Adaptive Output Feedback Nonlinear Control of a pH Process with an Input Constraint

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Abstract

In this paper an adaptive output feedback nonlinear control scheme is presented for a pH process, which is difficult to control due to the nonlinearities and uncertainties. A nonlinear controller is designed using a backstepping technique to handle the pH process with an input constraint. The backstepping-based controller is combined with state and parameter estimators to deal with unmeasurable state variables and unknown parameters, thereby resulting in an adaptive output feedback nonlinear controller. The input constraint mentioned above is considered in the controller and estimator designs. The stability properties of the proposed scheme are given, and simulation results show that uniform closed-loop performance is obtained despite the nonlinearity, the uncertainties of the parameters, and the unavailability of the state variables.

1 Introduction

The main objective of pH processes is to control the effluent pH value by manipulating the flow rate of titrating stream. The control of pH is important in the chemical industry, especially in wastewater treatment. However, such a pH process is difficult to control due to its nonlinear dynamics with uncertainties. For this reason, pH control by conventional PID controllers or advanced control techniques based on linear system theory is ineffective [7]. In addition the control input under consideration is physically constrained to remain non-negative. Thus, there is an obvious motivation to apply a constrained adaptive nonlinear control scheme for the pH process.

Among many nonlinear control techniques, especially two nonlinear control schemes have recently attracted researchers' attention: feedback linearization and backstepping design. Feedback linearization is for linearizing a nonlinear system by cancelling its nonlinearities in order to apply abundant linear control theories to the linearized system [10]. On the other hand, backstepping-based control can handle a wider class of uncertain systems by offering a systematic stabilizing process; in the backstepping design, unwanted cancellations of favorable nonlinearities can be avoided. Adaptive nonlinear control schemes can be designed by combining such nonlinear controllers mentioned above with parameter estimation algorithms in a variety of ways. For adaptive feedback linearization, see e.g. [1, 9, 11]. Adaptive backstepping design methods can be found in [5, 6, 8].

Recently, adaptive feedback linearizing control schemes have been applied to the pH process [2, 3]. In the studies, a nonlinear model of the pH process is obtained using the reaction invariants [13], and then adaptive input-output linearization is applied, where a buffer flow rate is considered an unknown parameter and thus is estimated. However, in practice, flow rates can often be measured but parameters such as the concentrations of the reaction invariants of inlet wastewater streams are hardly available on line. These concentrations are the crucial parameters for the behavior of the process. Hence, a backstepping-based adaptive nonlinear control scheme has been applied to the pH process, which estimates the concentrations of the reaction invariants of the inlet wastewater stream [14]. However, in [14], the state variables, i.e., the concentrations of the reaction invariants of the effluent stream are assumed to be measured, which is demanding in practice. For this reason, an adaptive output feedback nonlinear control scheme has been proposed for the process [15]. It is noted, however, that the physical constraint on the control input is not considered in [15].

In this paper, we extend the results of [15] to the case where the control input is physically constrained to remain non-negative. State and parameter estimators are proposed, which have better convergence properties than those of [15]. A nonlinear controller is designed using a backstepping procedure such that the input constraint is satisfied, and is combined with the state and parameter estimators. The stability properties of the overall adaptive scheme are given; simulations show that uniform closed-loop performance is obtained despite the nonlinearity, the uncertainties of the parameters, and the unavailability of the state variables.

2 pH Process Model

Consider a pH neutralization process as given in [14]. The flow rates of acid, buffer, titrating and effluent streams are denoted by q_1, q_2, q_3, and q_4, respectively. The output of the process is the pH value of the effluent stream (pH_4), and the flow rate of the titrating stream q_3 is the control input. A dynamic model is derived using the conservation equations of the reaction invariants and equilibrium relations [2, 3].

