A Performance Comparison of Dynamic vs. Static Load Balancing Policies in a Mainframe – Personal Computer Network Model

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Abstract

Distributed computer systems can share job processing in the event of overloads. Load balancing involves the distribution of jobs throughout a networked computer system, thus increasing throughput without having to obtain additional or faster computer hardware. Load balancing policies may be either static or dynamic. Static load balancing policies are generally based on the information about the average behavior of system; transfer decisions are independent of the actual current system state. Dynamic policies, on the other hand, react to the actual current system state in making transfer decisions. This makes dynamic policies necessarily more complex than static ones, and truly optimal dynamic policies are known only for special systems. This study focuses on performance comparison between static and dynamic load balancing policies in a distributed computer system where truly optimal solutions of both dynamic and static policies have been characterized. The system consists of two types of service facilities, a Mainframe node and an unlimited number of Personal Computer nodes. The results suggest that, in the model examined, the dynamic policy outperforms the static one in the mean response time, at most about 30 percent and for the range of parameter values such that the arrival rate is near the processing rate of the Mainframe.

1 Introduction

One of the advantages of distributed systems over the stand-alone systems is that balancing the workload of the system among the nodes can improve the performance of the system. Load balancing policies are often used for balancing the workload of distributed systems. The purpose of load balancing policies, either static or dynamic, is to improve performance by redistributing the workload among nodes. Dynamic load balancing policy reacts to the current system state, whereas static load balancing policy depends only on the average behavior of the system in order to balance the workload of the system. This makes dynamic policy necessarily more complex than static one. But, dynamic load balancing policies have been believed to have better performance than static ones [3].

In this paper, we consider *dynamic and static overall optimal policies* whereby job scheduling is determined so as to minimize the system mean response time. The goal of this paper is to examine to what extent the optimal dynamic load balancing policy outperforms the static one by an exhaustive numerical investigation on a model for which both policies are analytically studied. Optimal static policies have been analytically studied in a variety of models for distributed computer systems [4, 5, 6, 7, 9]. On the other hand, as far as we know, optimal dynamic policies have been studied only in very specific models: one is that of using an $M/M/m$ queueing model [2], and another is what we use here and is analytically studied in [1]. That is, the model studied here consists of a Mainframe node and an unlimited number of Personal Computer nodes. The objective of both policies is to minimize the overall mean response time. The model allows us to have exhaustive numerical investigations to gain insight into the problem. The results suggest that, in the model examined, the dynamic policy outperforms the static one in the mean response time, at most about 30 % and for the range of parameter values such that the arrival rate is near the processing rate of the Mainframe node.

Meanwhile there have been some studies of performance comparison of dynamic vs. static policies in more sophisticated models where overheads are considered but the truly optimal dynamic policy is not accurately obtained than ours [3, 9].

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This paper is organized as follows. Section 2 describes the system model of this paper. Section 3 presents two optimal load balancing policies: static and dynamic. Section 4 describes the results of numerical examination. Finally, Section 5 summarizes this paper.

2 The System Model

We consider a distributed computer system. The system consists of two types of service facilities, a Mainframe node (Q_{MF}) and unlimitedly many Personal Computer nodes (Q_{PC}), both of which are connected by a communication network. We call this system model an *MF-PC network model*. We assume that the expected communication delay between the MF node and each PC node is negligible. Jobs arrive at the system according to a time-invariant Poisson process, i.e. inter-arrival times of jobs are independent, identically and exponentially distributed with mean $1/\lambda$. Simultaneous arrivals are excluded. A job arriving at the system may be processed either by the MF node or by a PC node according to load balancing policies. We assume that the service rate at Q_{MF} is μ and that its service discipline is processor sharing so that the service intensity for each job equals $v(n) = \mu/n$, where n is the number of jobs in Q_{MF}. Every Q_{PC} offers a fixed expected service time θ^{-1}. In each Q_{PC} service starts immediately upon admission, and thus the sojourn time is identical with the service time. We assume that at both Q_{MF} and Q_{PC}, service times are independent, identically and exponentially distributed.

