Global Adaptive Output Feedback Controllers with Application to Nonlinear Friction Compensation

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Abstract

In this paper we consider a class of nonlinear systems in which a set of constant parameters is unknown and some state variables are not available for measurement. For such systems we provide a constructive procedure for the solution of the global adaptive tracking problem with dynamic partial state feedback. We illustrate an application of the control strategy to the adaptive nonlinear friction compensation of a DC motor servomechanism. We improve previous results in two directions: we allow for a subset of the unmeasurable states to enter in the system nonlinearly; we consider systems which are linearly parametrized with respect to a set of unknown constant parameters.

1 Introduction and motivation

Consider single input nonlinear systems that can be transformed in the interconnection of the four subsystems $\sum_\mu$, $\sum_\eta$, $\sum_\zeta$, and $\sum_\xi$:

$$\begin{array}{l}
\sum_\mu : \dot{\mu} = F(\zeta,\xi)\mu + G_1(\zeta,\xi)\theta + g_1(\zeta,\xi) \\
\sum_\eta : \dot{\eta} = S(\zeta,\xi)\eta + G_2(\zeta,\xi)\theta + g_2(\zeta,\xi) \\
\sum_\zeta : \left\{ \begin{array}{l} \dot{\zeta} = f(\zeta) + g(\zeta)\xi \\ z = h(\zeta) \end{array} \right. \\
\sum_\xi : \left\{ \begin{array}{rcl} \dot{\xi} & = & \phi_u(\zeta,\xi)u + \phi(\zeta,\xi) \\ & & +p^T(\mu)\bar{\psi}(\zeta,\xi)\theta + \eta^T \ell(\zeta,\xi) \\ & & +\bar{\phi}(\zeta,\xi)\theta + p^T(\mu)\psi(\zeta,\xi) \end{array} \right.
\end{array} \quad (1)$$

with tracking output $z \in \Re$, state $(\mu \times \eta \times \zeta \times \xi) \in (\Re^{\nu_1} \times \Re^{\nu_2} \times \Re^d \times \Re)$, measurable output $y = \left[\zeta^T \ \xi\right]^T \in \Re^{d+1}$, and input $u \in \Re$, with $\phi_u(\zeta,\xi) \neq 0$ for all $(\zeta \times \xi) \in (\Re^d \times \Re)$. We assume the sub-state vectors $\mu(t) \in \Re^{\nu_1}$, $\eta(t) \in \Re^{\nu_2}$, to be not available for measurement, and the constant vector $\theta \in \Re^q$ to be unknown. In what follows $\Re^+$ will denote the set of positive real numbers, and $\mathcal{N}$ the set of the natural numbers, including 0. We assume that $p(\mu) \in \Re^s$ is a vector function whose entries are products of entries of $\mu(t)$, i.e. $p(\mu) = \left[\prod_{j=1}^{\nu_1} \mu_j^{k_{1j}}, \ \prod_{j=1}^{\nu_1} \mu_j^{k_{2j}}, \ \ldots, \ \prod_{j=1}^{\nu_1} \mu_j^{k_{sj}}\right]^T$, with $s \in \mathcal{N}$, $k_{ij} \in \mathcal{N}$, $1 \leq i \leq s$, and $1 \leq j \leq \nu_1$. The assumption that subsystem $\sum_\xi$ is scalar is made only for sake of simplicity. The approach we propose applies to a more general class of systems in which $\sum_\xi$ is in strict feedback form as will be shown in the next section. We assume that:

(H1) The subsystem $\sum_\zeta$, with input ξ and output $z(t)$ is globally input-output linearizable with uniform global relative degree r, with $1 \leq r \leq d$; i.e. $L_g L_f^i h(\zeta) = 0$, for all $\zeta \in \Re^d$, $0 \leq i \leq r-2$, and $L_g L_f^{r-1} h(\zeta) \neq 0$ for all $\zeta \in \Re^d$. Besides, for every initial condition $\zeta(t_0)$ and every bounded input $\xi(t) \in \mathcal{L}_\infty[t_0,\infty)$ also the solution $\zeta(t)$ starting from initial condition $\zeta(t_0)$ is bounded, i.e. $\zeta(t) \in \mathcal{L}_\infty^d[t_0,\infty)$.

(H2) There exists a symmetric positive-definite matrix P_μ such that, for all $(\zeta,\xi) \in \Re^{d+1}$, $P_\mu F(\zeta,\xi) + F^T(\zeta,\xi)P_\mu \leq -I_\nu$.

(H3) There exists a symmetric positive-definite matrix Q such that for all $(\zeta,\xi) \in \Re^{d+1}$, $QS(\zeta,\xi) + S^T(\zeta,\xi)Q \leq 0$, and for every $\eta(t_0) \in \Re^{\nu_2}$, every bounded $(\zeta(t),\xi(t)) \in \mathcal{L}_\infty^d[t_0,\infty) \times \mathcal{L}_\infty[t_0,\infty)$ also the solution $\eta(t)$ of subsystem $\sum_\eta$ starting from initial condition $\eta(t_0)$ is bounded, i.e. $\eta(t) \in \mathcal{L}_\infty^{\nu_2}[t_0,\infty)$.

Our control objective is to track a smooth bounded reference signal $z_d(t)$ that is known together with its bounded time-derivatives $z_d^{(i)}(t) = \frac{d^i z_d(t)}{dt^i}$, $i = 1,2,\ldots r+1$. This leads us to a global adaptive tracking problem definition, as follows.