The state space model of the process is written as follows:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where

$$x = [W_{a_4} \; W_{b_4}]^T, \; u = q_3,$$

$$\begin{aligned} f(x) &= [f_1(x) \;\; f_2(x)]^T \\ &= \begin{bmatrix} -\frac{1}{V}(q_1+q_2)x_1 + \frac{1}{V}(q_1 W_{a_1} + q_2 W_{a_2}) \\ -\frac{1}{V}(q_1+q_2)x_2 + \frac{1}{V}(q_1 W_{b_1} + q_2 W_{b_2}) \end{bmatrix}, \end{aligned}$$

$$g(x) = \begin{bmatrix} g_1(x) \\ g_2(x) \end{bmatrix} = \begin{bmatrix} \frac{1}{V}(W_{a_3} - x_1) \\ \frac{1}{V}(W_{b_3} - x_2) \end{bmatrix},$$

W_{a_i} and W_{b_i} $(i = 1, \ldots, 4)$ are the concentrations of the two reaction invariants in each process stream, and V is the constant volume of tank. The nonlinear relation between the output y and the state variables x is given by

$$h(x, y) \;=\; 0, \tag{2}$$

where

$$\begin{aligned} h(x,y) &= x_1 + h_{x_2} x_2 + 10^{y-14} - 10^{-y}, \\ h_{x_2} &= \tfrac{1 + 2\times 10^{y-pK_2}}{1 + 10^{pK_1 - y} + 10^{y - pK_2}}, \\ pK_1 &= -\log_{10} K_{a_1}, \; pK_2 = -\log_{10} K_{a_2}. \end{aligned}$$

The nominal values of the parameters are the same as in [14]. Note that the control input should be nonnegative. The model (1) can be rewritten as follows:

$$\dot{x} = \bar{f} + \bar{g}(x)v + F\theta, \tag{3}$$

where

$$v = \tfrac{1}{V}[q_1 + q_2 + u],$$

$$\bar{f} = \begin{bmatrix} \bar{f}_1 \\ \bar{f}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{V}\{-q_1 W_{a_3} + q_2(W_{a_2} - W_{a_3})\} \\ \frac{1}{V}\{-q_1 W_{b_3} + q_2(W_{b_2} - W_{b_3})\} \end{bmatrix},$$

$$\bar{g}(x) = \begin{bmatrix} W_{a_3} - x_1 \\ W_{b_3} - x_2 \end{bmatrix}, \; F = \begin{bmatrix} \frac{q_1}{V} & 0 \\ 0 & \frac{q_1}{V} \end{bmatrix}, \; \theta = \begin{bmatrix} W_{a_1} \\ W_{b_1} \end{bmatrix}.$$

Note that the vector θ contains the unknown parameters W_{a_1}, W_{b_1} to be estimated, and equation (3) is linear in θ.

3 Design of Adaptive Output Feedback Control

In this section, we design an adaptive output feedback nonlinear controller for the pH process with the input constraint on the basis of modular backstepping design.

3.1 Observability of the pH process

In this subsection, we show that it is possible to design a closed-loop state estimator by checking the local observability of the nonlinear process. The explicit form of the output equation (2) can be written as follows:

$$y = \bar{h}(x), \tag{4}$$

where $\bar{h}(x)$ is not known, however its differential can be obtained as

$$d\bar{h} = -\left(\tfrac{\partial h}{\partial y}\right)^{-1}\left(\tfrac{\partial h}{\partial x_1}\right)dx_1 - \left(\tfrac{\partial h}{\partial y}\right)^{-1}\left(\tfrac{\partial h}{\partial x_2}\right)dx_2. \tag{5}$$

Suppose a point $x_0 \in R^2$ is given. From [8, p. 209, 12, p. 418], the pH process given in (1) and (4) is locally observable in a neighborhood of x_0 if

$$\mathrm{rank}\{d\bar{h},\; dL_f\bar{h},\; dL_g\bar{h}\} = 2 \text{ at } x_0, \tag{6}$$

where

$$\begin{aligned} &L_f\bar{h} := \tfrac{\partial \bar{h}}{\partial x} f, \; L_g\bar{h} := \tfrac{\partial \bar{h}}{\partial x} g, \\ dL_f\bar{h} &= (\tfrac{\partial L_f\bar{h}}{\partial x_1} + \tfrac{\partial L_f\bar{h}}{\partial y}\tfrac{\partial \bar{h}}{\partial x_1})dx_1 + (\tfrac{\partial L_f\bar{h}}{\partial x_2} + \tfrac{\partial L_f\bar{h}}{\partial y}\tfrac{\partial \bar{h}}{\partial x_2})dx_2, \\ dL_g\bar{h} &= (\tfrac{\partial L_g\bar{h}}{\partial x_1} + \tfrac{\partial L_g\bar{h}}{\partial y}\tfrac{\partial \bar{h}}{\partial x_1})dx_1 + (\tfrac{\partial L_g\bar{h}}{\partial x_2} + \tfrac{\partial L_g\bar{h}}{\partial y}\tfrac{\partial \bar{h}}{\partial x_2})dx_2. \end{aligned}$$