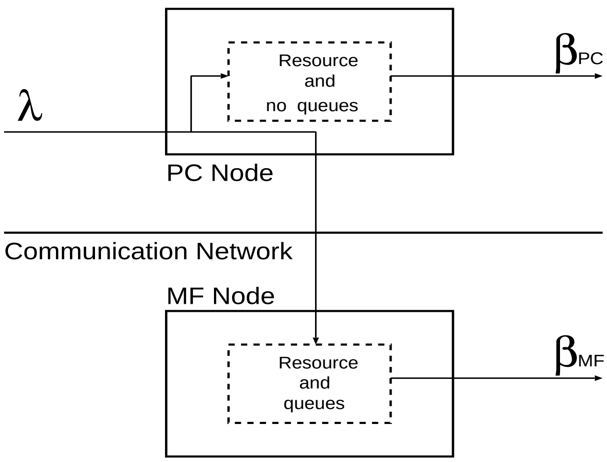

Figure 1: A model of an MF-PC network system

3 Two Optimal Load Balancing Policies

In the following two subsections, we describe optimal static and dynamic polices and present their solutions.

3.1 Optimal Static Load Balancing Policy

In this policy, the decision by the policies of transferring a job does not depend on the state of the system, and hence is *static* in nature. Also, we assume that a job transferred from one node to another receives its service there and is not transferred further. In this section, we consider an optimal static load balancing policy that determines the optimal load at each node so as to minimize the mean job response time in our system model.

We use the following notation:

- β_{MF} Job processing rate (load) at node MF
- $F_{MF}(\beta_{MF})$ Expected node delay of job processed at node MF

$$F_{MF}(\beta_{MF}) = \begin{cases} \dfrac{1}{\mu - \beta_{MF}} & if\ \beta_{MF} < \mu, \\ \infty & \text{otherwise.} \end{cases}$$

The problem of minimizing the system mean job response time is expressed as

$$\begin{aligned} & \text{minimize} \quad D(\beta_{MF}) \\ = \quad & \frac{1}{\lambda}[\beta_{MF} F_{MF}(\beta_{MF}) + (\lambda - \beta_{MF})\theta^{-1}] \qquad (1) \end{aligned}$$

with respect to β_{MF} such that $0 \leq \beta_{MF} \leq \lambda$.

Define β_0 ($0 \leq \beta_0 < \mu$) such that

$$\frac{\mu}{(\mu - \beta_0)^2} = \theta^{-1}.$$

The optimal β_{MF} is given as follows:

$$\beta_{MF} = \begin{cases} \beta_0 & if\ \beta_0 < \lambda, \\ \lambda & if\ \lambda \leq \beta_0. \end{cases}$$

3.2 Optimal Dynamic Load Balancing Policy

By this policy, each arriving job may observe the current load, and then choose whether to join the shared mainframe or to remain at a PC node. Also, in this policy the overall optimization problem is considered, where the goal is to minimize the expected average sojourn time per job. Observing that this social or overall system cost does not depend on the service discipline in Q_{MF} (PS, FCFS, etc.), the problem reduces to that of a standard queuing control. We use $[L, q]$ threshold rule as a dynamic policy. In this rule, an arriving job will go to the MF node with probability of, respectively, 0, q, and 1, if the job finds that the MF node has, more than, equal to, and less than, L jobs. We consider a formula $E\left[W_{[L,q]}\right]$ for the expected sojourn time of the system with respect to $[L, q]$ threshold rule and minimize $E\left[W_{[L,q]}\right]$. The expected sojourn time of a job arriving at the system with threshold $[L, q]$, $E\left[W_{[L,q]}\right]$, is obtained as follows:

$$E\left[W_{[L,q]}\right] = P\theta^{-1} + Q\lambda^{-1},$$

where, if $\rho \neq 1$ (i.e. $\lambda \neq \mu$),

$$\begin{aligned} P &= P_0(1-q+q\rho)\rho^L, \\ Q &= P_0\rho\frac{(-(L+1)\rho^L)(1-\rho)+(1-\rho^{L+1})}{(1-\rho)^2} \\ &\quad +(L+1)P_0 q\rho^{L+1}, \\ P_0 &= \frac{1-\rho}{1-\rho^{L+1}(1-q)-q\rho^{L+2}}, \end{aligned}$$

and if $\rho = 1$ (i.e. $\lambda = \mu$),

$$P = \frac{1}{L+1+q}, \quad Q = \frac{(L+1)(L+2q)}{2(L+1+q)}.$$

(For the derivation of the above, see Appendix A.)

We obtain numerically the values of L and q that give the minimum response time for each combination of the values of λ, μ and θ.