Definition 1 *Consider system (1). Given a reference trajectory $z_d(t) \in C^\infty$ that is known together with its time-derivatives, the **global adaptive tracking problem** is said to be solvable if a dynamic control law $\dot{\chi} = \Theta(\chi,y,t)$, $\chi \in \Re^{\nu_4}$, $\nu_4 \in \mathcal{N}$, and $u = u(\chi,y,t)$, can be found such that the closed loop system trajectories are bounded, and for every initial condition $(\mu(t_0) \times \eta(t_0) \times \zeta(t_0) \times \xi(t_0)) \in (\Re^{\nu_1} \times \Re^{\nu_2} \times \Re^d \times \Re)$, every initial condition $\chi(t_0) \in \Re^{\nu_4}$, and every $\theta \in \Re^q$,* $\lim_{t\to\infty}[z_d(t) - z(t)] = 0$.

Control strategies for systems with partly unmeasured dynamics involve the use of nonlinear filters, in order to

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find estimates of the system unmeasured states and unknown parameters. A survey of research efforts on this issue can be found in [12], [8]. In a similar context, [4] design a global tracking controller for a class of systems that is similar to (1), namely when $\mu = 0$, and when on subsystem $\sum_\eta$ in (1) more restrictive assumptions are made than (H3) -as shown later- while $\sum_\xi$ is a vector system in strict feedback form.

In this paper we solve the global adaptive tracking problem with dynamic partial state feedback for (1), and provide a construction procedure that improves previous results in two directions: we allow for the subset $\mu(t)$ of the unmeasurable states to enter in the $\sum_\xi$ subsystem nonlinearly, via the vector function $p(\mu)$; we consider systems which are linearly parametrized with respect to a set of unknown constant parameters $\theta \in \Re^q$, in order to render the global asymptotic tracking strategy adaptive with respect to them. Our approach allows to find global adaptive tracking controllers even for those systems with $\mu(t) = 0$, but not complying with the hypotheses in [4], such as the Lund-Grenoble model of nonlinear friction systems (see [2] and [5]), that arises in accurate modelling of systems involving Coulomb friction, viscous and static friction, and the Stribeck effect (see [1], for a survey). The Lund-Grenoble model can be formalized as follows.

Definition 2 ***Lu-Gre friction model.*** *(see [2] and [5]). Consider a class of servo motor systems with friction effects, for the control of high precision tracking table. Friction between two sliding surfaces can be modelled as contact between sets of bristles, and is described with a state variable bristle model, leading to the following dynamics:*

$$\begin{array}{rcl} J\ddot{x} & = & Ki - F(\dot{x}, z, \dot{z}) \\ \dot{z} & = & -\frac{|\dot{x}|}{g(\dot{x})} z + \dot{x} \end{array} \qquad (2)$$

where the average bristle deflection is z, the total motor and load inertia is J, the motor shaft angular position is x, the torque provided by a D.C. servo motor is Ki. The friction torque is $F(\dot{x}, z, \dot{z}) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x}$, where σ_0 is the bristle stiffness coefficient, σ_1 is related to damping during bristle deflection transients, and σ_2 is the viscous damping coefficient. The dynamics of the state variable bristle deflection z includes the nonlinear positive bounded term $g(\dot{x})$ that models the Stribeck effect, (see [3])

$$g(\dot{x}) = \frac{1}{\sigma_0}\left[\alpha_0 + \alpha_1 e^{-\left(\frac{\dot{x}}{v_s}\right)^2}\right],$$

where $\alpha_0 + \alpha_1$, α_0, are respectively static and Coulomb friction coefficients, and v_s is the characteristic Stribeck velocity.

If numerical values can be provided for all the scalar parameters in the model dynamic equations, including σ_1, while bristle deflection z is not measured when the system is moving, tracking can be achieved for the system considered with a control strategy proposed in previous literature (see for instance [2], [3]). We will show that system (2) can be expressed as a particularization of (1), and will find a global tracking control which is adaptive with respect to σ_1, assuming that only the position x and its derivative $\dot{x}$ are available for measurement. In Section 2 we show the main result, by introducing suitable time-varying global coordinate transformations to express the tracking error system in a form which is instrumental for the solution of problem in Definition 1. In Subsection 2.1 we describe global tracking control laws for (1). In Subsection 2.2 we suggest how the adaptive tracking strategy can be extended to systems $\sum_\xi$ in strict feedback form. In Section 3 we show the application to the Lund-Grenoble friction model of the tracking control law proposed and draw the conclusions in Section 4.

2 Main result

We define now suitable time-varying global coordinate transformations to obtain the tracking error system. We can set $\Phi_1(\zeta, t) = h(\zeta) - z_d(t)$, $\Phi_2(\zeta, t) = L_f h(\zeta) - z_d^{(1)}(t)$, $\ldots$ $\Phi_r(\zeta, t) = L_f^{r-1} h(\zeta) - z_d^{(r-1)}(t)$, and define

$$\begin{array}{rcl} \tilde{h}(\zeta, t) & = & L_f^r h(\zeta) - z_d^{(r)}(t) \\ & & + \sum_{i=0}^{r-1} c_i \left(L_f^i h(\zeta) - z_d^{(i)}(t)\right) \\ \tilde{k}(\zeta) & = & L_g L_f^{r-1} h(\zeta), \end{array} \qquad (3)$$

$$\begin{array}{rcl} \tilde{\xi}(\zeta, \xi, t) & = & \tilde{h}(\zeta, t) + \tilde{k}(\zeta)\xi \\ \tilde{u}(\zeta, \xi, u) & = & \phi_u(\zeta, \xi, t)\tilde{k}(\zeta) u, \end{array} \qquad (4)$$

where $c_0, \ldots c_{r-1}$ are positive reals such that the polynomial $p(s) = s^r + c_{r-1}s^{r-1} + \ldots c_0$ is Hurwitz. By hypothesis (H1) there exist (see [12]) $d-r$ nonlinear functions $\Phi_i(\zeta, t)$, $r+1 \leq i \leq d$ such that the subsystem $\sum_\zeta$ in (1) is globally diffeomorphic, via the time-varying global coordinate change $\tilde{\zeta} = [\Phi_1(\zeta, t)\ ,\ \ldots,\ \Phi_r(\zeta, t)]^T$ and $\varpi = [\Phi_{r+1}(\zeta, t),\ \ldots,\ \Phi_d(\zeta, t)]^T$ to the two subsystems $\dot{\varpi} = F\left(\tilde{\zeta}, \varpi, t\right)$, and $\dot{\tilde{\zeta}} = A\tilde{\zeta} + b\tilde{\xi}$, where $F\left(\tilde{\zeta}, \varpi, t\right)$ is a suitable smooth vector function, $b = [0,\ \ldots\ 0, 1]^T$, and the matrix