The set M satisfying the condition (6) is found as follows:

$$\begin{aligned} M \;\; &= \{x \;|\; \tfrac{h_{y_2}}{(h_{y_1} + x_2 h_{y_2})^3}(f_1(x) + h_{x_2} f_2(x)) \neq 0 \\ &\quad \text{or } \tfrac{h_{y_2}}{(h_{y_1} + x_2 h_{y_2})^3}(g_1(x) + h_{x_2} g_2(x)) \neq 0\}, \end{aligned}$$

where

$$h_{y_1} = (\ln 10)(10^{y-14} + 10^{-y}),$$

$$h_{y_2} = (\ln 10)\tfrac{10^{pK_1 - y} + 10^{y - pK_2} + 4\times 10^{pK_1 - pK_2}}{(1 + 10^{pK_1 - y} + 10^{y - pK_2})^2}.$$

This implies that there exists a closed-loop state estimator for each point in the set M.

Remark 1 As shown in [2], the linearized model for the pH process given in (1) and (2) is unobservable at every operating point because the rank condition of the observability matrix of the linearized model is not satisfied. That is, the pH process is locally observable at most points, but its linearized model is unobservable at all points.

3.2 Design of state and parameter estimators

From (2), the output equation is rewritten as

$$\psi_1 x_1 + \psi_2 x_2 + \psi_3 = 0, \tag{7}$$

where $\psi_1 = 1 + 10^{pK_1 - y} + 10^{y - pK_2}$, $\psi_2 = 1 + 2 \times 10^{y-pK_2}$, $\psi_3 = (10^{y-14} - 10^{-y})(1 + 10^{pK_1-y} + 10^{y-pK_2})$. From (3) and (7), we can derive a state estimator as follows:

$$\dot{\widehat{x}} = \bar{f} + \bar{g}(\widehat{x})v + F\widehat{\theta} + L(\psi_1 \widehat{x}_1 + \psi_2 \widehat{x}_2 + \psi_3), \tag{8}$$

where $\widehat{(\cdot)}$ is the estimate of $(\cdot)$, $L = [L_1 \quad L_2]^T$,

$$\begin{aligned} L_1 &= \begin{cases} -\frac{(p_1\psi_2 - 2p_2\psi_1)(v-\bar{v})}{2p_2\psi_1^2 - 2p_1\psi_1\psi_2 + 2\psi_2^2} & ,\text{if } \widehat{x}_2(t) > W_{b_3}, \\ 0 & ,\text{if } \widehat{x}_2(t) = W_{b_3}, \end{cases} \\ L_2 &= \begin{cases} \frac{(2\psi_2 - p_1\psi_1)(v-\bar{v})}{2p_2\psi_1^2 - 2p_1\psi_1\psi_2 + 2\psi_2^2} & ,\text{if } \widehat{x}_2(t) > W_{b_3}, \\ 0 & ,\text{if } \widehat{x}_2(t) = W_{b_3}, \end{cases} \end{aligned} \tag{9}$$

and p_1, p_2, and $\bar{v}$ are such that

$$p_2 - \tfrac{p_1^2}{4} > 0, \ \bar{v} > v. \tag{10}$$

Note that $\bar{v}$ in (10) can be determined such that L_1 and L_2 are continuous, in which case the solution of (8) is well defined.

Remark 2 Assuming that $\widehat{\theta} = \theta$, the condition (10) implies that the performance of the closed-loop state estimator is better than that of the open-loop state estimator, i.e., $L_1 = 0$, $L_2 = 0$. Note that, when $\widehat{x}_2(t) = W_{b_3}$, L_1 and L_2 are both set to zero so as to obtain the following inequality:

$$\widehat{x}_2(t) \geq W_{b_3}, \ \forall t, \tag{11}$$

which is necessary for the feasibility of the parameter estimator and nonlinear controller to be designed.

