String Stability of Stochastic Composite Systems

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Abstract

The sufficient conditions of exponential string stability for a few class of nonlinear composite stochastic systems are established. The excitations are assumed to be parametric white noises. In this case the objective is to analyze composite systems in their lower order subsystems and in terms of their interconnecting structure. The cases of exponential string stability for weak coupling systems, vehicle-following systems and l_2 string stability for weak coupling systems are considered.

1. Introduction

The problem of string stability of interconnected deterministic systems was studied earlier, for instance, in [1], [5], [6], and recently in [2],[10], [11]. For further references the reader is referred to the survey paper by Shladover [7] and the references therein. In particular, there have been several definitions for string stability, for instance, Chu defined string stability in the context of vehicle following [1], Swaroop and Hedrick introduced a kind of Lyapunov stability for interconnected systems [11]. In this last paper the authors have obtained the sufficient conditions of string stability for nonlinear weakly coupled subsystems. They have also discussed their robustness to structural and singular perturbations. In contrast, to our knowledge the stability analysis of nonlinear composite stochastic systems has not been developed. This paper is devoted to a detailed study of sufficient conditions of exponential string stability of nonlinear composite stochastic systems. To derive these conditions the idea of the exponential p-stability of stochastic systems is combined with the concept of string stability for nonlinear composite deterministic systems presented in [11].

2. Mathematical preliminaries

We consider the nonlinear autonomous composite stochastic system which is described by the Ito equation

$$dx^i = f(x^i, x^{i-1}, \dots, x^{i-r+1})dt + q(x^i, x^{i-1}, \dots, x^{i-r+1})d\xi^i, \quad \mathbf{(1)}$$

where $i\in\mathbb{N}$, $t\in[t_0, +\infty)$, x^i is the state of each subsystem, $x^i \in R^n$, $x^{i-j}\equiv 0$ for all $i\le j$, f and q are vector nonlinear functions $f,q\colon R^n\times\dots\times R^n \to R^n$ such that $f(0,\dots,0) = q(0,\dots,0) = 0$, and ξ^i are independent standard Wiener processes. For simplicity we assume that $t_0 = 0$.

We use the following notation: $|.|$ is the Euclidean norm; for all $p<\infty$ $\quad ||f_i||^p_\infty = ||f_i(.)||^p_\infty = \sup_{t\ge 0} E[|f_i(t)|^p]$,

$$||f(0)||^p_\infty = \sup_{i\in\mathbb{N}} E[|f_i(0)|^p] ,$$

$$||f_i||_p = ||f_i(.)||_p = \left(\int_0^\infty E|f_i(t)|^p dt\right)^{\frac{1}{p}} ,$$

$$||f(0)||_p = \left(\sum_{i=1}^\infty E|f_i(0)|^p\right)^{\frac{1}{p}} .$$

System (1) is treated as an interconnection of isolated subsystems described by

$$dx^i = f(x^i, 0, \dots, 0)dt + q(x^i, 0, \dots, 0)d\xi^i, \ i\in\mathbb{N} \quad \mathbf{(2)}$$

Definition 1. The equilibrium $x^i = 0$, $i\in\mathbb{N}$ of system (1) is p-mean string stable if given $\varepsilon\ge 0$ there exists a $\delta>0$ such that by

$$||x^i(0)||^p_\infty < \delta \ \Rightarrow \ \sup_{i\in\mathbb{N}} ||x^i(.)||^p_\infty < \varepsilon \quad \mathbf{(3)}$$

Definition 2. The equilibrium $x^i = 0$, $i\in\mathbb{N}$ of system (1) is exponentially string p-stable if it is p-mean string stable and if there exist positive constants c_i and α_i, such that by

$$E|x^i(t)|^p < c_i |x_0^i|^p \exp\{-\alpha_i(t-t_0)\} \quad \mathbf{(4)}$$

for all $i\in\mathbb{N}$. In particular cases for p=1 and p=2 it is called mean and mean-square string stability, respectively.

Definition 3. The equilibrium $x^i = 0$, $i\in\mathbb{N}$ of system (1) is l_p string mean stable if for all $\varepsilon\ge 0$ there exists a $\delta>0$ such that by

$$||x^i(0)||_p < \delta \ \Rightarrow \ \sup_{t\ge 0} \left(\sum_{i=1}^\infty E[|x^i(t)|^p]\right)^{\frac{1}{p}} < \varepsilon \quad \mathbf{(5)}$$

To assure the readability of this paper we will quote the basic results on exponential p-stability obtained in [3], [4].