4 Results and Discussion

We estimate the mean response time of the MF-PC network system for each combination of the values of job arrival rate λ to the system, job processing rate μ at node MF, and job processing rate θ at node PC. Since we have only three system parameters λ, μ and θ, we scale down θ to 1 and thus we have only two independent parameters. We denote by T_D and T_S, respectively, the mean response times of the dynamic and static policies. Figures 2 and 3 show the mean response time of the system by the static and dynamic policies, respectively, for various combinations of the values of λ and μ. We examined the ratio of improvement in the mean response time by the dynamic policy over the static policy. Figure 4 shows the ratio of improvement for various combinations of the values of λ and μ. Figure 5 shows the maximum ratio of improvement with respect to μ for each given value of λ. The results naturally confirmed our forecast that dynamic load balancing policy is more effective than static one. On the other hand, we see that the mean response time is improved by the optimal dynamic policy over that of the optimal static one at most about 30% in the range of parameter values examined. Figure 6 shows the corresponding value of μ that gives the maximum ratio of improvement for each value of λ. From this figure, we see that the maximum ratio of improvement is achieved for the cases where $\lambda \sim \mu$ for rather large values of both λ and μ.

These three figures, 4, 5 and 6, show seemingly peculiar behaviors concerning the ratio of improvement as the values of system parameters change. It would need much argument to understand this peculiarity, but here we only indicate that the peculiar behavior may be related to the dependence of the mean response time by the dynamic policy on the values of the threshold-rule parameters L and q as exemplified by the Figure 7.

5 Conclusion

We have studied two optimal load balancing policies, static and dynamic, for a system consisting of a single-server central(Q_{MF}) node and an infinite-server satellite (Q_{PC}) node connected by a communication network. By numerical examination, we have estimated the difference in the effects on the mean response time between an optimal dynamic load balancing policy using threshold $[L, q]$ and a static optimal load balancing policy. We have observed that the ratio of improvement in the mean response time by the dynamic optimal policy over the static one is at most about 30% in the model examined. The difference is of a certain magnitude for the cases where $\lambda \sim \mu$ for rather large values of both.

Appendix A: Derivation of $E\left[W_{[L,q]}\right]$

We derive here the expected sojourn time of a job arriving at the system with threshold $[L, q]$, $E\left[W_{[L,q]}\right]$. Let P_k be the probability that the number of jobs in the MF node is k. The state transition diagram is shown in Figure 8. With this state transition diagram we have the following equations:

$$\begin{aligned} \lambda P_0 &= \mu P_1 \\ \lambda P_1 &= \mu P_2 \\ \cdots \;\; &\cdots \;\; \cdots \\ \lambda P_{L-1} &= \mu P_L \\ \lambda q P_L &= \mu P_{L+1}. \end{aligned} \qquad \text{(A.1)}$$

Let $\rho = \lambda/\mu$. From (A.1), we can easily have the recursions:

$$\begin{aligned} P_1 &= \rho P_0 \\ P_2 &= \rho^2 P_0 \\ \cdots \;\; &\cdots \;\; \cdots \\ P_L &= \rho^L P_0 \\ P_{L+1} &= \rho^{L+1} q P_0, \end{aligned} \qquad \text{(A.2)}$$

and if $\rho = 1$,

$$P_1 = P_2 = \cdots = P_L = P_0, \quad P_{L+1} = qP_0. \qquad \text{(A.3)}$$

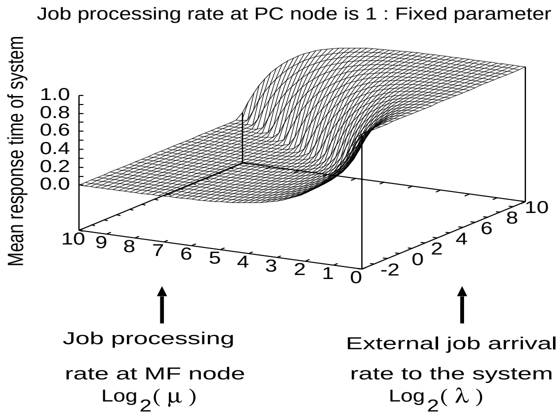

Figure 2: The mean response time T_S by the static optimal policy for each combination of the values of λ and μ.