A parameter estimator is designed here to satisfy the following requirement:

$$\widetilde{\theta} := \theta - \widehat{\theta} \in L_\infty. \tag{12}$$

To find an estimator satisfying the condition (12), consider the dynamic output equation:

$$\dot{y} = \tfrac{1}{h_y}(l_1 u + l_2) - \tfrac{h_x F\theta}{h_y}, \tag{13}$$

and define the following prediction equation:

$$\dot{\widehat{y}} = k_e \widetilde{y} + \tfrac{1}{\widehat{h}_y}(l_1 u + l_2) - \tfrac{h_x F\widehat{\theta}}{\widehat{h}_y} - \chi_e, \ (k_e > 0), \tag{14}$$

where h_x and h_y denote the partial derivatives of $h(x, y)$ with respect to x and y obtained as

$$h_x = [1 \quad h_{x_2}], \ h_y = h_{y_1} + x_2 h_{y_2},$$
$$\widetilde{y} = y - \widehat{y}, \ \widehat{h}_y = h_{y_1} + \widehat{x}_2 h_{y_2}.$$

Also χ_e is a nonlinear damping term to counteract the effects of the state estimation errors, and is obtained as

$$\chi_e = -k_1 \widetilde{y} l_3^2 - k_2 \widetilde{y} \|l_4\|^2, \tag{15}$$

where l_i, $(i = 1, \ldots, 4)$ are appropriate nonlinear functions defined in the appendix and k_i, $(i = 1, 2)$ are positive constants. Using (13)-(15) and the so-called σ-modification method [4], we can derive the parameter estimator:

$$\dot{\widehat{\theta}}_1 = -\gamma_1 \tfrac{q_1 \widetilde{y}}{V \widehat{h}_y} - \gamma_1 \sigma_1 \widehat{\theta}_1, \tag{16}$$

$$\begin{aligned} \dot{\widehat{\theta}}_2 &= \text{Proj}\{-\gamma_2 \tfrac{q_1 \widetilde{y} h_{x_2}}{V \widehat{h}_y} - \gamma_2 \tfrac{2q_1^2 p_1 (p_2+1)}{d(q_1+q_2)\lambda_m(P)V} \widehat{\theta}_1 \\ &\quad -\gamma_2 \sigma_2 \widehat{\theta}_2\} \end{aligned} \tag{17}$$

where

$$P = \begin{bmatrix} 1 & \frac{p_1}{2} \\ \frac{p_1}{2} & p_2 \end{bmatrix},$$

$$\sigma_1 > \tfrac{q_1^2(p_1^2+4)}{2d(q_1+q_2)\lambda_m(P)V}, \ \sigma_2 > \tfrac{q_1^2(p_1^2+4p_2^2)}{2d(q_1+q_2)\lambda_m(P)V},$$

$\lambda_m(P)$ is the minimum eigenvalue of P, γ_1 and γ_2 are positive constants, and $\text{Proj}\{\cdot\}$ is a projection operator used to ensure

$$\widehat{\theta}_2(t) \geq \underline{\theta}_2, \ \forall t \geq 0, \tag{18}$$

where $\underline{\theta}_2$ is defined as $\underline{\theta}_2 := \inf\{W_{b_1} \mid q_1(t)(W_{b_1} - W_{b_3}) + q_2(t)(W_{b_2} - W_{b_3}) \geq 0, \ \forall t\}$. It is assumed that the value of $\underline{\theta}_2$ is available. Note that the inequality (18) is necessary for (11), and that this parameter estimator is guaranteed to satisfy the condition (12) as is shown in Lemma 4.2 of section 4.

3.3 Nonlinear controller design

The standard backstepping procedure is slightly modified in this section so as to achieve a desired rise time.

Step 1: Define z_1 as

$$z_1 = \int_0^t (y - y_d)\bar{1}(\bar{u})d\tau, \tag{19}$$

i.e.

$$\dot{z}_1 = (y - y_d)\bar{1}(\bar{u}), \tag{20}$$

where y_d is the set-point for y, $\bar{u}$ is a nonlinear function to be designed by the backstepping procedure without considering the physical constraint, and $\bar{1}(\bar{u})$ is defined such that $\bar{1}(\bar{u}) = 1$ if $\bar{u} > 0$, and $\bar{1}(\bar{u}) = 0$ if $\bar{u} \leq 0$. Considering $y - y_d$ as the so-called virtual control for (20), define z_2 as follows:

$$z_2 = \tfrac{1}{\eta}(y - y_d - \alpha_1), \tag{21}$$

where the stabilizing function α_1 is given by $-c_1z_1$ and c_1 and η are positive constants. Note that η is introduced as the standard backstepping design may fail to satisfy desired closed-loop specifications. Using (21), $\dot{z}_1$ is derived as

$$\dot{z}_1 = (-c_1z_1 + \eta z_2)\bar{1}(\bar{u}). \tag{22}$$

Step 2: Differentiating z_2 results in

$$\begin{aligned}\dot{z}_2 \;&= \tfrac{1}{\eta}\{\tfrac{l_1}{\hat{h}_y}u + \tfrac{l_2}{\hat{h}_y} - \tfrac{h_xF\theta}{\hat{h}_y} - \dot{y}_d \\ &+(-c_1^2z_1 + c_1\eta z_2)\bar{1}(\bar{u})\}.\end{aligned} \tag{23}$$