Theorem 1: The equilibrium $x^i=0$, $i\in\mathbb{N}$ of system (2) is exponentially p-stable if there exists a positive definite function $V(t,x^i)$ $i\in\mathbb{N}$ continuously differentiable with respect to t and twice differentiable with respect to x^i and there exist positive constants γ_{ik}, $i\in\mathbb{N}$, $k=1,2,3$ such that

$$\gamma_{i1}|x^i|^p \le V(t,x^i) \le \gamma_{i2}|x^i|^p \quad (6)$$

$$\mathfrak{L}^*_{(2)}(V(t,x^i)) \le -\gamma_{i3}|x^i|^p \quad (7)$$

where $\mathfrak{L}^*_{(2)}(.)$ is the operator associated with equation (2) defined by

$$\mathfrak{L}^*_{(2)}(.) = \frac{\partial(.)}{\partial t} + \sum_{j=1}^{n} f_j(x^i,0...,0)\frac{\partial(.)}{\partial x_j^i} + \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\sigma_{lj}(x^i,0...,0)\frac{\partial^2(.)}{\partial x_j^i\partial x_l^i} \quad (8)$$

$$\sigma_{lj}(x^i,0...,0) = q_l(x^i,0...,0)q_j(x^i,0....,0)$$

3. Weak coupling systems

In proof of main criterion we will use the following lemmas.

Lemma 1 [11]: Let r be a constant positive integer. Define $P_r(z)=z^r-\sum_{j=1}^{r}\beta_j z^{r-j}$, $\beta_j>0$. If $\sum_{j=1}^{r}\beta_j<1$ the r-th degree polynomial $P_r(z)$ has all its roots inside the unit circle.

Lemma 2. Let $V_i(t)=V_i(t,x^i(t))\ge 0$ for all $i\in\mathbb{N}$, $t\ge 0$, $x^i\in R^n$ and if

$$\mathfrak{L}^*_{(1)}V_i(t) \le \left(-\beta_0 V_i(t) + \sum_{j=1}^{\infty}\beta_j V_{i-j}(t)\right)\exp\{\beta_0 t\} \quad (9)$$

with $\beta_0>0$ and $\beta_j\ge 0$ for j=1,2,... and $\beta_0>\sum_{j=1}^{\infty}\beta_j$; $V_j(t)=0$ for all $j\le 0$. Then given any $\varepsilon\ge 0$ there exists a $\delta>0$ such that by

$$||V_i(0)||_\infty<\delta \;\Rightarrow\; \sup_{i\in\mathbb{N}}||V_i(.)||_\infty<\varepsilon \quad (10)$$

Proof: We observe that for the function $W_i(t)=W_i(t,x^i(t))$ defined by

$$W_i(t,x^i(t)) = V_i(t,x^i(t))\exp\{\beta_0 t\} - \int_0^t \exp\{\beta_0 s\}\sum_{j=1}^{\infty}\beta_j V_{i-j}(s,x^{i-j}(s))\,ds \quad (11)$$

condition (9) implies that $\mathfrak{L}^*_{(1)}W_i(t)\le 0$. Hence, using the properties of operator $\mathfrak{L}^*$ [4] we obtain

$$E[V_i(t)\exp\{\beta_0 t\}] - E[V_i(0)] - \int_0^t \exp\{\beta_0 s\}\sum_{j=1}^{\infty}\beta_j E[V_{i-j}(s)]\,ds \le 0 \quad (12)$$

or

$$E[V_i(t)] \le E[V_i(0)]\exp\{-\beta_0 t\} + \int_0^t \exp\{-\beta_0(t-s)\}\sum_{j=1}^{\infty}\beta_j E[V_{i-j}(s)]ds \quad (13)$$

From inequality (13) we find

$$\sup_{t>0} E[V_i(t)] \le E[V_i(0)] + \sum_{j=1}^{\infty}\frac{\beta_j}{\beta_0}\sup_{t>0} E[V_{i-j}(t)] \quad (14)$$

or

$$||V_i||_\infty \le E[V_i(0)] + \sum_{j=1}^{\infty}\frac{\beta_j}{\beta_0}||V_{i-j}||_\infty \quad (15)$$

To prove condition (10) it suffices to show that $||V_i||_\infty \le M||V_i(0,x_0^i)||_\infty$ where $M^{-1}=1-\sum_{j=1}^{\infty}\frac{\beta_j}{\beta_0}<1$. It can be done by induction as in [11] with small changes. Hence it follows that $\sup_{i\in\mathbb{N}}||V_i||_\infty \le M||V_i(0,x_0^i)||_\infty$.