$$A = \begin{bmatrix} 0 & I_{r-1} \\ 0 & 0 \end{bmatrix} - b\begin{bmatrix} c_0 & c_1 & \ldots & c_{r-1} \end{bmatrix}$$

is Hurwitz. On the other hand, the subsystem $\sum_\xi$ in (1) is diffeomorphic via (4) to the system

$$\sum_{\tilde{\xi}} : \left\{ \begin{array}{rcl} \dot{\tilde{\xi}} & = & \tilde{u} + p^T(\mu)\tilde{\psi}(\tilde{\zeta}, \varpi, \tilde{\xi}, t) \\ & & + p^T(\mu)\breve{\psi}(\tilde{\zeta}, \varpi, \tilde{\xi}, t)\theta + \eta^T \tilde{\ell}(\tilde{\zeta}, \varpi, \tilde{\xi}, t) \\ & & + \breve{\phi}(\tilde{\zeta}, \varpi, \tilde{\xi}, t)\theta + \tilde{\phi}(\tilde{\zeta}, \varpi, \tilde{\xi}, t) \end{array} \right. \qquad (5)$$

where $\tilde{\phi} = \left(\frac{\partial \tilde{h}}{\partial \zeta} + \xi \frac{\partial \tilde{k}}{\partial \zeta}\right)(f + g\xi) + \frac{\partial \tilde{h}}{\partial t} + \tilde{k}\phi$, and $\tilde{\psi} = \tilde{k}\psi$, $\breve{\phi} = \tilde{k}\bar{\phi}$, $\breve{\psi} = \tilde{k}\bar{\psi}$, $\tilde{\ell} = \tilde{k}\ell$. We now introduce the filters

$$\begin{array}{rcll} \dot{\hat{\mu}}_0 & = & F(\zeta,\xi)\hat{\mu}_0 + g_1(\zeta,\xi) & \hat{\mu}_0 \in \Re^{\nu_1} \\ \dot{\hat{\mu}}_i & = & F(\zeta,\xi)\hat{\mu}_i + G_{1i}(\zeta,\xi) & \hat{\mu}_i \in \Re^{\nu_1},\ 1 \le i \le q \\ \dot{\hat{\eta}}_0 & = & S(\zeta,\xi)\hat{\eta}_0 + g_2(\zeta,\xi) & \hat{\eta}_0 \in \Re^{\nu_2} \\ \dot{\hat{\eta}}_i & = & S(\zeta,\xi)\hat{\eta}_i + G_{2i}(\zeta,\xi) & \hat{\eta}_i \in \Re^{\nu_2},\ 1 \le i \le q \end{array} \quad (6)$$

where $G_{ij}(\zeta,\xi)$ denotes the j-th column of the matrix $G_i(\zeta,\xi)$, $i = 1,2$, and define the parameter-dependent filtered transformation

$$\begin{array}{rcl} \tilde{\mu} & = & \mu - \hat{\mu}\theta - \hat{\mu}_0 \\ \tilde{\eta} & = & \eta - \hat{\eta}\theta - \hat{\eta}_0 \end{array} \quad (7)$$

with $\hat{\mu} = \left[\hat{\mu}_1, \hat{\mu}_2, \ldots \hat{\mu}_q\right]$, $\hat{\eta} = \left[\hat{\eta}_1, \hat{\eta}_2, \ldots \hat{\eta}_q\right]$, and

$$\omega = \left[\hat{\mu}_0^T,\ \hat{\mu}_1^T,\ \ldots,\ \hat{\mu}_q^T, \hat{\eta}_0^T, \hat{\eta}_1^T,\ \ldots, \hat{\eta}_q^T, \varpi^T\right]^T. \quad (8)$$

If we express system (5) in terms of $\tilde{\mu} = \tilde{\mu}(\omega,\theta)$ and $\tilde{\eta} = \tilde{\eta}(\omega,\theta)$, as defined in (7), by expanding $p(\tilde{\mu} + \hat{\mu}\theta + \hat{\mu}_0)$, the error system in the new coordinates becomes

$$\begin{array}{l} \sum_{\tilde{\mu}} : \dot{\tilde{\mu}} = F(\tilde{\zeta},\tilde{\xi},\omega,t)\tilde{\mu}, \\ \sum_{\tilde{\eta}} : \dot{\tilde{\eta}} = S(\tilde{\zeta},\tilde{\xi},\omega,t)\tilde{\eta}, \\ \sum_{\omega} : \dot{\omega} = \Omega(\omega,\tilde{\zeta},\tilde{\xi},t), \\ \sum_{\tilde{\zeta}} : \dot{\tilde{\zeta}} = A\tilde{\zeta} + b\tilde{\xi}, \\ \sum_{\tilde{\xi}} : \left\{ \begin{array}{rcl} \dot{\tilde{\xi}} & = & \tilde{u} + \pi(\tilde{\zeta},\tilde{\xi},\omega,t) + \tilde{\eta}^T\tilde{\ell}(\tilde{\zeta},\tilde{\xi},t) \\ & & +\bar{\pi}(\tilde{\zeta},\tilde{\xi},\omega,t)\bar{\theta} + \bar{p}^T(\tilde{\mu})\delta(\tilde{\zeta},\tilde{\xi},\omega,t) \\ & & +\bar{p}^T(\tilde{\mu})\bar{\delta}(\tilde{\zeta},\tilde{\xi},\omega,t)\bar{\theta} \end{array} \right. \end{array} \quad (9)$$