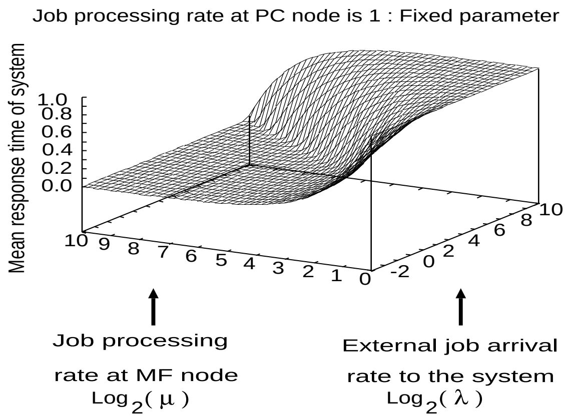

Figure 3: The mean response time T_D by the dynamic optimal policy for each combination of the values of λ and μ.

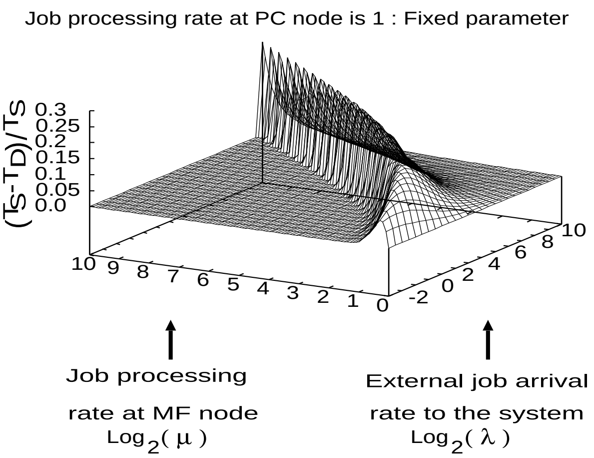

Figure 4: The ratio of improvement in the mean response time by the dynamic policy over the static policy for each combination of the values of λ and μ.

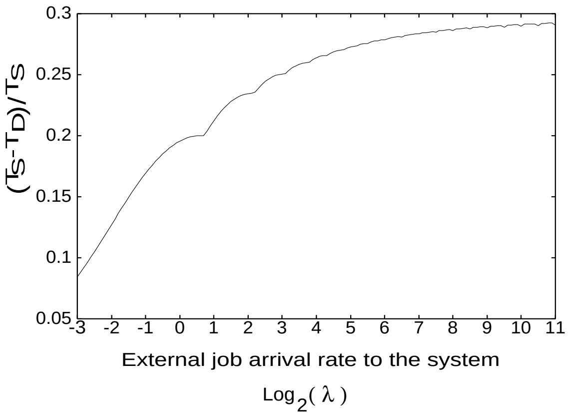

Figure 5: The maximum ratio of improvement in the mean response time (with respect to μ) by the dynamic policy over the static policy for each value of λ.

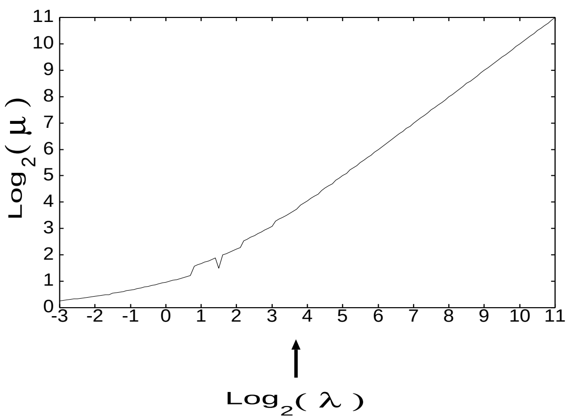

Figure 6: The value of μ that gives the maximum ratio of improvement in the mean response time by the dynamic policy over the static policy for each value of λ.

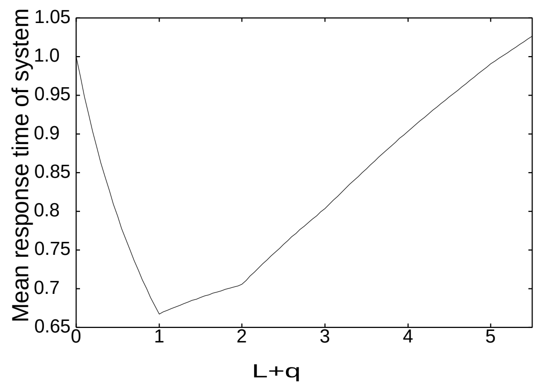

Figure 7: The mean response time by the dynamic policy for each combination of L and q for the case of $\lambda = 1.4142135$ and $\mu = 2.2028464$.