We introduce the following assumptions

Assumption 1. The functions f and q are continuously differentiable with respect to their arguments and there exist a real number $M_1\ge 0$ such that for all $x^i\in R^n$, $i\in\mathbb{N}$

$$\left|\frac{\partial f_l}{\partial x_j^i}\right| \le M_1, \quad \left|\frac{\partial q_l}{\partial x_j^i}\right| \le M_1, \quad l,j=1,...,n,\ i\in\mathbb{N} \quad (16)$$

Assumption 2. There exists a positive definite function $V_i=V(x^i)$, $i\in\mathbb{N}$ continuously twice differentiable with respect to x_j^i and there exist positive constants α_x and γ_k, $k=1,...,4$ such that the following inequalities are satisfied

$$\gamma_1|x^i|^2 \le V(x^i) \le \gamma_2|x^i|^2 \quad (17)$$

$$\mathfrak{L}^*_{(2)}V(x^i) \le -2\alpha_x V(x^i) \quad (18)$$

$$\left|\frac{\partial V(x^i)}{\partial x_j^i}\right| \le \gamma_3|x|, \quad \left|\frac{\partial^2 V(x^i)}{\partial x_l^i\partial x_j^i}\right| \le \gamma_4, \quad l,j=1,...,n,\ i\in\mathbb{N} \quad (19)$$

Theorem 2. Suppose that Assumptions 1 and 2 (for system (2)) hold and additionally the following conditions are satisfied.

Assumption 3. There exist positive constants k_l^f and k_l^q l=1,..., r such that f and q are globally Lipschitz in their arguments i.e.

$$|f(y_1,...,y_r) - f(z_1,...,z_r)| \le \sum_{l=1}^{r} k_l^f |y_l - z_l| \qquad \textbf{(20)}$$

$$|q(y_1,...,y_r) - q(z_1,...,z_r)| \le \sum_{l=1}^{r} k_l^q |y_l - z_l| \qquad \textbf{(21)}$$

Then for sufficiently small k_l^f and k_l^q l=1,..., r the composite system (1) is globally exponentially mean-square string stable.

Proof: For simplicity we denote $V_i = V(x^i)$. We calculate $\mathfrak{L}^*(V)$ for each subsystem of composite system (1)

$$\begin{aligned}\mathfrak{L}^*_{(1)}(V_i) &= \sum_{j=1}^{n} \frac{\partial V_i}{\partial x_j^i} f_j(x^i,0,...,0) \\ &+ \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 V_i}{\partial x_l^i \partial x_j^i} \sigma_{lj}(x^i,0,...,0) \\ &+ \sum_{j=1}^{n} \frac{\partial V_i}{\partial x_j^i}[f_j(x^i,x^{i-1},...,x^{i-r+1}) - f_j(x^i,0,...,0)] \\ &+ \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 V_i}{\partial x_l^i \partial x_j^i} \\ &\times[\sigma_{lj}(x^i,x^{i-1},...,x^{i-r+1}) - \sigma_{lj}(x^i,0,...,0)]\end{aligned} \qquad \textbf{(22)}$$

where

$$\sigma_{lj} = q_l(x^i,x^{i-1},...,x^{i-r+1})q_j(x^i,x^{i-1},...,x^{i-r+1}) \qquad \textbf{(23)}$$

Using equation (22) and Assumptions 1-3 one can show that

$$\mathfrak{L}^*_{(1)}(V_i) \le -\delta_0 V_i + \sum_{j=2}^{r} \delta_j V_{i-j+1} \qquad \textbf{(24)}$$

where

$$\begin{aligned}\delta_0 &= 2\alpha_x - \frac{\gamma_3 n}{2\gamma_1}\sum_{j=2}^{r} k_j^f - M_1 n^2 \frac{\gamma_4}{\gamma_1}\sum_{j=2}^{r} k_j^q \\ \delta_j &= \frac{\gamma_3 n}{2\gamma_1}k_j^f + M_1 n^2 \frac{(r+1)\gamma_4}{2\gamma_1}k_j^q + M_1 n^2 \frac{\gamma_4}{2\gamma_1}\sum_{s=2}^{r} k_s^q\end{aligned} \qquad \textbf{(25)}$$

If k_l^f and k_l^q l=1,..., r are sufficiently small then

$$\delta_0 > \sum_{j=2}^{r} \delta_j \qquad \textbf{(26)}$$

From conditions (24)-(26) and lemmas 1 and 2 follows mean string stability.

