Blockwise Subspace Identification for Active Noise Control[1]

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Abstract

In this paper, a subspace identification solution is provided for Active Noise Control (ANC) problems. The solution is related to so-called *block updating methods*, where instead of updating the (feedforward) controller on a sample by sample base, it is updated each time based on a block of N samples. The use of the subspace identification based ANC methods enables *non-iterative* derivation and *updating* of MIMO compact state space models for the controller. The robustness property of subspace identification methods forms the basis of an accurate model updating mechanism, using small size data batches. The design of a feedforward controller via the proposed approach is illustrated for an acoustic duct benchmark problem, supplied by TNO Institute of Applied Physics (TNO-TPD), the Netherlands. We also show how to cope with intrinsic feedback. A comparison study with various ANC schemes, such as block Filtered-U demonstrates the increased robustness of a subspace derived controller.

Keywords: Active noise control, active vibration control, subspace identification, state-space model, block Filtered-U LMS, acoustical duct.

1 Introduction

The Active Noise Control (ANC) problem considers the rejection of a noise signal (sound or vibration noise) in a particular region by actively producing anti-noise. Figure 1 shows the ANC-setup studied in this paper. Here $x(n)$ represents the noise produced by the noise-source and the error signal $e(n)$ the residual noise. The primary noise represented by $d(n)$ is the noise via the primary path, characterized by the operator P. The secondary or artificially produced noise $y(n)$, via the controller W and the secondary path S, should counteract the primary noise. The summation of both signals is the measured error signal $e(n)$. The influence of the control signal on the measurable disturbance signal, called intrinsic feedback and characterized by the operator F, is an additional difficulty in ANC. Intrinsic feedback causes a disturbance $x_f(n)$ on the noise $x(n)$ from the noise source, resulting the measured disturbance signal $x_m(n) = x(n) + x_f(n)$. In the paper, all operators are assumed to be linear (time-varying) dynamical systems.

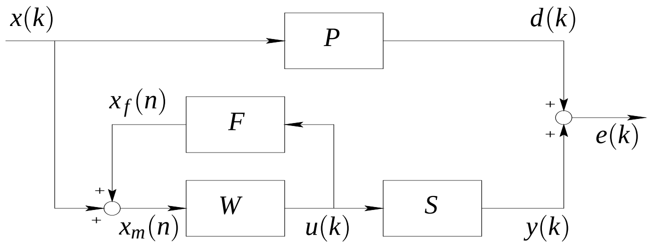

Figure 1: Blockscheme of feedforward ANC, with disturbance $d(n)$, noise source signal $x(n)$, $x_f(n)$ the influence of intrinsic feedback on $x(n)$, measured disturbance $x_m(n)$, control signal $u(n)$, secondary noise $y(n)$ and error signal $e(n)$. The operators P, S, F and W are the primary and the secondary path, the intrinsic feedback and the feedforward controller respectively.

Many strategies to design the (feedforward) controller W, have been developed in the past. One major family of ANC methods have their roots in the field of (recursive) system identification and aim to minimize the variance of the error $e(n)$ in the scalar case, and $\mathrm{tr}(\mathrm{E}[e(n)e^T(n)])$ in the vector case. Using a model of the primary path P and secondary

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path S, neglecting intrinsic feedback F, the H_2 optimal control design procedure developed by Wiener [7], in case all operators are LTI, can be used to design W [8]. Because a model of the primary path is usually not available, recursive methods like the classical LMS algorithm [14], and its variants like Filtered-X LMS (FxLMS) [15, 2] for FIR controllers and the Filtered-U LMS (FuLMS) [4] for IIR controllers, are widely used in ANC [16, 9]. Recently the link with off-line system identification has been tightened by the development of block LMS type updating algorithms. These block update algorithms reduce the computational complexity of the sample-by-sample LMS algorithms by implementations in the frequency domain using the FFT [5].

However, both sample-by-sample and block LMS algorithms can give rise to slow convergence, stepsize choice and significant filter lengths and in case of IIR controllers the possibility of becoming unstable. Another drawback is the large number of coefficients of multiple-input/multiple output (MIMO) controllers, which can be reduced by using state-space controllers as is explained in [11].

