Design of Robust Repetitive Control System for Multiple Periods

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Abstract

In this paper, a new approach to regulate quickly any periodic signals with multiple periods is proposed. The contributions are as follows. First, the proposed multiple repetitive controller not only can be implemented with much less memory elements than the previous ones but also can provide much faster convergence of the controlled error to zero. Secondly, the proposed repetitive controller can assign all poles of the system on a disk with a given radius whose center is the origin. Thirdly, the proposed controller is obtained in an explicit form and the design method requires to solve no equation. Fourthly, the robustness of the system is improved by introducing a low pass zero phase filter.

1. Introduction

In many tracking control problem, a desired output or disturbance input includes periodic signals with some known periods. The repetitive control system is a servo system that achieves zero steady-state tracking error for any periodic desired outputs and any periodic disturbance inputs with fixed periods.

Tomizuka et al. [5] have dealt with the repetitive control system for a single period and have presented a repetitive controller to assure the stability of the system in an explicit form. Since it is required to solve no equation, the design effort is small even if the period is large [2]. In [6,7], the repetitive controller of [5] has been extended from the viewpoint of the pole placement. However, the periodic generator using in these all previous methods is restricted to that with a single period. Consequently, in order to regulate any periodic signals with multiple periods, the dead-time length of the generator should be equal to the least common multiple (*l.c.m*) of all the periods. This is undesirable, since the size of memory for implementation of the dead-time element should be very large.

In this paper, a new approach to regulate quickly any periodic signals with multiple periods is proposed and a useful multiple repetitive control system is presented from the practical viewpoint. The contributions are as follows: First, the dead-time length of the periodic generator is reduced to the sum of all the periods. Therefore, the proposed multiple repetitive controller not only can be implemented with much less memory element than the previous ones but also can provide much faster convergence of the controlled error to zero. Secondly, the proposed repetitive controller not only can assure the stability of the multiple repetitive control system but also can assign all poles of the closed loop system on the disk with a given radius whose center is the origin. Thirdly, the proposed controller is obtained in an explicit form and the design method requires to solve no equation. The design effort is very small even if the periods are very large. Fourthly, the robustness of the system is improved by introducing a low pass zero phase filter. Finally, the effectiveness is demonstrated by simulation.

The following notations are used throughout this paper. $\mathbf{R}$, $\mathbf{C}$ and $\mathbf{I}_+$ denote the set of real numbers , the set of complex ones and the set of positive integers including 0, respectively. $\mathbf{R}[z]$ and $\mathbf{R}[z^{-1}]$ denote the set of real polynomials in z and z^{-1}, respectively.

2. Multiple Repetitive Control System

Figure 1 depicts the overall structure of the single-input single-output discrete time control system studied in this paper. $r[k], e[k], u[k], d[k]$ *and* $y[k]$ are the desired output, the controlled error, the control input, the disturbance input and the controlled output, respectively. $P(z^{-1})$ and $C(z^{-1})$ denote the proper discrete-time plant to be controlled and the proper discrete time controller to be designed, respectively. We assume that $P(z^{-1})$ has no zero on the unit circle. Let $L_i \in \mathbf{I}_+, \quad i = 1 \cdots h$ be the periods of the periodic signals to be regulated. Without loss of generality, we assume that

$$L_1 \ge L_2 \ge \cdots \ge L_h . \tag{2.1}$$

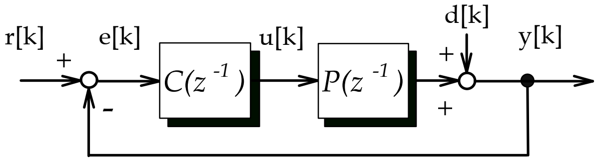

Fig.1 Control System

The z-transform of the periodic signals $r_i[k]$, $i = 1 \cdots h$ with period L_i, $i = 1 \cdots h$ are represented as