If we consider the composite system (1) and we define for him the Lyapunov function $V(d^{-1},t) = \sum_{i=1}^{\infty} V_i(t) d^{-i}$, where $d>1$ then we find

$$\mathfrak{L}^*_{(1)}(V) = \sum_{i=1}^{\infty} \mathfrak{L}^*_{(1)}(V_i) d^{-i} \le \sum_{i=1}^{\infty}\left[-\delta_0 V_i d^{-i} + \sum_{l=2}^{r} \delta_l V_{i-l+1} d^{-i}\right] \qquad \textbf{(27)}$$

Using similar arguments as in [11] one can show that $E[V] \to 0$ exponentially and hence $E[V_i(t)], E[|x^i(t)|^2] \to 0$ exponentially.

4. Vehicle-following system

A simple class of interconnected deterministic systems that arise in the in the context of vehicle-following systems is given by [11]

$$\acute{x}^i = f(x^i, x^{i-1}, \acute{x}^{i-1}) \qquad \textbf{(28)}$$

The natural generalization of equation (28) to the stochastic model leads to the following Ito equation

$$dx^i = f^i(x^i,x^{i-1},f^{i-1}(.))dt + q^i(x^i,x^{i-1},q^{i-1}(.))d\xi^i, \qquad \textbf{(29)}$$

where $i \in \mathbb{N}$, $t \in [t_0, +\infty)$, x^i is the state of each subsystem, $x^i \in R^n$, $x^{i-j} \equiv 0$ for all $i \le j$, f^i and q^i are vector nonlinear functions $f^i = f, q^i = q$: $R^n \times R^n \times R^n \to R^n$ such that $f^i(0,0,0) = q^i(0,0,0) = 0$, and ξ^i are independent standard Wiener processes. System (29) is treated as an interconnection of isolated subsystems described by

$$dx^i = f(x^i,0,0)dt + q(x^i,0,0)d\xi^i, \; i \in \mathbb{N} \qquad \textbf{(30)}$$

The following theorem establish the sufficient conditions of exponential mean-square string stability

Theorem 3. Suppose that Assumptions 1 and 2 (for system (30)) hold and additionally the following conditions are satisfied.

Assumption 4. There exist positive constants $k_1^f, k_2^f, d_1^f, k_1^q, k_2^q$ and d_1^q such that f and q are globally Lipschitz in their arguments i.e.

$$\begin{aligned}&|f(y_1,y_2,y_3) - f(z_1,z_2,z_3)| \\ &\le k_1^f|y_1 - z_1| + k_2^f|y_2 - z_2| + d_1^f|y_3 - z_3|\end{aligned} \qquad \textbf{(31)}$$

$$\begin{aligned}&|q(y_1,y_2,y_3) - q(z_1,z_2,z_3)| \\ &\le k_1^q|y_1 - z_1| + k_2^q|y_2 - z_2| + d_1^q|y_3 - z_3|\end{aligned} \qquad \textbf{(32)}$$

Then for sufficiently small $k_1^f, k_2^f, d_1^f, k_1^q, k_2^q$ and d_1^q the composite system (29) is globally exponentially mean-square string stable.

Proof: We calculate $\mathfrak{L}^*(V)$ for each subsystem of composite system (29)

$$\begin{aligned}\mathfrak{L}^*_{(29)}(V_i) &= \sum_{j=1}^{n} \frac{\partial V_i}{\partial x_j^i} f_j(x^i,0,0) \\ &+ \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 V_i}{\partial x_l^i \partial x_j^i} \sigma_{lj}(x^i,0,0) \\ &+ \sum_{j=1}^{n} \frac{\partial V_i}{\partial x_j^i}[f_j(x^i,x^{i-1},f(.)) - f_j(x^i,0,0)] \\ &+ \frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 V_i}{\partial x_l^i \partial x_j^i}[\sigma_{lj}(x^i,x^{i-1},f(.)) - \sigma_{lj}(x^i,0,0)]\end{aligned} \qquad \textbf{(33)}$$

where

$$\sigma_{lj} = q_l(x^i, x^{i-1}, q(.))q_j(x^i, x^{i-1}, q(.)) \quad \textbf{(34)}$$