In this paper, we propose the use of recently developed subspace model identification (SMI) methods [13] for the determination of a *low order* MIMO state-space controller and its update based on a block of measurements. Though SMI doesn't minimize the variance of $e(n)$ explicitly, it gives a better estimation of the optimal H_2 controller than the ARX identification method. This is because subspace identification gives a controller which is accurate in the whole frequency band which is excited, while ARX identification gives only an accurate estimate of the controller at the higher frequencies in this band at the expense of accuracy at lower frequencies [10]. Though SMI algorithms are able to identify a LTI state-space model under very general noise conditions in a non-iterative manner, its real-time performances may be hampered by a high computational requirement. However, this 'mental obstacle' might be dealt with in practical applications by fully exploiting the robustness properties of SMI tools to extract accurate MIMO model of complex systems based on small size data batches [1] and exploiting the structure of the matrices in SMI algorithms to develop fast algorithms, such as in [3]. In the paper we further show, that intrinsic feedback can be incorporated using the Youla parameterization [17]. The main focus of the current paper is on deriving a *robust* procedure for tuning IIR controllers for ANC. Numerical issues and (measurement) noise issues are studied later.

The paper is organized as follows. Section 2 briefly describes the key concepts of SMI. Section 3 describes the identification and updating of the feedforward controller by SMI. Section 4 compares this method with block LMS type methods. Section 5 describes simulation results of the method on a realistic acoustical duct model and Section 6 presents some preliminary conclusions.

2 Subspace identification

Consider the state-space representation of the controller W

$$\begin{aligned} z(n+1) &= A_w z(n) + B_w x(n) \\ u(n) &= C_w z(n) + D_w x(n) \end{aligned}$$

with $x(n) \in \mathbb{R}^p$ the input, $u(n) \in \mathbb{R}^m$ the output and $z(n) \in \mathbb{R}^r$ the state and r the order of the controller, and the system matrices A_w, B_w, C_w and D_w of appropriate dimensions. The subspace identification problem is to estimate a quadruple (A_w, B_w, C_w, D_w) (up to a similarity transformation) based on the block of measured input/output data $\{x(n), u(n)\}_{n=1}^{N}$ such that the predicted output $u_p(n)$ approximates the measured output $u(n)$. The subspace identification schemes yield a consistent estimate of the order r of the system and a quadruple (A_w, B_w, C_w, D_w) (up to a similarity transformation). The main steps of the identification of the state-space model via the MOESP approach [13, 6] are:

1. Using input/output data estimate the column space of the extended observability matrix Γ_0, given by

$$\Gamma_0 = \begin{bmatrix} C_w \\ C_w A_w \\ \vdots \\ C_w A_w^{s-1} \end{bmatrix}$$

2. Compute A_w and C_w from Γ_0;

3. Obtain B_w, D_w and the initial condition $z(0)$ from the estimated pair (A_w, C_w) and the given input/output data.

The only parameters to be chosen by the user are the number of blockrows s of the extended observability matrix and the delay in the system. Usually, the parameter s can be chosen twice the expected order of the sytem.

3 Feedforward controller identification and update

Subspace identification is used to derive a block-updating scheme for the controller W, first without and then with intrinsic feedback. We remark that the controller and its update can also be estimated by other (off-line) identification methods, such as the ARX identification method. However, ARX identification doens't give an accurate estimate of the controller at low frequencies, which reduces its performance. This is supported by experiments in the simulations Section 5.

3.1 Without intrinsic feedback

3.1.1 Startup: We start with collecting a block of N measurements of $x(n) = x_m(n)$ and $e(n)$ for $n = 1, \ldots, N$ without control, hence $e(n) = d(n)$. The controller at startup, denoted by $W^{(1)}$, should have functioned such that $SW^{(1)}x(n)$ approached $-d(n)$ as close as possible. Initially assuming that S is a single-input/single-output (SISO) LTI

system and is accurately modelled by $\hat{S}$, the feedforward controller problem based on subspace identification can be stated as:
Given the block of input/output data

$$\{\hat{S}x(n), -d(n)\}_{n=1}^{N}$$

find a quadruple $(A_{w1}, B_{w1}, C_{w1}, D_{w1})$ *of the system*

$$\begin{aligned} q(n+1) &= A_{w1}q(n) + B_{w1}x'(n) \\ y(n) &= C_{w1}q(n) + D_{w1}x'(n) \end{aligned}$$

with $x'(n) = \hat{S}x(n)$, *such that the output* $y(n)$ *approximates* $-d(n)$ *over* $n = 1, \ldots, N$. *The quadruple* $(A_{w1}, B_{w1}, C_{w1}, D_{w1})$ *gives a realization of the controller* $W^{(1)}$ *(up to a similarity transformation).*
This identification problem of deriving $W^{(1)}$ is solved with subspace identification. Remark, that doing so, the equality $WS = SW$ for scalar LTI systems was implicitly used.

