β^*. This time β^* is achieved at $\mu = 0.16376$, but the controller and nominal tracking error are the same as the previous result obtained with no uncertainty. Increasing the uncertainty level to $\varepsilon_B = \varepsilon_A = 0.05$ gives an optimum at $\mu = 0.718$, and a different controller,

$$\frac{g}{(1-z)f} = \frac{2.744 - 2.276z - 1.780z^2 + 1.112z^3}{(1-z)(1-2.221z-2.224z^2)},$$

giving a nominal tracking error,

$$\phi(z) = 1 - 12.721z + 26.101z^2 + 12.243z^3 - 11.119z^4,$$

with $\beta^* = 93.0$, $\nu_0 = 26.1$.

In the above examples, the nominal tracking errors, obtained by minimizing β or its upper bound, are FIR. It is possible to find examples where β is minimized with different closed-loop poles. For instance, with plant

$$P = \frac{-10.5z + 26z^2 - 10z^3}{1 - 10.75z + 7.625z^2 - 1.25z^3}$$

which has zeros at $z \in \{0, 0.5, 2.1\}$ and poles at $z \in \{0.1, 2, 4\}$, using $F = G = 3$, a feasible solution to the problem in Theorem 1 can be found for $\mu = 0$, but β is minimized at $\mu \approx 0.05$, giving a nominal tracking error

$$\phi(z) = \frac{1 - 12.58z + 23.95z^2 + 21.25z^3 - 23.95z^4 + 4.51z^5}{1 - 0.00481147z + 0.0451885z^7}.$$

In this example, the plant has a stable pole and stable zero near each other. In such situations, controller design by closed-loop pole placement can be problematic since the pole placement Diophantine equation can be poorly conditioned, giving rise to large controller coefficient magnitudes unless a closed-loop pole is chosen carefully near the offending plant pole and zero [12]. The automatic closed-loop pole placement arising from the minimization of β, which to some extent penalizes large controller coefficients, can alleviate this problem.

6 Conclusions

We have extended results on the design of super stable systems to the problem of tracking a fixed command. Our approach involves minimizing an upper bound on the l_∞ norm of the tracking error and readily allows the incorporation of plant uncertainty to enable the design of fixed order controllers for guaranteed tracking performance under l_1 bounded plant uncertainty.

In this paper we have focussed on the case of a step command ($w(z) = 1/(1-z)$) but other commands can be treated similarly. Although we have restricted our analysis to SISO systems, MIMO systems can also be considered in the same framework.

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