Using equation (33) and Assumptions 1-3 one can show that

$$\mathfrak{L}^*_{(36)}(V_i) \le -\delta_0 V_i + \delta_1 V_{i-j} + \delta_2 V_{i+1} \quad \textbf{(35)}$$

where

$$\delta_0 = 2\alpha_x - \frac{n\gamma_3}{2\gamma_1}(k_2^f + d_1^f k_1^f)\sum_{j=1}^{i-1}(d_1^f)^{j-1} - n^2\frac{\gamma_4}{4\gamma_1}(k_1^q + nM_1)(k_2^q + d_1^q k_1^q)\sum_{j=1}^{i-1}(d_1^f)^{j-1}$$

$$\delta_j = \frac{n\gamma_3}{2\gamma_1}(k_2^f + d_1^f k_1^f)(d_1^f)^{j-1} + n^2\frac{\gamma_4}{4\gamma_1}(k_2^q + d_1^q k_1^q)^2(d_1^q)^{j-1}\sum_{j=1}^{i-1}(d_1^f)^{j-1} + n^2\frac{\gamma_4}{4\gamma_1}(k_1^q + nM_1)(k_2^q + d_1^q k_1^q)(d_1^q)^{j-1} \quad \textbf{(36)}$$

For sufficiently small $k_1^f, k_2^f, d_1^f, k_1^q, k_2^q$ and d_1^q one can show (similarly to previous proof) global exponential string mean-square stability.

5. Weak coupling for l_2 string stability

Consider the following composite system in which every subsystem is connected to its neighbouring subsystems

$$dx^i = f(x^{i-1}, x^i, x^{i+1})dt + q(x^{i-1}, x^i, x^{i+1})d\xi^i, \quad \textbf{(37)}$$

where $i\in\mathbb{N}$, $t\in[t_0, +\infty)$, x^i is the state of each subsystem, $x^i\in R^n$, $x^{i-j}\equiv 0$ for all $i\le j$, f and q are vector nonlinear functions $f,q:\ R^n\times R^n\times R^n \to R^n$ such that $f(0,0,0) = q(0,0,0) = 0$, and ξ^i are independent standard Wiener processes.
System (37) is treated as an interconnection of isolated subsystems described by

$$dx^i = f(0, x^i, 0)dt + q(0, x^i, 0)d\xi^i, \ i\in\mathbb{N} \quad \textbf{(38)}$$

The following theorem establish the sufficient conditions of exponential l_2 mean string stability

Theorem 4. Suppose that Assumptions 1 and 2 (for system (38)) hold and additionally the following conditions are satisfied.

Assumption 5. There exist positive constants k_l^f and k_l^q l=1,2,3 such that f and q are globally Lipschitz in their arguments i.e.

$$|f(y_1,y_2,y_3) - f(z_1,z_2,z_3)| \le \sum_{l=1}^{3} k_l^f|y_l - z_l| \quad \textbf{(39)}$$

$$|q(y_1,y_2,y_3) - q(z_1,z_2,z_3)| \le \sum_{l=1}^{3} k_l^q|y_l - z_l| \quad \textbf{(40)}$$

Then for sufficiently small $k_1^f, k_2^f, d_1^f, k_1^q, k_2^q$ and d_1^q the composite system (37) is globally exponentially l_2 mean string stable.

Proof: As in Theorem 2 we find for each subsystem of composite system (37)

$$\mathfrak{L}^*_{(36)}(V_i) \le -\delta_0 V_i + \delta_1 V_{i-j} + \delta_2 V_{i+1} \quad \textbf{(41)}$$

where

$$\delta_0 = 2\alpha_x - \frac{n\gamma_3}{2\gamma_1}(k_1^f + k_3^f) - \frac{n^3\gamma_4}{2\gamma_1}M_1(k_1^q + k_3^q)$$

$$\delta_1 = \frac{n\gamma_3}{2\gamma_1}k_1^f + \frac{n^3\gamma_4}{2\gamma_1}M_1[(k_1^q)^2 + k_1^q k_3^q + k_1^q] \quad \textbf{(42)}$$

$$\delta_2 = \frac{n\gamma_3}{2\gamma_1}k_3^f + \frac{n^3\gamma_4}{2\gamma_1}M_1[(k_3^q)^2 + k_1^q k_3^q + k_3^q]$$

We define for the composite system (37) the Lyapunov function $V(d^{-1},t) = \sum_{i=1}^{\infty} V_i(t)d^{-i}$, where $d>1$. Then using similar arguments as in [11] one can show that for sufficiently small k_1^f, k_3^f, k_1^q and k_3^q the composite system (37) is globally exponentially l_2 mean string stable.