$$R_i(z^{-1}) = \frac{\hat{R}_i(z^{-1})}{1 - z^{-L_i}}, \quad i = 1 \cdots h, \qquad (2.2)$$

respectively, where $\hat{R}_i(z^{-1}) \in \mathbf{R}[z^{-1}]$ is the z-transform of the finite sequence consisting of the first period of $r_i[k]$, *i.e.,*

$$\hat{R}_i(z^{-1}) = \sum_{k=0}^{L_i - 1} r_i[k] z^{-k}.$$

The purpose of this paper is to present a multiple repetitive control system to achieve zero steady-state tracking error for any desired outputs and any periodic disturbance inputs consisting of the periodic signals $r_i[k]$, $i = 1 \cdots h$ with the plural number of periods L_i, $i = 1 \cdots h$. The previous approach is to introduce the generator $1/(1 - z^{-L})$ of the periodic signal with the single period given by

$$L = LCM\{L_i, i = 1 \cdots h\} \in \boldsymbol{I}_+ \qquad (2.3)$$

into the controller $C(z^{-1})$. However, this is undesirable from the viewpoint of implementing the repetitive system, since the size of memory for implementation of the dead-time element should be very large. To overcome this problem, we suggest the multiple repetitive controller as follows;

$$C(z^{-1}) = \tilde{C}(z^{-1}) \prod_{i=1}^{h} \frac{1}{(1 - z^{-L_i})}, \qquad (2.4)$$

where $\tilde{C}(z^{-1})$ is a controller to be designed. Then the following lemma holds.

Lemma 2.1

Consider the control system with $C(z^{-1})$ of eq.(2.4). Then the steady-state controlled error becomes zero for any periodic desired outputs and any periodic disturbance inputs with the periods L_i, $i = 1 \cdots h$, provided that the closed-loop system is asymptotically stable.

Proof : The proof is straightforward from the internal model principle, and is omitted. Q.E.D.

Remark : The dead-time length of the generator in eq.(2.4) is reduced to the sum of all the periods. As a result, the multiple repetitive controller of eq.(2.4) can be implemented with much less memory element than the previous controller based on a generator with the single period L of eq.(2.3).

3. Stabilizing Controller

In this section, we will present a new and useful class of controllers that stabilize the multiple repetitive control system in an explicit form. Let a plant described by

$$P(z^{-1}) = \frac{z^{-d} n(z^{-1})}{d(z^{-1})}, \qquad (3.1)$$

where $n(z^{-1}) \in \mathbf{R}[z^{-1}]$ and $d(z^{-1}) \in \mathbf{R}[z^{-1}]$ are coprime and $d \in \mathbf{I}_+$. Without loss of generality, it is assumed that the plant is asymptotically stable. $n(z^{-1}) \in \mathbf{R}[z^{-1}]$ is factored as follows:

$$n(z^{-1}) = n_s(z^{-1}) n_u(z^{-1}), \qquad (3.2)$$

where $n_s(z^{-1}) \in \mathbf{R}[z^{-1}]$ contains all asymptotically stable zeros and $n_u(z^{-1}) \in \mathbf{R}[z^{-1}]$ contains the other zeros, *e.g.,* unstable zeros. Then the following controller is proposed.

$$\tilde{C}(z^{-1}) = \sum_{i=1}^{h} z^{-L_i + d + m_{F_i} + m} \left\{ z^{-m_{F_i}} F_i(z) \right\} \left\{ z^{-m} n_u(z) \right\} \times \prod_{j=1}^{i-1} (1 - z^{-L_j}) \prod_{j=i+1}^{h} \left(1 - a(z, F_j) z^{-L_j} \right) \frac{d(z^{-1})}{n_s(z^{-1})}, \qquad (3.3)$$

where

$$a(z, F_j) = 1 - F_j(z) n_u(z) n_u(z^{-1}), \qquad (3.4)$$

$n_u(z) \in \mathbf{R}[z]$ is a polynomial in z obtained by replacing z^{-1} in $n_u(z^{-1}) \in \mathbf{R}[z^{-1}]$ by z, and m denotes the highest degree of $n_u(z)$ in terms of z. $F_i(z)$'s are zero phase FIR filters to be designed and m_{F_i} denotes the highest degree of the $F_i(z)$ in terms of z.