Since the control input of the pH process is the flow rate of the titrating stream, it cannot be negative. Therefore we define the following control law

$$u \;=\; \max\{0,\; \bar{u}\}, \tag{24}$$

where

$$\begin{aligned}\bar{u} \;&= \tfrac{\hat{h}_y}{l_1}\{(c_1^2-\eta^2)z_1 - \eta(c_1+c_2)z_2 - \tfrac{l_2}{\hat{h}_y} \\ &+\tfrac{h_xF\hat{\theta}}{\hat{h}_y} + \dot{y}_d \;+\chi_u\},\end{aligned}$$

$$\chi_u = -k_3\eta\frac{l_5^2}{\hat{h}_y}z_2 - k_4\eta\frac{\|\eta^{-1}(h_xF)^T\|^2}{\hat{h}_y}z_2, \tag{25}$$

c_2 and k_i, $(i\;=\;3,\;4)$ are positive constants, and l_5 is an appropriate nonlinear function defined in the appendix. Note that the presence of z_1 in (25) implies integral action in the controller and the term χ_u is a nonlinear damping term to counteract the effects of the estimation errors $\widetilde{x}_2$ and $\widetilde{\theta}$, and also that $\bar{u}$ is the unconstrained backstepping control input. This controller is combined with the state and parameter estimators obtained in the previous subsection, thereby leading to the adaptive output feedback nonlinear controller. Stability properties of the resulting controller are investigated in the next section.

4 Stability Analysis

Consider the following assumptions which are needed for the analysis of stability and feasibility.

(A1) $q_1(t)(W_{a_1} - W_{a_3}) + q_2(t)(W_{a_2} - W_{a_3}) \geq 0,\ \forall t$
(A2) $q_1(t)(W_{b_1} - W_{b_3}) + q_2(t)(W_{b_2} - W_{b_3}) \geq 0,\ \forall t$
(A3) $x_1(0) > W_{a_3}$
(A4) $x_2(0) \geq W_{b_3}$
(A5) $W_{b_3} \geq 0$
(A6) $0 < q_1(t) < \infty,\ 0 < q_2(t) < \infty,\ \forall t$

Assumptions (A1)-(A4) are the minimum requirements for the operation of the given process [14, Remark 1]. Assumptions (A1)-(A5) are used to guarantee the feasibility of the parameter estimator and nonlinear controller, as is shown below. Assumption (A6) is used to obtain the stability properties of the closed-loop system, which is always satisfied in practice.

Lemma 4.1 *Suppose that Assumptions (A1)-(A5) and the following conditions are satisfied*

$$\widehat{x}_2(0) \geq W_{b_3},\ \widehat{\theta}_2(0) \geq \underline{\theta}_2. \tag{26}$$

Then the parameter estimator given in (16) and (17) and nonlinear controller (24) are feasible.

Proof. The proof of this lemma is similar to that of Theorem 1 in [14], and therefore is omitted. □

The next lemma concerns the stability properties of the estimation algorithms given in (8), (16), and (17).

Lemma 4.2 *Suppose that the state and parameter estimators given in (8), (16), and (17) are employed to estimate the concentrations of the reaction invariants of the effluent and acid flows of the pH process, and that* $u \geq 0$. *If Assumptions (A5) and (A6) and the conditions given in (26) are satisfied, then the following properties are guaranteed to hold:*

$$\|\widetilde{x}(t)\| < \infty,\;\; |\widetilde{y}(t)| < \infty,\;\; \|\widetilde{\theta}(t)\| < \infty,\;\; \forall t, \tag{27}$$

where $\widetilde{(\cdot)} = (\cdot) - \widehat{(\cdot)}$.