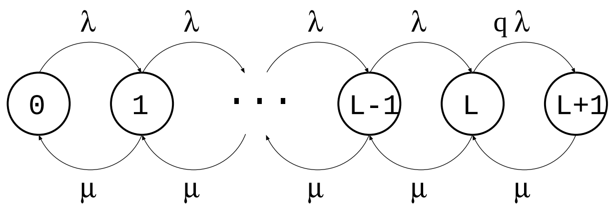

Figure 8: State transition diagram

From (A.2), we have

$$\begin{aligned} P_1 + P_2 + \cdots + P_L &= P_0(\rho + \rho^2 + \cdots + \rho^L) \\ &= P_0 \frac{\rho - \rho^{L+1}}{1-\rho}. \end{aligned} \quad (A.4)$$

Note that $\sum_{i=0}^{L+1} P_i = 1$. We have

$$P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{L+1}(1-q) - q\rho^{L+2}} & \text{if } \rho \neq 1, \\ \dfrac{1}{L+1+q} & \text{if } \rho = 1. \end{cases} \quad (A.5)$$

Substituting relation (A.5) to (A.2) or (A.3), we can have the probability that the number of jobs in the MF node is k, $P_k (0 \leq k \leq L)$. With the above relations, we proceed to calculate the expected sojourn time of a job arriving at the system. Let P be the probability that a job arriving at the system goes to the PC node. With $[L, q]$ threshold rule, the arriving job will go to a PC node with probability of 1 if the job finds the MF node with states $L+1, L+2, \cdots$, and with probability of $1-q$ if the job finds the MF node with state L. Then P is expressed as

$$P = (1-q)P_L + P_{L+1}. \quad (A.6)$$

The expected sojourn time of a job that goes to PC node is θ^{-1}. Let Q be the expected number of jobs (which includes the jobs in service) in the MF node from state 0 to state $L+1$ in the state transition diagram. By the Little's Law, the expected sojourn time of a job arriving at the system goes to the MF node is

$$QV^{-1},$$

where, V is the actual load rate to the MF node, and is given by $V = \lambda(1-P)$. Therefore, the total expected sojourn time of a job arriving at the system with threshold $[L, q]$, $E\left[W_{[L,q]}\right]$, is

$$\begin{aligned} E\left[W_{[L,q]}\right] &= P\theta^{-1} + (1-P)QV^{-1} \quad (A.7) \\ &= P\theta^{-1} + Q\lambda^{-1}. \end{aligned}$$

From (A.4), Q can be calculated as follows:

$$Q = \sum_{i=1}^{L} iP_i + (L+1)P_{L+1} \quad (A.8)$$

By substituting relations (A.6) and (A.8) into (A.7), we obtain the expected sojourn time of a job arriving at the system with threshold $[L, q]$, $E\left[W_{[L,q]}\right]$. The relation is as follows:

$$E\left[W_{[L,q]}\right] = ((1-q)P_L + P_{L+1})\theta^{-1} + Q\lambda^{-1}, \quad (A.9)$$

where, if $\rho \neq 1$,

$$\begin{aligned} P_L &= \rho^L P_0, \\ P_{L+1} &= q\rho^{L+1} P_0, \\ Q &= \sum_{i=1}^{L} iP_i + (L+1)P_{L+1} \\ &= P_0 \rho \frac{(-(L+1)\rho^L)(1-\rho) + (1-\rho^{L+1})}{(1-\rho)^2} \\ &\quad +(L+1)P_0 q \rho^{L+1}, \\ P_0 &= \frac{1-\rho}{1-\rho^{L+1}(1-q) - q\rho^{L+2}}, \end{aligned}$$

and if $\rho = 1$,

$$\begin{aligned} P_L &= P_0, \\ P_{L+1} &= qP_0, \\ Q &= \sum_{i=1}^{L} iP_i + (L+1)P_{L+1} \\ &= \left(\sum_{i=1}^{L} i + (L+1)q\right) P_0 \\ &= \left(\frac{L(L+1)}{2} + (L+1)q\right) P_0 \\ &= \frac{(L+1)(L+2q)}{2(L+1+q)}, \\ P_0 &= \frac{1}{L+1+q}. \end{aligned}$$

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