where $\bar{p}(\tilde{\mu}) = \left[\prod_{j=1}^{\nu} \tilde{\mu}_j^{\bar{k}_{1j}},\ \prod_{j=1}^{\nu} \tilde{\mu}_j^{\bar{k}_{2j}},\ \ldots,\ \prod_{j=1}^{\nu} \tilde{\mu}_j^{\bar{k}_{\bar{s}j}}\right]^T$, $\bar{p}(\tilde{\mu}) \in \Re^{\bar{s}}$, and $\bar{\theta} = \left[\prod_{j=1}^{\nu} \theta_j^{\ell_{1j}},\ \prod_{j=1}^{\nu} \theta_j^{\ell_{2j}},\ \ldots,\ \prod_{j=1}^{\nu} \theta_j^{\ell_{\bar{q}j}}\right]^T$, $\bar{\theta} \in \Re^{\bar{q}}$, $\bar{s}, \bar{q} \in \mathcal{N}$, whose entries collect all the different terms like $\left(\prod_{j=1}^{\nu} \tilde{\mu}_j^{\bar{k}_{ij}}\right)$ and $\left(\prod_{h=1}^{q} \theta_h^{\ell_{ih}}\right)$ respectively, and $\Omega(\cdot)$, $\pi(\cdot)$, $\bar{\pi}(\cdot)$, $\delta(\cdot)$, $\bar{\delta}(\cdot)$, are suitable smooth matrix functions of their arguments. The tracking error is $z(t) - z_d(t) = \tilde{\zeta}_1$.

2.1 Controller design

We describe now the control strategy for system (9). Since for any $a, b, c \in \Re$, $ab \le \frac{a^2}{4c^2} + c^2b^2$, it follows that there exists a positive real $\tilde{\lambda} \in \Re^+$ and a positive integer $\bar{s} \in \mathcal{N}$ such that

$$\|\bar{p}(\tilde{\mu})\|^2 \le \tilde{\lambda} \sum_{\ell=1}^{\bar{s}} \|\tilde{\mu}\|^{2\ell} \text{ for all } \tilde{\mu} \in \Re^{\nu_1}. \quad (10)$$

Let $\lambda_1 \in \Re^+$ be a positive real to be specified later, and introduce the positive definite function

$$\begin{array}{rcl} V_1(\tilde{\mu},\tilde{\zeta},\tilde{\xi},\tilde{\theta}_e) & = & \lambda_1 \sum_{i=1}^{\bar{s}} \left(\tilde{\mu}^T P_\mu \tilde{\mu}\right)^i + \tilde{\zeta}^T P_\zeta \tilde{\zeta} \\ & & +\frac{1}{2}\left[\tilde{\xi} + \tilde{\theta}_e^T \Gamma_e \tilde{\theta}_e\right], \end{array}$$

where $\tilde{\theta}_e = \theta_e - \hat{\theta}_e$, and $\hat{\theta}_e$ is an estimate of $\theta_e \in \Re^{\nu_3}$, with $\nu_3 = \frac{3}{2}\bar{q} + \frac{\bar{q}^2}{2} + \nu_2$, given by

$$\theta_e(t) = \left[\bar{\theta}_1, \ldots \bar{\theta}_{\bar{q}}, \bar{\theta}_1^2, \bar{\theta}_1\bar{\theta}_2, \ldots, \theta_{\bar{q}}^2, \tilde{\eta}_1, \ldots, \tilde{\eta}_{\nu_2}\right], \quad (11)$$

and

$$\Gamma_e = \left[\begin{array}{cc} \Gamma^{-1} & 0 \\ 0 & Q \end{array}\right],$$

Γ is a symmetric positive definite matrix, and P_ζ is such that $P_\zeta A + A^T P_\zeta = -I_r$. The computation of the time-derivative of V_1 yields

$$\begin{array}{rcl} \dot{V}_1 & = & \lambda_1 \sum_{i=1}^{\bar{s}} i\left(\tilde{\mu}^T P_\mu \tilde{\mu}\right)^{i-1} \tilde{\mu}^T\left[P_\mu F + F^T P_\mu\right]\tilde{\mu} \\ & & +\frac{1}{2}\tilde{\theta}_e^T\left[\Gamma_e S_e + S_e^T \Gamma_e\right]\tilde{\theta}_e - \left\|\tilde{\zeta}\right\|^2 + 2\tilde{\zeta}^T P_\zeta b\tilde{\xi} \\ & & +\tilde{\theta}_e^T \Gamma_e\left(S_e\hat{\theta}_e - \dot{\hat{\theta}}_e\right) + \tilde{\xi}\dot{\tilde{\xi}}, \end{array} \quad (12)$$

with $S_e = \left[\begin{array}{cc} 0 & 0 \\ 0 & S \end{array}\right]$. By hypothesis (H2) there exists a suitable $\lambda_m \in \Re^+$ such that

$$\begin{array}{l} \lambda_1 \sum_{i=1}^{\bar{s}} i\left(\tilde{\mu}^T P_\mu \tilde{\mu}\right)^{i-1} \tilde{\mu}^T\left[P_\mu F + F^T P_\mu\right]\tilde{\mu} \le \\ -\lambda_1\lambda_m \sum_{i=1}^{\bar{s}} \|\tilde{\mu}\|^{2i}. \end{array} \quad (13)$$

By expanding the term $\tilde{\xi}\dot{\tilde{\xi}}$ we obtain

$$\begin{array}{rcl} \tilde{\xi}\dot{\tilde{\xi}} & = & \tilde{\xi}\tilde{u} + \tilde{\xi}\pi + \tilde{\xi}\bar{p}^T(\tilde{\mu})\delta + \tilde{\xi}\bar{\pi}\bar{\theta} \\ & & +\tilde{\xi}\bar{p}^T(\tilde{\mu})\bar{\delta}\bar{\theta} + \tilde{\xi}\tilde{\eta}^T\tilde{\ell}. \end{array} \quad (14)$$