Proof. Assumption (A5) and the conditions given in (26) imply that the parameter estimator is feasible (from Lemma 4.1). Using (13)-(15), we have

$$\begin{aligned}\dot{\widetilde{y}} \;&= -k_e\widetilde{y} - \tfrac{1}{\hat{h}_yh_y}(l_1u + l_2)h_{y_2}\widetilde{x}_2 \\ &+\tfrac{1}{\hat{h}_yh_y}h_{y_2}\widetilde{x}_2h_xF\theta - \tfrac{1}{\hat{h}_y}h_xF\widetilde{\theta} + \chi_e \\ &= -k_e\widetilde{y} + \chi_e - l_3\tfrac{\widetilde{x}_2}{\hat{h}_y} + l_4^T\tfrac{\theta\widetilde{x}_2}{h_y} - \tfrac{1}{\hat{h}_y}h_xF\widetilde{\theta}.\end{aligned}$$

It follows that

$$\tfrac{d}{dt}\left(\tfrac{1}{2}\widetilde{y}^2\right) \leq -k_e\widetilde{y}^2 + l_6\widetilde{x}_2^2 - \widetilde{y}\tfrac{1}{\hat{h}_y}h_xF\widetilde{\theta}.$$

where l_6 is an appropriate nonlinear function defined in the appendix. Consider the following function

$$V_e = \tfrac{1}{d}\widetilde{x}^TP\widetilde{x} + \tfrac{1}{2}\widetilde{y}^2 + \tfrac{1}{2\gamma_1}\widetilde{\theta}_1^2 + \tfrac{1}{2\gamma_2}\widetilde{\theta}_2^2,$$

where d is a positive constant such that $d < \frac{(q_1+q_2)\lambda_m(P)}{2V}l_6^{-1}$. Since $u \geq 0$, differentiating this gives

$$\begin{aligned}\dot{V}_e \;&= -\tfrac{2v}{d}\widetilde{x}^TP\widetilde{x} - \tfrac{1}{d}\tfrac{(\bar{v}-v)(\psi_1\widetilde{x}_1+\psi_2\widetilde{x}_2)^2}{(2p_2\psi_1^2-2p_1\psi_1\psi_2+2\psi_2^2)} \\ &+\tfrac{1}{d}\left(2\tfrac{q_1}{V}\widetilde{\theta}_1 + p_1\tfrac{q_1}{V}\widetilde{\theta}_2\right)\widetilde{x}_1 + \tfrac{1}{d}\left(p_1\tfrac{q_1}{V}\widetilde{\theta}_1 + 2p_2\tfrac{q_1}{V}\widetilde{\theta}_2\right)\widetilde{x}_2 \\ &+\tfrac{d}{dt}\left(\tfrac{1}{2}\widetilde{y}^2\right) + \tfrac{1}{\gamma_1}\widetilde{\theta}_1(-\dot{\widehat{\theta}}_1) + \tfrac{1}{\gamma_2}\widetilde{\theta}_2(-\dot{\widehat{\theta}}_2) \\ &\leq -\tfrac{(q_1+q_2)}{2dV}\widetilde{x}^TP\widetilde{x} - k_e\widetilde{y}^2 + \widetilde{\theta}_1\left(-\tfrac{q_1\widetilde{y}}{V\hat{h}_y} - \tfrac{1}{\gamma_1}\dot{\widehat{\theta}}_1\right) \\ &+\widetilde{\theta}_2\left(-\tfrac{q_1\widetilde{y}h_{x_2}}{V\hat{h}_y} - \tfrac{2q_1^2p_1(p_2+1)}{d(q_1+q_2)\lambda_m(P)V}\widetilde{\theta}_1 - \tfrac{1}{\gamma_2}\dot{\widehat{\theta}}_2\right) \\ &+\tfrac{q_1^2(p_1^2+4)}{2d(q_1+q_2)\lambda_m(P)V}\widetilde{\theta}_1^2 + \tfrac{q_1^2(p_1^2+4p_2^2)}{2d(q_1+q_2)\lambda_m(P)V}\widetilde{\theta}_2^2.\end{aligned}$$

It then follows from (A6), (16), and (17) that

$$\tfrac{d}{dt}\sqrt{V_e} \leq -\kappa_1\sqrt{V_e} + \kappa_2,$$

where κ_1 and κ_2 are positive constants. Hence $V_e(t) < \infty,\ \forall t$, which guarantees the properties in (27). □

Having shown the stability properties of the state and parameter estimators, we now prove the stability of the overall adaptive output feedback nonlinear control system.