6. Example

Consider a special case of equations (1) and (2) for scalar quantities x^i, $i\in\mathbb{N}$, $t\in[t_0, +\infty)$: for composite system

$$dx^i = [f(x^i) + \sum_{j=1}^{r} a_{i-j+1}x^{i-j+1}]dt + [g(x^i) + \sum_{j=1}^{r} q_{i-j+1}x^{i-j+1}]d\xi^i, \ i\in\mathbb{N} \quad \textbf{(43)}$$

and for isolated subsystems

$$dx^i = [f(x^i) + a_i x^i]dt + [g(x^i) + q_i x^i]d\xi^i, \ i\in\mathbb{N} \quad \textbf{(44)}$$

where a_i and q_i are constant parameters, f and q are nonlinear functions such that $0\le f(x^i)\le M_2(x^i)^2$, $f(0)=0$, $0\le g(x^i)\le M_3(x^i)^2$, $g(0)=0$, $\left|\frac{\partial g(x)}{\partial x}\right| < M_1$ and ξ^i are independent standard Wiener processes.
Assume that the Lyapunov function for each subsystem has the form

$$V_i = V(x^i) = \alpha_i(x^i)^2, \ i\in\mathbb{N} \quad \textbf{(45)}$$

where $\alpha_i>0$ are constant parameters. The sufficient conditions for exponential mean-square stability for each subsystem have the form

$$2M_2 + 2a_i + M_3^2 + 2M_3 q_i + q_i^2 \le 0 \qquad \textbf{(46)}$$

Repeating considerations given in section 3 we obtain

$$\begin{aligned}\mathfrak{L}^*_{(43)}(V_i) &= 2\alpha_i[f(x^i) + \sum_{j=1}^{r} a_{i-j+1}x^{i-j+1}] \\ &+ \alpha_i[g(x^i) + \sum_{j=1}^{r} q_{i-j+1}x^{i-j+1}]^2 \qquad \textbf{(47)} \\ &\le -\delta_0 V_i + \sum_{j=2}^{r} \delta_j V_{i-j+1}, \quad i\in\mathbb{N}\end{aligned}$$

where

$$\begin{aligned}\delta_0 &= -2M_2 + 2\sum_{j=1}^{r} a_{i-j+1} + M_3^2 \\ &+ 2M_3 q_i + q_i^2 + (M_1 + q_i)\sum_{j=2}^{r} q_{i-j+1} \qquad \textbf{(48)}\end{aligned}$$

$$\delta_j = \frac{\alpha_i}{\alpha_{i-j+1}}\left[2a_{i-j+1} + (M_1 + \sum_{l=1}^{r} q_{i-l+1})q_{i-j+1}\right]$$

If constant parameters $M_1, M_2, M_3, \alpha_i, a_i$ and q_i, $i\in\mathbb{N}$ satisfy condition (46) for each isolated subsystem and

$$\delta_0 > \sum_{j=2}^{r} \delta_j \qquad \textbf{(49)}$$

then the composite system (43) is globally exponentially mean-square string stable. If we assume, for instance, $M_1 = M_2 = M_3 = M$, $\alpha_i = \alpha$, $a_i = a$ and $q_i = q$, $i\in\mathbb{N}$ the sufficient conditions of stability (46) and (48), (49) reduce to the following

$$a < -M(q+1) - \frac{1}{2}(M^2 + q^2) \qquad \textbf{(50)}$$

and

$$a < -\frac{1}{2r-1}[(M(qr+1) + \frac{1}{2}(M^2 + rq^2(r-1))] \qquad \textbf{(51)}$$

7. Conclusions and final remarks

In this paper the problem of exponential string stability of autonomous composite nonlinear systems with parametric white noise excitations has been studied. Sufficient conditions of exponential string stability for weak coupling systems, vehicle-following systems and l_2 string stability for weak coupling systems have been derived. Although we have considered only the systems described by Ito stochastic differential equations the proposed approach can be applied directly for the system with wide band noise. Then instead of Ito equations (1), (29) or (37) we consider corresponding Stratonovich stochastic differential equation which first we transform to Ito form and next we apply the proposed approach. Several extensions can be done, for instance, the main criterion (theorem 2) can be generalized to nonautonomous composite systems. The string stability can be considered for nonlinear systems with parametric excitations which obey the law of large numbers using the stability criteria obtained in [8]. Another extension can be done for nonlinear stochastic composite systems with singular perturbations where the results obtained in [9] can be used.

8.References

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