Theorem 3.1

For $L_i - d - m_{F_i} - m \geq 0$, $i = 1 \cdots h$, the controller of eq.(3.3) is proper. Then the repetitive control system with the controller is asymptotically stable for any zero phase FIR filters $F_i(z)$, $i = 1 \cdots h$ that satisfy

$$0 < F_i(e^{jwT}) < \frac{2}{\left| n_u(e^{jwT}) \right|^2}, \qquad (3.5)$$

$$\forall wT \in [0, p], \quad i = 1 \cdots h.$$

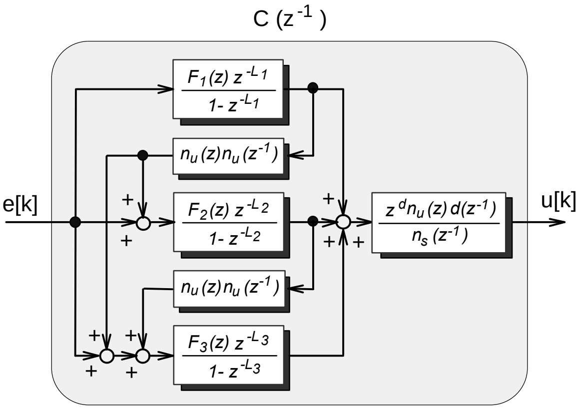

Fig.2 Structure of proposed multiple repetitive controller (*h* = *3*)

Proof : It is obvious that the controller of eq.(3.3) is proper. For the controller, the closed loop characteristic polynomial $j\,(z^{-1})$ as follows.

$$j\,(z^{-1}) = d(z^{-1})n_s(z^{-1})\prod_{i=1}^{h}\left(1-a(z,F_i)z^{-L_i}\right). \tag{3.6}$$

It follows that

$$\left\| a(z,F_i)z^{-L_i} \right\|_\infty < 1 \tag{3.7}$$

for any zero phase FIR filters $F_i(z),\ i=1\cdots h$ that satisfy eq.(3.5). From the small gain theorem, the $j\,(z^{-1})$ of eq.(3.6) is an asymptotically stable polynomial in z^{-1}. The proof is completed. Q.E.D.

Remark 1 : The class of stabilizing controllers for the multiple repetitive control system is obtained by eq.(3.3) in an explicit form, and the free parameter are characterized by zero phase FIR filters $F_i(z),\ i=1\cdots h$ that satisfy eq.(3.5). Letting $h=1$, the controller of eq.(3.3) is exactly equal to that of [6,7].

Remark 2 : From eqs.(2.4) and (3.3), the proposed multiple repetitive controller is represented as follows,

$$C(z^{-1}) = \frac{z^d n_u(z)d(z^{-1})}{n_s(z^{-1})}\sum_{k=1}^{h}\frac{F_k(z)z^{-L_k}}{1-z^{-L_k}} \times\prod_{j=k+1}^{h}\left(1+\frac{F_j(z)z^{-L_j}}{1-z^{-L_j}}n_u(z)n_u(z^{-1})\right). \tag{3.8}$$

Figure 2 shows the structure of the proposed controller $C(z^{-1})$ for the case of $h=3$. We can see that the proposed controller has a simple and compact structure.