By "completing the squares" of the terms in (14) together with (10) we have

$$\tilde{\xi}\bar{p}^T(\tilde{\mu})\delta \le \frac{1}{4\bar{\gamma}}\tilde{\xi}^2\|\delta\|^2 + \bar{\gamma}\sum_{i=1}^{\bar{s}}\|\tilde{\mu}\|^{2i} \quad (15)$$

$$\tilde{\xi}\bar{p}^T(\tilde{\mu})\bar{\delta}\bar{\theta} \le \frac{1}{4\gamma}\tilde{\xi}^2\bar{\theta}^T\bar{\delta}^T\bar{\delta}\bar{\theta} + \gamma\sum_{i=1}^{\bar{s}}\|\tilde{\mu}\|^{2i} \quad (16)$$

where $\gamma, \bar{\gamma} \in \Re^+$ are any positive reals. Notice that in (16) appear terms in the form $\bar{\theta}_i\bar{\theta}_j q_{ij}(t)$ where $q_{ij}(t)$, $i, j = 1, \ldots \bar{q}$, are suitable time-dependent functions, and there exists a suitable vector function $\beta(\gamma, \tilde{\zeta}, \tilde{\xi}, \omega, t) \in \Re^{\nu_3}$ available for measurement such that

$$\tilde{\xi}\beta^T\theta_e = \tilde{\xi}\bar{\pi}\bar{\theta} + \frac{1}{4\gamma}\tilde{\xi}^2\bar{\theta}^T\bar{\delta}^T\bar{\delta}\bar{\theta} + \tilde{\xi}\tilde{\eta}^T\tilde{\ell}. \quad (17)$$

By substituting (16), (17) in (14) and rearranging terms,

$$\begin{array}{rcl} \tilde{\xi}\dot{\tilde{\xi}}_1 & \le & \tilde{\xi}\beta^T\theta_e(t) + \tilde{\lambda}\left(\bar{\gamma} + \gamma\right)\sum_{i=1}^{\bar{s}}\|\tilde{\mu}\|^{2i} \\ & & +\tilde{\xi}\tilde{u} + \tilde{\xi}\pi + \frac{1}{4\bar{\gamma}}\tilde{\xi}^2\|\delta_1\|^2. \end{array} \quad (18)$$

By substituting (18), (13), in (12), since $2\tilde{\zeta}^T P_\zeta b\tilde{\xi} \le \left\|\tilde{\zeta}\right\|^2 + \left\|P_\zeta b\tilde{\xi}\right\|^2$, and by the definitions of Γ_e, S_e, $\Gamma_e S_e + S_e^T\Gamma_e \le 0$, by rearranging terms, we have

$$\begin{array}{l} \dot{V}_1 \le \left[-\lambda_1\lambda_m + \tilde{\lambda}\left(\bar{\gamma} + \gamma\right)\right]\sum_{i=1}^{\bar{s}}\|\tilde{\mu}\|^{2i} - \left\|\tilde{\zeta}\right\|^2 \\ +\tilde{\xi}\left(\tilde{u} + \beta^T\theta_e + \frac{1}{4\bar{\gamma}}\tilde{\xi}\|\delta\|^2 + \pi + \tilde{\xi}\|P_\zeta\|^2\right). \\ +\tilde{\theta}_e^T\Gamma_e\left(S_e\hat{\theta}_e - \dot{\hat{\theta}}_e\right) \end{array}$$

By setting

$$\begin{array}{rcl}\tilde{u} & = & -\beta^T\hat{\theta}_e - \frac{1}{4\tilde{\gamma}}\tilde{\xi}\left\|\delta\right\|^2 - \pi - \tilde{\xi}\left\|P_\zeta\right\|^2 - \tilde{\gamma}\tilde{\xi} \\ \dot{\hat{\theta}}_e & = & S_e\hat{\theta}_e + \Gamma_e^{-1}\beta\tilde{\xi}\end{array} \qquad (19)$$

where $\tilde{\gamma} \in \Re^+$, $\Upsilon = [c_0, \ \ldots, \ c_{r-1}, \gamma, \bar{\gamma}, \tilde{\gamma}]^T$, and choosing λ_1 such that $\lambda_1 > \frac{\tilde{\lambda}}{\lambda_m}(\bar{\gamma}+\gamma)$we have that

$$\dot{V}_1 \leq -\kappa_{\min}\left(\left\|\tilde{\mu}\right\|^2 + \left\|\tilde{\zeta}\right\|^2 + \tilde{\xi}^2\right) \qquad (20)$$

with $\kappa_{\min} = \min\left\{\lambda_1\lambda_m - \tilde{\lambda}(\bar{\gamma}+\gamma), 1, \tilde{\gamma}\right\} > 0$. Taking (20) into account and using Barbalat's Lemma (see [14]), it follows that $\lim_{t\to\infty}\left\|\left[\tilde{\mu}^T(t), \tilde{\zeta}^T(t), \tilde{\xi}^T(t)\right]\right\| = 0$. This result can be summarized in the following

Proposition 1 *Consider system (1). If the hypotheses (H1)-(H2) hold, then the global tracking problem in Definition 1 is solvable.*