3.1.2 MIMO controller: When $\hat{S}$ and $W^{(1)}$ are MIMO LTI models, $W^{(1)}$ and $\hat{S}$ do not commute. To identify the MIMO controller $W^{(1)}$ such that $SW^{(1)}x(n)$ approaches $-d(n)$ over $n = 1, \ldots, N$, let

$$\begin{aligned} X_N &= \begin{bmatrix} x(1) & \cdots & x(N) \end{bmatrix} \\ D_N &= \begin{bmatrix} d(1) & \cdots & d(N) \end{bmatrix} \end{aligned}$$

with $x(n) \in \mathbb{R}^p$ and $d(n) \in \mathbb{R}^l$. Further, we define

$$Y_N = \hat{S}W^{(1)}X_N$$

hence, Y_N should approach $-D_N$. Note, that the filter $\hat{S}W^{(1)}$ acts on the sequence X_N. Using $Y_N^T = X_N^T(W^{(1)})^H\hat{S}^H$ (with $(.)^H$ the adjoint operator) and the matrix property $\text{vec}(ABC) = (C^T \otimes A)\text{vec}(B)$, we can write

$$\text{vec}\left(Y_N^T\right)^T = \text{vec}\left((W^{(1)})^T\right)^T \left(\hat{S}^T \otimes X_N\right) \qquad (1)$$

where $\text{vec}\left((W^{(1)})^T\right)^T$ and $\text{vec}(Y_N^T)^T$ are the concatenations of the rows in $W^{(1)}$ and Y_N respectively. For example, let $\hat{S}$ and $W^{(1)}$ be 2×2 and 2×1 systems respectively (hence $p = 1$ and $l = 2$). Then, we have

$$\begin{aligned} Y_N &= \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} \\ \hat{S}_{21} & \hat{S}_{22} \end{bmatrix} \begin{bmatrix} W_1^{(1)} \\ W_2^{(1)} \end{bmatrix} X_N \\ &= \begin{bmatrix} W_1^{(1)}\hat{S}_{11} + W_2^{(1)}\hat{S}_{12} \\ W_1^{(1)}\hat{S}_{21} + W_2^{(1)}\hat{S}_{22} \end{bmatrix} X_N \end{aligned}$$

where $Y_N \in \mathbb{R}^{2\times N}$. Then the i^{th} row of Y_N, indicated with $Y_N^i \in \mathbb{R}^{1\times N}$, can be written as

$$Y_N^i = \begin{bmatrix} W_1^{(1)} & W_2^{(1)} \end{bmatrix} \begin{bmatrix} \hat{S}_{i1}X_N \\ \hat{S}_{i2}X_N \end{bmatrix}$$

for $i = 1, 2$. Concatenating the sequences Y_N^1 and Y_N^2 using this expression for Y_N^i $(i = 1, 2)$, gives

$$\begin{aligned} \begin{bmatrix} Y_N^1 & Y_N^2 \end{bmatrix} &= \begin{bmatrix} W_1^{(1)} & W_2^{(1)} \end{bmatrix} \begin{bmatrix} \hat{S}_{11}X_N & \hat{S}_{21}X_N \\ \hat{S}_{12}X_N & \hat{S}_{22}X_N \end{bmatrix} \\ &= \begin{bmatrix} W_1^{(1)} & W_2^{(1)} \end{bmatrix} \left(\begin{bmatrix} \hat{S}_{11} & \hat{S}_{21} \\ \hat{S}_{12} & \hat{S}_{22} \end{bmatrix} \otimes X_N \right) \end{aligned}$$

where $\left(\begin{bmatrix} \hat{S}_{11}X_N \\ \hat{S}_{12}X_N \end{bmatrix}, Y_N^1\right)$ is the first and $\left(\begin{bmatrix} \hat{S}_{21}X_N \\ \hat{S}_{22}X_N \end{bmatrix}, Y_N^2\right)$ the second input/output data block.
Because Y_N should approach $-D_N$, the controller $W^{(1)}$ can be derived by identification of system $\text{vec}((W^{(1)})^T)^T$ using the input/output data block

$$\{\hat{S}^T \otimes X_N, \text{vec}(-D_N^T)^T\}$$

Subspace identification methods, as developed in [13] can easily cope with such concatenated data sets. This commutation of the secondary path and the controller is often done in ANC, see [9]. However, in a sample-by-sample update the controller W is timevariant, hence this commutation introduces additional errors.