Theorem 4.1 *Suppose that the set-point is constant, and that Assumptions (A1)-(A6) and the conditions given in (26) are satisfied. If the overall adaptive output feedback nonlinear control scheme consisting of the estimators given in (8), (16), and (17) and the controller (24) is employed to control the pH process, then all the state variables of the closed-loop are bounded.*

Proof. From Lemma 4.1, the parameter estimator and nonlinear controller are feasible. Also from Assumptions (A2), (A4), and (A5), we have $x_2(t) \geq 0$, which implies that

$$h_y(t) = h_{y_1}(t) + x_2(t)h_{y_2}(t) > 0, \ \forall t. \qquad (28)$$

Consider the following positive definite function $V_c = \frac{1}{2}\|z\|^2$, where $z = [z_1 \ z_2]^T$.

(i) If $\bar{u} \leq 0$, then $u = 0$. The derivative of the function is

$$\dot{V}_c = \frac{1}{\eta}\left(\frac{l_2}{h_y} - \frac{h_x F\theta}{h_y}\right) \leq -l_7 V_c + \frac{l_8^2}{2l_7} + \frac{l_7}{2}{z_1}^2,$$

where z_1 remains unchanged when $u = 0$ and l_7, l_8 are appropriate nonlinear functions defined in the appendix. From (28) and Assumption (A6), the functions $l_7(>0)$, l_8 are bounded.

(ii) If $\bar{u} > 0$, then $u = \bar{u}$. The derivative of the function along the solution with the control law (24) is

$$\begin{aligned}\dot{V}_c &= -c_1 z_1^2 - c_2\frac{\widehat{h}_y}{h_y}z_2^2 + \frac{1}{\eta}z_2\frac{\widehat{h}_y}{h_y}\chi_u + l_5 z_2\frac{\widetilde{x}_2}{h_y} - \frac{1}{\eta}z_2\frac{h_x F\widetilde{\theta}}{h_y} \\ &\leq -2c_0 V_c + \left(\frac{\widetilde{x}_2^2}{4k_3 h_y} + \frac{\|\widetilde{\theta}\|^2}{4k_4 h_y}\right),\end{aligned}$$

where $c_0 = \min\{c_1, \ c_2 \inf_{t\geq 0}\{\widehat{h}_y(t)/h_y(t)\}\}$.

From (i) and (ii), $\dot{V}_c \leq -\kappa_3 V_c + \kappa_4$, where $\kappa_3 = \min\{l_7, \ 2c_0\}$, $\kappa_4 = \max\{\frac{l_8^2}{2l_7} + \frac{l_7}{2}z_1^2, \ \frac{\|\widetilde{x}_2^2\|_\infty}{4k_3 h_y} + \frac{\|\widetilde{\theta}\|_\infty^2}{4k_4 h_y}\}$. Then, from the property (27) given in Lemma 4.2 and (28), we have $\|z(t)\| < \infty$.

Consider the following positive definite function $V_x = \frac{1}{2}(x_1 - W_{a_3})^2 + \frac{1}{2}(x_2 - W_{b_3})^2$. Since $u \geq 0$, differentiating the function gives

$$\dot{V}_x \leq -\frac{1}{V}(q_1 + q_2 + u)V_x + \kappa_5 \leq -\frac{1}{V}(q_1 + q_2)V_x + \kappa_5,$$

where κ_5 is a constant. This leads to the boundedness of x_1 and x_2, which in turn results in the boundedness of the signals in the estimators (from Lemma 4.2). □

5 Simulation Results

All the parameters of the nonlinear controller and estimators are given in the appendix and $\bar{v}$ is set to $7v$. The initial values of the state variables are assumed to be $x_1(0) = -4.4997 \times 10^{-4}$, $x_2(0) = 5.5025 \times 10^{-4}$, and their initial estimates are set to $\widehat{x}_1(0) = -5.0 \times 10^{-3}$, $\widehat{x}_2(0) = 2.5 \times 10^{-4}$. The proposed adaptive output feedback nonlinear control scheme is simulated under the assumption that the concentrations of the reaction invariants in the acid stream are varying in range of $\pm$ 30% as that of Figure 2 in [14]. Figure 1 gives simulation results for the adaptive output feedback control scheme. The dotted lines indicate the set-point for y and the true parameter values W_{a_1} and W_{b_1} in Figure 1a, and the true values of x_1 and x_2 in Figure 1b. As demonstrated in Figure 1a, the regulation performance is as good as that in [14, Figure 5] where the regulation performance in the case of using the state measurements is presented. It is seen that the proposed adaptive output feedback controller leads to uniform closed-loop performance despite the nonlinearity, the uncertainties of the parameters, and the unavailability of the state variables.