2.2 Extension to systems in strict feedback form

Consider now the system

$$\begin{array}{l}\sum_\mu : \dot{\mu} = F(\zeta,\xi_1)\mu + G_1(\zeta,\xi_1)\theta + g_1(\zeta,\xi_1) \\ \sum_\eta : \dot{\eta} = S(\zeta,\xi_1)\eta + G_2(\zeta,\xi_1)\theta + g_2(\zeta,\xi_1) \\ \sum_\zeta : \left\{\begin{array}{l}\dot{\zeta} = f(\zeta) + g(\zeta)\xi_1 \\ z = h(\zeta)\end{array}\right. \\ \sum_\xi \left\{\begin{array}{rcl}\dot{\xi}_1 & = & \xi_2 + \phi_1(\zeta,\xi_1) + p^T(\mu)\psi_1(\zeta,\xi_1) \\ & & +\eta^T\ell_1(\zeta,\xi_1) + \bar{\phi}_1(\zeta,\xi_1)\theta \\ & & +p^T(\mu)\bar{\psi}_1(\zeta,\xi_1)\theta, \\ \dot{\xi}_2 & = & \xi_3 + \phi_2(\zeta,\xi_1,\xi_2) \\ & & +p^T(\mu)\psi_2(\zeta,\xi_1,\xi_2) + \bar{\phi}_2(\zeta,\xi_1,\xi_2)\theta \\ & & +p^T(\mu)\bar{\psi}_2(\zeta,\xi_1,\xi_2)\theta + \eta^T\ell_2(\zeta,\xi_1,\xi_2) \\ \vdots & \vdots & \vdots \\ \dot{\xi}_n & = & \phi_u(\zeta,\xi)u + \phi_n(\zeta,\xi) + p^T(\mu)\psi_n(\zeta,\xi) \\ & & +\bar{\phi}_n(\zeta,\xi)\theta + p^T(\mu)\bar{\psi}_n(\zeta,\xi)\theta + \eta^T\ell_n(\zeta,\xi).\end{array}\right.\end{array} \qquad (21)$$

that is a generalization both of (1), for the vector structure of $\sum_\xi$, and of a system in "strict feedback form". Assume that (21) complies with hypotheses (H1), (H2), (H3). If in a " *strict feedback form*" system there are no unmeasured dynamics, i.e. $\mu = 0$, $\eta = 0$,and $\theta \neq 0$, the problem of global adaptive tracking can be solved (see [7]) with a dynamic control law whose dimension is equal to the dimension of the vector of constant parameters $\theta \in \Re^q$. In recent years there has been considerable interest in deriving control laws for systems that are extensions of "benchmark" strict feedback form systems to systems with unmeasurable dynamics, as [10], following [9], ansd in [13], where global regulation can be achieved via a nonlinear mapping and a simple integrator, following the research line in [11].

It can be shown that problem 1 can be solved for system (21), by adapting to our case the strategy in [7], for the adaptive tracking without overparametrization. The key tool in the controller construction is the "backstepping" approach, according to which the solution of the problem for $n = i-1$ it is used to construct the control strategy for $n = i$. It can be obtained an iterative procedure for the synthesis of n smooth functions $\alpha_j(\tilde{\zeta}, \tilde{\xi}_1 \ldots, \tilde{\xi}_{j-1}, \omega, \hat{\theta}_e, t) \in \Re$, $j = 1, \ldots, n$, and n smooth vectors $\beta_j(\tilde{\zeta}, \tilde{\xi}_1, \ldots, \tilde{\xi}_j, \omega, t) \in \Re^{\nu_3}$ $j = 1, \ldots, n$. By defining $V_n = \lambda_1\sum_{\ell=1}^{\bar{s}}\left(\tilde{\mu}^T P_\mu\tilde{\mu}\right)^\ell + \tilde{\zeta}^T P_\zeta\tilde{\zeta} + \frac{1}{2}\left[\tilde{\theta}_e^T\Gamma_e\tilde{\theta}_e + \sum_{j=1}^n x_j^2\right]$, where $x_1 = \tilde{\xi}_1$, $x_j = \tilde{\xi}_j - \alpha_{j-1}$; $2 \leq j \leq n$, and by setting $\dot{\hat{\theta}}_e = S_e\hat{\theta}_e + \Gamma_e^{-1}\sum_{j=1}^n\beta_j x_j$, $\tilde{u} = \alpha_n$, inequality

$$\dot{V}_n \leq -\kappa_{\min}\left(\left\|\tilde{\mu}\right\|^2 + \left\|\tilde{\zeta}\right\|^2 + \sum_{j=1}^n x_j^2\right) \leq 0 \qquad (22)$$

holds, by a suitable $\kappa_{\min} > 0$. By (22), with arguments similar to the ones used when $\dim(\xi) = 1$ in previous subsection, it is straightforward to show that $\lim_{t\to\infty}\left\|\left[\tilde{\mu}^T(t), \tilde{\zeta}^T(t), x_1, \ldots, x_n\right]\right\| = 0$, and which implies that $\lim_{t\to\infty}(z_d(t) - z(t)) = 0$.

3 Controller implementation for the Lu-Gre model.

Consider the machine model in Definition 2. If we choose as control input the D. C. motor rotor current $u = i$, set $c_{\eta_1}\sigma_1 z = \eta_1$, $c_{\eta_2}z = \eta_2$, $x = \zeta$, $\dot{x} = \xi$, $\varphi(\xi) = \frac{|\xi|}{g(\xi)}$, $\theta_1 = c_\theta\sigma_1$, where c_{η_1}, c_{η_2}, c_θ are positive real rescaling parameters, then the system model becomes

$$\begin{array}{l}\sum_\eta : \dot{\eta} = S(\xi)\eta + G(\xi)\theta + g_1(\xi) \\ \sum_\zeta : \dot{\zeta}_1 = \xi \\ \sum_\xi : \dot{\xi}_1 = \frac{K}{J}u + \eta_1\ell_1(\xi) + \eta_2\ell_2(\xi) + \phi(\xi) + \bar{\phi}(\xi)\theta_1\end{array} \qquad (23)$$

with $\eta = [\eta_1, \eta_2]^T$, tracking output $\zeta \in \Re$, measurable states (ζ, ξ),

$$S(\xi) = \left[\begin{array}{ll}-\varphi(\xi) & 0 \\ 0 & -\varphi(\xi)\end{array}\right]; \qquad (24)$$