3.1.3 Update mechanism: In practise the primary path P and the secondary path S may vary in time, therefore the controller should be updated accordingly. In this paper we assume for the sake of brievity, only variations in P and assume an accurate model of S. The update is based on the block of the last N measurements of $x(n) = x_m(n)$ and $e(n)$ for $n = (k-1)N, \ldots, kN$ and $k > 1$. In this current window the controller $W^{(k+1)}$ should have functioned such that $SW^{(k+1)}x(n)$ approached $-d(n)$ as close as possible. Let $y(n) = SW^{(k)}x(n)$, hence $e(n) = d(n) + y(n)$, and

$$W^{(k+1)} = W^{(k)} + \Delta W^{(k)}$$

then, constraining $SW^{(k+1)}x(n)$ to approximate $-d(n)$ in the current window is equivalent to constraining $S\Delta W^{(k)}x(n)$ to approximate $-d(n)$ in the current window. Initially assuming again, that S is a SISO LTI system and accurately modelled by $\hat{S}$, the problem of updating the controller, denoted by $\Delta W^{(k)}$, based on subspace identification can be stated as:
Given the block of input/output data

$$\{\hat{S}x(n), -e(n)\}_{n=(k-1)N+1}^{kN}$$

find a quadruple $(\Delta A_{wk}, \Delta B_{wk}, \Delta C_{wk}, \Delta D_{wk})$ *of the system*

$$\begin{aligned} z(n+1) &= \Delta A_{wk}z(n) + \Delta B_{wk}x'(n) \\ \Delta y(n) &= \Delta C_{wk}z(n) + \Delta D_{wk}x'(n) \end{aligned}$$

with $x'(n) = \hat{S}x(n)$, *such that the output* $\Delta y(n)$ *approximates* $-e(n)$ *over* $n = (k-1)N + 1, \ldots, kN$. *The quadruple* $(\Delta A_{wk}, \Delta B_{wk}, \Delta C_{wk}, \Delta D_{wk})$ *gives a realisation of the controller* $\Delta W^{(k)}$ *(up to a similarity transformation).*
The problem of deriving $\Delta W^{(k)}$ is written as a subspace identification problem, which can be solved by the methods mentioned in Section 2. The state-space controller $W^{(k+1)}$ can be inferred from the state-space realisations of $W^{(k)}$ and $\Delta W^{(k)}$ by state-augmentation, possibly in combination with state reduction. The controller $W^{(k+1)}$ can also be obtained by summation of the FIR realizations of $W^{(k)}$ and $\Delta W^{(k)}$. The extension to MIMO systems is analogue to Section 3.1.2. The controller $W^{(k+1)}$ can also be inferred, without explicitly calculating $\Delta W^{(k)}$, by just repeating the startup

described in Section 3.1.1 using the block of input/output data

$$\{\hat{S}x(n), -\hat{d}(n)\}_{n=(k-1)N+1}^{kN}$$

with $\hat{d}(n) = e(n) - \hat{S}u(n)$.

3.2 Incorporation of intrinsic feedback

The intrinsic feedback gives rise to a closed loop, which should be internally stabilized by the controller W. We assume that the intrinsic feedback F has no unstable poles, and is accurately modelled by $\hat{F}$.
The closed loop is internally stable if and only if

$$\begin{bmatrix} I & -F \\ -W & I \end{bmatrix}^{-1} = \begin{bmatrix} I+F(I-WF)^{-1}W & F(I-WF)^{-1} \\ (I-WF)^{-1}W & (I-WF)^{-1} \end{bmatrix}$$

is stable. Let W_0 be a stabilizing controller, with coprime factorization

$$W_0 = U_0 V_0^{-1}$$

with U_0, V_0 stable LTI operators. Then, all stabilizing controllers $W(Q)$ are given by the Youla parameterization [12, 17]