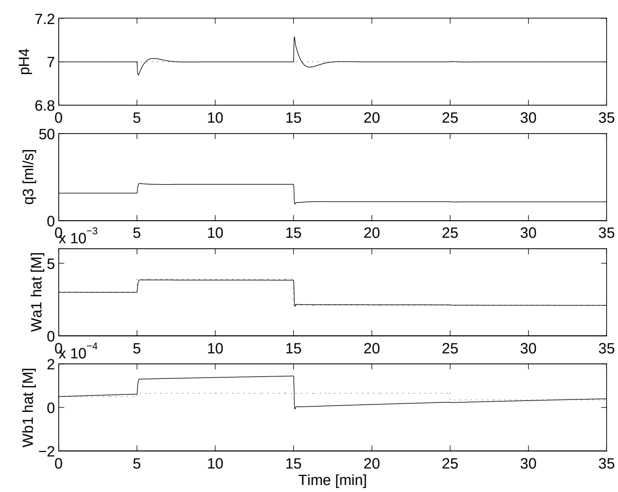

Figure 1a: The output, input, and parameter estimates.

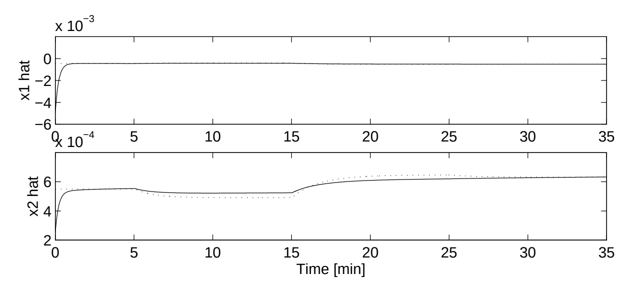

Figure 1b: The state variables and the estimates

6 Conclusions

This paper has presented an adaptive output feedback control scheme for a pH process with an input constraint on the basis of modular backstepping design. State and parameter estimators are designed to guarantee the boundedness of the estimation errors, and a nonlinear controller is obtained using a backstepping technique considering the physical constraint. Then the estimators and controller are combined with the nonlinear damping terms so that the all state variables of the closed-loop are bounded. The stability properties of the overall adaptive system are presented. Simulations show that uniform closed-loop performance is obtained despite the nonlinearity, the uncertainties of the parameters, and the unavailability of the state variables. More research is needed in the future for the convergence of the output tracking error.

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Appendix

Table 1: Parameters of the controller and estimators

Parameters	Values	Parameters	Values
c_1	0.0255	d	1.0
c_2	0.0255	p_1	-1.5
η	0.0255	p_2	25
k_e	1.0	γ_1	4.0×10^{-4}
k_1	1.0×10^{-5}	γ_2	4.0×10^{-5}
k_2	1.0×10^{-10}	σ_1	0.1
k_3	1.0×10^{-5}	σ_2	10.0
k_4	1.0×10^{-9}		

Table 2: Nonlinear functions used for the controller and estimator designs

Nonlinear functions	Definitions
l_1	$-\frac{1}{V}(10^{y-14} - 10^{-y} + W_{a_3} + h_{x_2} W_{b_3})$
l_2	$-\frac{1}{V}(q_1 + q_2)(10^{y-14} - 10^{-y})$ $-\frac{1}{V} q_2 (W_{a_2} + h_{x_2} W_{b_2})$
l_3	$-\frac{h_{y2}}{\hat{h}_y}(l_1 u + l_2)$
l_4	$-\frac{h_{y2}}{\hat{h}_y}(h_x F)^T$
l_5	$-\frac{1}{\eta} h_{y_2}(-\eta^2 z_1 + c_1^2 z_1 - c_1 \eta z_2)$
l_6	$\frac{1}{4k_1 h_y^2} + \frac{\lVert\theta\rVert^2}{4k_2 h_y^2}$
l_7	$\frac{(q_1+q_2)}{\eta V h_y}(10^{-y} + 10^{y-14})$
l_8	$-\frac{1}{\eta V h_y}\{\frac{1}{V} q_2 (W_{a_2} + h_{x_2} W_{b_2}) + \frac{1}{V} q_1 \theta_1$ $+\frac{1}{V} q_1 h_{x_2} \theta_2\}$