$$G(\xi) = \left[\begin{array}{c}c_{\eta_2}\xi \\ 0\end{array}\right]; \ g_1(\xi) = \left[\begin{array}{c}0 \\ \frac{c_{\eta_1}\xi}{c_\theta}\end{array}\right] \qquad (25)$$

where $\ell_1(\xi) = \dfrac{\varphi(\xi)}{Jc_{\eta_1}}$, $\ell_2(\xi) = \dfrac{-\sigma_0}{Jc_{\eta_2}}$, $\phi(\xi) = -\dfrac{\xi\sigma_2}{J}$, $\bar{\phi}(\xi) = -\dfrac{\xi}{Jc_\theta}$. System (23) is a particularization of (1), with $\mu = 0$, with the minor difference that $\varphi(\xi) \in C^\infty$ for all $\xi \in \Re$ except for $\xi = 0$. In this case global tracking can be obtained in the light of the arguments presented in this paper. The resulting control law will be therefore piece-wise smooth, leaving the entire procedure unchanged. If the trajectory $x_r(t)$ has to be tracked, then setting $\tilde{\zeta} = \zeta - x_r(t)$, $\tilde{\xi} = \xi - \dot{x}_r(t) + c_0 (\zeta - x_r(t))$, with $c_0 \in \Re^+$, and introducing the filtered transformation $\omega = \left[\hat{\eta}_0^T,\ \hat{\eta}_1^T\right]^T$, with

$$\begin{array}{rcll} \dot{\hat{\eta}}_0 &=& -\varphi(\xi)\hat{\eta}_0 + \frac{c_{\eta_1}\xi}{c_\theta} & \hat{\eta}_0 \in \Re \\ \dot{\hat{\eta}}_1 &=& -\varphi(\xi)\hat{\eta}_1 + c_{\eta_2}\xi & \hat{\eta}_1 \in \Re, \end{array} \tag{26}$$

system (23) in the new coordinates can be written as a particularization of (9) with $\mu = 0$, and $n = 1$. In this case the subsystem $\sum_{\tilde{\zeta}}$ is $\dot{\tilde{\zeta}} = -c_0\tilde{\zeta} + \tilde{\xi}$, and $\sum_{\tilde{\xi}}$ is expressed as in (9) with $\mu = 0$, $\pi = c_0(\xi - \dot{x}_r(t)) - \ddot{x}_r(t) + \phi(\xi) + \ell_2(\xi)\hat{\eta}_0$, $\bar{\pi} = \bar{\phi}(\xi) + \ell_1(\xi)\hat{\eta}_1$, and $\tilde{u} = \frac{K}{J}u$; the adaptive global tracking controller can be derived following subsection 2.1. We notice that does not apply to (23) the global adaptive tracking controller given in [4], which requires more restrictive assumptions. In fact, by setting $\eta_e = [\eta_1\ \eta_2\ \ \theta_1]^T$, subsystem $\sum_\eta$ can be rearranged as

$$\sum_{\eta_e} : \dot{\eta}_e = S_e(\xi)\eta_e + g_e(\xi) \tag{27}$$

$$S_e(\xi) = \left[\begin{array}{cc} S(\xi) & G(\xi) \\ 0 & 0 \end{array}\right];\ g_e(\xi) = \left[\begin{array}{c} g_1(\xi) \\ 0 \end{array}\right]$$

In [4] it is proposed a tracking control law assuming that there exists a symmetric positive definite matrix P_e -which is used to construct the tracking controller- such that

$$P_e S_e(\xi) + S_e^T(\xi)P_e \leq 0, \qquad \text{for all } \xi \in \Re. \tag{28}$$

It is also assumed that for every initial condition $\eta_e(t_0)$ and every bounded $(\zeta(t), \xi(t)) \in \mathcal{L}_\infty[t_0, \infty) \times \mathcal{L}_\infty[t_0, \infty)$ also the solution $\eta_e(t)$ starting from initial condition $\eta_e(t_0)$ is bounded, i.e. $\eta_e(t) \in \mathcal{L}_\infty^2[t_0, \infty)$. However, in this case a positive definite matrix P_e that complies with (28) does not exist. By contradiction, if the converse holds, (28) implies that for any $\xi(t)$

$$\|\eta_e(t)\| \leq \sqrt{\frac{\lambda_M}{\lambda_m}}\|\eta_0\| \tag{29}$$

with $\eta_e(t)$ solution of $\dot{\eta}_e = S_e(\xi(t))\eta_e$, λ_M and λ_m being the largest and smallest eigenvalue of P_e. The particularization of $\dot{\eta}_e = S_e(\xi(t))\eta_e$ to its second entry $\dot{\eta}_{e2}$ yields

$$\dot{\eta}_{e2} = -\sigma_0\varphi(\xi)\eta_{e2} + \xi\eta_{e3} \tag{30}$$

with scalar $\eta_{e2} = \eta_2$ and constant $\eta_{e3} = \theta$. System (30) with state $\eta_{e2}(t)$ and input ξ is reachable for any $\eta_{e3} \neq 0$; thus for any $\bar{\eta}_1 \in \Re$, and $T \in \Re^+$, there exists a function $\xi(t)$ such that $\eta_{e2}(T) = \bar{\eta}_1$. This contradicts (29) and in turn, hypothesis (28)

3.1 Tracking control simulation

The adaptive tracking control strategy (19) (26) has been simulated for system (2) with tracking trajectory $z_d(t) = 5.6\left[\sin(0.4\pi t)\right]\left[\sin(0.02\pi t)\right]$, setting $c_0 = 50$, $\tilde{\gamma}_1 = 10$, choosing Γ_e , P_ζ as identity matrices, model parameters proposed in [3]. numerical values of systems parameters are:

system parameter	nominal value
$\sigma_0\ \left(N\ m\ rad^{-1}\right)$	260.0
$\sigma_1\ \left(N\ m\ s\ rad^{-1}\right)$	0.6
$\sigma_2\ \left(N\ m\ s\ rad^{-1}\right)$	0.018
$\alpha_0\ (N\ m)$	0.285
$\alpha_1\ (N\ m)$	0.05
$v_s\ \left(rad\ s^{-1}\right)$	0.01
$J\ (Kg\ m^{-2})$	0.0022
$K\ (N\ m\ A^{-1})$	0.352