$$W(Q) = U(Q)V(Q)^{-1},$$
$$U(Q) = U_0 + Q, \qquad V(Q) = V_0 + \hat{F}Q$$

with Q any stable LTI operator. If $\hat{F}$ is stable, a stabilizing controller is $W_0 = 0$ (by $U_0 = 0$ and $V_0 = I$), and all stabilizing controllers are given by

$$W(Q) = Q(I + \hat{F}Q)^{-1}$$

with Q any stable LTI operator. The set of controllers $W(Q)$ can be implemented as indicated in Figure 2. Here the estimate of $x_f(n)$ is defined as $\hat{x}_f(n) = \hat{F}u(n)$ and the estimate of $x(n)$ as $\hat{x}(n) = x_m(n) - \hat{x}_f(n)$. If $\hat{F}$ is an accurate model of F, we can assume that the intrinsic feedback is completely canceled, and we have $\hat{x}(n) = x(n)$. Hence the control signal $u(n)$ is given by $u(n) = Qx(n)$ and the LTI operator Q can be identified and updated in the same way as the identification and the update of the complete controller W without intrinsic feedback in the previous subsection, by replacing the measurable disturbance signal $x(n)$ with its estimate $\hat{x}(n)$ in the subspace identification problem defined in Section 3.1.

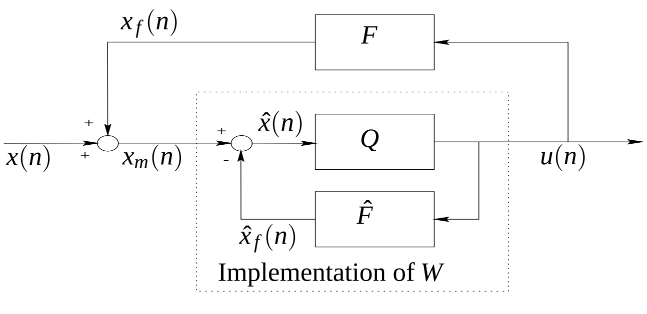

Figure 2: Intrinsic feedback compensation by Youla parameterization $W = Q(I+\hat{F}Q)^{-1}$, with $x(n)$ the noise from the source and estimate $\hat{x}(n)$, $x_f(n)$ the influence from intrinsic feedback and estimate $\hat{x}_f(n)$, measured disturbance $x_m(n)$ and control output $u(n)$.

4 Comparison with block LMS methods

The data used in the Subspace Identification ANC (SIANC) method, outlined in Section 3 is equivalent with that used in block LMS update algorithms. In the following, we first describe the block Filtered-U LMS (FuLMS) algorithm [9, 4], neglecting intrinsic feedback, by assuming $x(n) = x_m(n)$. Then block FuLMS will be compared with the SIANC method.

4.1 Block Filtered-U LMS

For simplicity we only consider the SISO case. The controller has the following IIR structure

$$W^{(n)}(z) = \frac{a_0(n) + a_1(n)z^{-1} + \cdots + a_{L-1}(n)z^{-L+1}}{1 + b_1(n)z^{-1} + \cdots + b_M(n)z^{-M}}$$

with $M+1 \geq L$ the controller order and n the current time instant for sample-by-sample FuLMS and the current block number for block FuLMS. The control signal $u(n)$ is given by:

$$u(n) = \mathbf{a}^T(n)\mathbf{x}(n) + \mathbf{b}^T(n)\mathbf{u}(n-1)$$

with

$$\mathbf{a}(n) = \begin{bmatrix} a_0(n) & a_1(n) & \cdots & a_{L-1}(n) \end{bmatrix}^T$$
$$\mathbf{b}(n) = \begin{bmatrix} b_1(n) & b_2(n) & \cdots & b_M(n) \end{bmatrix}^T$$

and the vectors $\mathbf{x}(n) \in \mathbb{R}^L$ and $\mathbf{u}(n-1) \in \mathbb{R}^M$ given by:

$$\mathbf{x}(n) = \begin{bmatrix} x(n) & x(n-1) & \cdots & x(n-L+1) \end{bmatrix}^T$$
$$\mathbf{u}(n-1) = \begin{bmatrix} u(n-1) & u(n-2) & \cdots & u(n-M) \end{bmatrix}^T$$