The simulation has been carried on for time $t : 0 \leq t \leq 50$ seconds, with all initial conditions set to zero. In Figure 1 it is shown the reference trajectory, in Figure 2 the tracking error trajectory, in Figure 3 the D.C. rotor current i that provides the input $\tilde{u} = \dfrac{Ki}{J}$, and in Figure 4 the first entry of $\hat{\theta}_e$, i.e. the estimate of $\sigma_1 = \frac{\theta_1}{c_\theta}$.

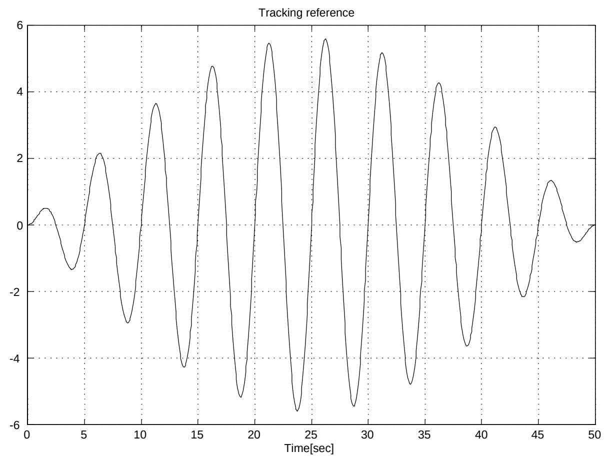

Figure 1: Reference position for $0 \leq t \leq 50$

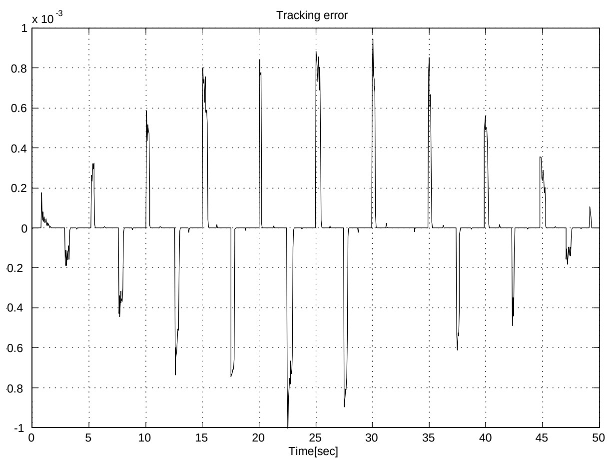

Figure 2: Tracking position error for $0 \leq t \leq 50$.

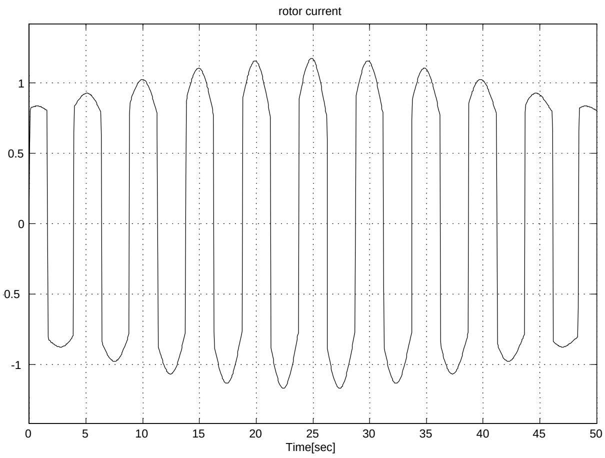

Figure 3: D.C. motor rotor current for $0 \leq t \leq 50$

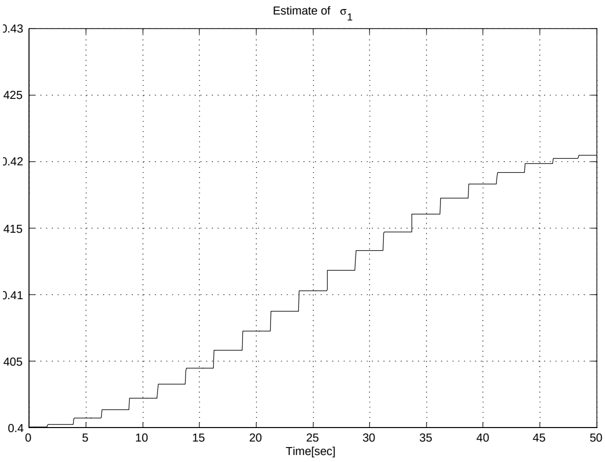

Figure 4: estimate of σ_1 for $0 \leq t \leq 50$.

Notice that while (as expected) the trajectory error is kept "small", estimate of σ_1 is instead far from *fast* convergence, since this property relies on suitable "persistency of excitation" conditions on the tracking trajectory.

4 Conclusion

We have provided a family of dynamic partial state feedback controllers for global asymptotic tracking. We have assumed that a subset of the unmeasurable states enters in the system nonlinearly, via a vector whose entries are polynomial functions. We have also rendered the global asymptotic tracking strategy adaptive with respect to a vector of unknown constant parameters, extending previous results on this subject. The strategy proposed in this note can be successfully applied to those systems, like in the nonlinear friction example, in which global tracking has to be generated even though there are unmeasured states and unknown parameters. By properly tuning scalar controller's parameters it is possible to select within the family of controllers a "practically implementable" control law, with a satisfactory transient behavior.

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