The parameter vector $\mathbf{w}(k)$ of the controller $W^{(n)}$ and the generalized reference vector $\mathbf{q}(n)$[1] are defined by

$$\mathbf{w}(n) = \begin{bmatrix} \mathbf{a}(n) \\ \mathbf{b}(n) \end{bmatrix}, \quad \mathbf{q}(n) = \begin{bmatrix} \mathbf{x}(n) \\ \mathbf{u}(n-1) \end{bmatrix}$$

Then, the sample-by-sample update of the controller by the FuLMS algorithm is given by [9]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{q}'(n)e(n)$$

where $\mathbf{q}'(n) = \hat{S}\mathbf{q}(n)$, and μ a user defined stepsize. The block FuLMS algorithm can be derived by averaging the update over a block of data [5], resulting in

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu_B}{N}\sum_{i=1}^{N} \mathbf{q}'((k-1)N+i)\, e((k-1)N+i)$$

with k the block number and μ_B a user defined stepsize.

[1] Usually $\mathbf{q}(n)$ is indicated with $\mathbf{u}(n)$, hence the name Filtered-U.

4.2 Comparison

The controller update of the block FuLMS algorithms as well as the SIANC method are both based on a block of measured data. However, there are some differences between both methods. First, the block FuLMS is a gradient search method for the controller which minimizes the variance of the error $e(n)$. The SIANC approach gives an optimal estimate of the controller, which should have minimized the error signal in a window over measurements in the past, but not *explicitly* the variance of the error signal. Second, because of the ARX structure of the controller, the block FuLMS algorithms do not minimize a *quadratic* criterion function. Hence the criterion function can have multiple local minima [9], which can give convergence problems. Third, the number of parameters of the ARX controller of the block FuLMS method can be very large, especially in the case of MIMO controllers. The SIANC approach enables the use of state-space controllers, which can reduce the number of parameters, due to their minimal parameterization [11]. Fourth, the tuning of the stepsize parameter μ_B and the selection of the controller order $M+1$ in the block FuLMS algorithm can be rather cumbersome to achieve accurate tracking and fast convergence without making the update algorithm unstable. In the SIANC approach the user gets information through the singular values [13] about the order of the controller. Further, the experiments with this method in the next Section show fast convergence.

5 Simulations on a realistic duct model

5.1 Setup

The SIANC approach is illustrated on a 19^{th} order discrete time realistic acoustical duct model suplied by TNO Institute of Applied Physics (TNO-TPD), the Netherlands. In this model P, S and F are discrete-time SISO IIR models derived by a sampling frequency of 1kHz, all with same denominator polynomial. We assume perfect models: $\hat{S}=S$ and $\hat{F}=F$. Because the intrinsic feedback is perfectly modeled, the performances of SIANC for the configuration with or without intrinsic feedback are the same (the effect of intrinsic feedback is completely cancelled with the internal model). Further, the primary path P changes at time instant $n=5001$. $P^{(1)}$ is the primary path before $n=5001$, the primary path after $n=5001$ is given by $P^{(2)}=1.1P^{(1)}$. The noise source signal $x(n)$ is bandlimitted white noise with a bandwith of 200Hz, and causes the disturbance signal $d(n)$. Please note, that we didn't include noise on the measurements.

5.2 Simulation results

For the block FuLMS method, we choose the controller of order 15, because lower order controllers showed slower convergence. With the blocksize $N=100$, by trial and error we found the stepsize $\mu_B=6$ which did not lead to instability of the update algorithm. The resulting error signal $e_{bl}(n)$ using the block FuLMS algorithm is given in the top part of Figure 3. We can see, that $e_{bl}(n)$ is converging slowly after about 2000 samples. The influence of the change in the primary path at $n=5001$ is negligible, because the error is not converged yet. The method described in Section 3 is illustrated by identification of the controller by ARX identification and by SMI. The error signal $e_{arx}(n)$ is given by the second plot of Figure 3. The first controller $W^{(1)}$ is a 6^{th} order ARX controller identified using the first 100 measurements. At $n=5001$, the error increases due to the change in the primary path. Another 100 measurements are collected and a 6^{th} order $\Delta W^{(1)}$ ARX controller is identified which is added to $W^{(1)}$, resulting in the 12^{th} order $W^{(2)}$ ARX controller. The controller $W^{(1)}$ and $\Delta W^{(1)}$ are also identified using SMI (according to the SIANC method). Here, $W^{(1)}$ and $\Delta W^{(1)}$ are both 6^{th} order state-space controllers. $W^{(2)}$ is formed by combining $W^{(1)}$ and $\Delta W^{(1)}$ and removing 6 states by model order reduction applying `minreal` of MATLAB, hence $W^{(2)}$ is again 6^{th} oder. The resulting error $e_{ss}(n)$ is given by the last plot of Figure 3.

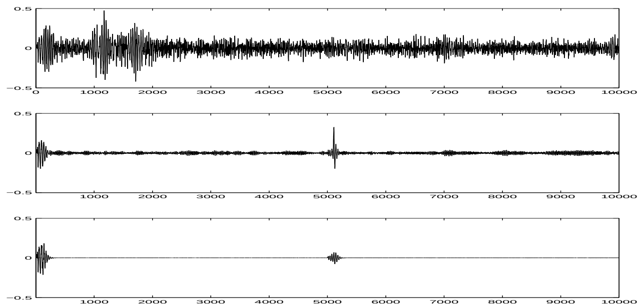

Figure 3: From up to down: error using FuLMS (15^{th} order); error using ARX identification (6^{th} order controller, after change at $n=5001$ identification of 6^{th} order ΔW); error using SMI (6^{th} order controller, after change: identification of ΔW, model reduction gives again 6^{th} order controller).

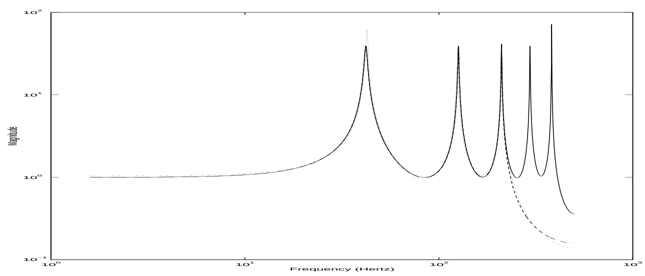

Figure 4: Magnitude of optimal controller (solid), up to $\approx$ 215Hz accurately approximated by the 6^{th} order state-space controller obtained by SMI (dashed), and magnitude of the 6^{th} order ARX controller obtained by Output Error identification (dotted).

5.3 Comparison

Comparing these results, we conclude that the identification of $W^{(1)}$ and $\Delta W^{(1)}$ gives better performance of the error signal than the block FuLMS algorithm. Though the identification of $W^{(1)}$ and $\Delta W^{(1)}$ is computationally more complex than the update of block FuLMS, *only 100 measurements* were necessary for an accurate update. Comparing, the ARX and SMI results, we conclude that the SMI method, though it does not explicitly minimize the variance, gives a much better estimate of the optimal controller which minimizes the variance of the error than the ARX identification method. This can be explained by the fact that ARX identification gives an accurate estimate of the controller at high frequencies at the expense of loss of accuracy at lower frequencies [10]. This is confirmed by Figure 4, which shows the magnitude of the optimal controller (solid), the magnitude of the 6^{th} order controller derived with SMI (dashed) and the 6^{th} order controller derived with ARX identification, both using 100 samples. One can see, that the controller based on ARX identification is less accurate at low frequencies (see the peak at $\approx$ 42Hz) than at high frequencies in the excited frequency band (i.e. up to $\approx$ 215Hz).

6 Conclusions

In this paper, we proposed a subspace identification and update approach for active noise control (ANC), which enables the use of compact MIMO state-space controllers to reject disturbance signals. The identification and the update of the controller could be based on *small* data batches, which reduce the computational complexity. In a simulation example, we could identify and update a 6^{th} order SISO controller based on a block of 100 samples with much better performance than the 15^{th} order controller which was derived by the block Filtered-U algorithm after more than 2000 measurements. Compared with the controller derived with ARX identification, the controller derived with subspace identification gave much better performance in the simulation, because ARX identification gives a more accurate estimate of the controller at high frequencies at the expense of accuracy at lower frequencies. Intrinsic feedback is incorporated by the Youla parameterization, assuming an accurate model of the intrinsic feedback is given. Further, the Subspace Identification Active Noise Control (SIANC) approach enabled the update of state-space controllers in ANC, which can reduce the number of coefficients of MIMO controllers.

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