4. Pole of Closed Loop System

In this section, an interesting method is proposed to determine the zero phase FIR filters from the viewpoint of the pole assignment of the closed loop system. For the controller of eq.(3.3), the transfer function $G_e(z^{-1})$ from *r* to *e* becomes

$$G_e(z^{-1}) = \prod_{i=1}^{h}\frac{1-z^{-L_i}}{1-a(z,F_i)z^{-L_i}}. \tag{4.1}$$

Denote the maximum value of the gain of $[a(z,F_i)]_{z=e^{jwT}}$ as follows.

$$m(F_i) = \max_{wT\in[0,p]}\left|[a(z,F_i)]_{z=e^{jwT}}\right|. \tag{4.2}$$

For any $F_i(z),\ i=1\cdots h$ that satisfy eq.(3.5), it is obvious that

$$m(F_i) < 1,\ i=1\cdots h. \tag{4.3}$$

The following lemma shows a relationship between the $m(F_i)$ and the poles of the closed loop system.

Lemma 4.1

For any zero phase FIR filters $F_i(z),\ i=1\cdots h$ that satisfy eq.(3.5), all poles of $1/(1-a(z,F_i)z^{-L_i})$ are on the region of $D[r_i(F_i)]$, which denotes the disk with the radius of $r_i(F_i)$ whose center at the origin, where

$$r_i(F_i) = \sqrt[L_i+m_{F_i}+m]{m(F_i)}. \tag{4.4}$$

Moreover all poles of the $G_e(z^{-1})$ of eq.(4.1) are on the region of $D[r_{\max}]$, where

$$r_{\max} = \max_{i=1\cdots h} r_i(F_i). \tag{4.5}$$

Proof : omitted.

Next, we present a zero phase FIR filter to make the $r_i(F_i)$ small in an explicit form. The following zero phase FIR filter is proposed,

$$F_i(z) = f_{opt}\sum_{i=0}^{N_i}a(z,f_{opt})^i, \tag{4.6}$$

where $N_i \in \mathbf{I}_+$ is a parameter to be designed. The $a(z,f_{opt})$ is given by eq.(3.4), where

$$f_{opt} = 2\ /\ (n_{\max}+n_{\min}) \in \mathbf{R} \tag{4.7}$$

$$n_{\max} = \max_{wT\in[0,p]}\left|n_u(e^{jwT})\right|^2 \tag{4.8}$$

$$n_{\min} = \min_{wT\in[0,p]}\left|n_u(e^{jwT})\right|^2. \tag{4.9}$$

Then we have

$$m_{F_i} = mN_i\ . \tag{4.10}$$

The following lemma is obtained.

Lemma 4.2

The repetitive control system with both $\tilde{C}(z^{-1})$ of eq.(3.3) and $F_i(z)$ of eq.(4.6) is asymptotically stable for any $N_i \in \mathbf{I}_+$. Moreover, for a given N_i, all poles of $1/(1-a(z,F_i)z^{-L_i})$ are on the region of $D[r_i(F_i)]$, where the $r_i(F_i)$ is given by

$$r_i(F_i) = \left(\frac{n_{\max} - n_{\min}}{n_{\max} + n_{\min}}\right)^{\frac{1}{\frac{L_i}{N_i+1}+m}}\ . \tag{4.11}$$

Proof : omitted.

Remark 1 : As the N_i increases, the $r_i(F_i)$ of eq.(4.11) monotonically decreases. Consequently, by an appropriate choice of the design parameters N_i's, the proposed controller of eq.(3.3) with the FIR filter of eq.(4.6) can assign all poles of the system on a disk with a given radius whose center is the origin. For detail, see [8, Theorem 2].

Remark 2 : In order to guarantee the properness of the controller of eq.(3.3) with the FIR filter of eq.(4.6), the admissible range of N_i's are given by

$$0 \le N_i \le \frac{L_i - d - m}{m},\ \ i = 1 \cdots h\ . \tag{4.12}$$

5. Robustness

The aim of this section is to analyze and improve the robustness of the proposed control system. Consider the following multiplicative perturbation of P

$$\tilde{P} \in \mathcal{P}(P,W) = \Big\{ P(1+\Delta)\ \Big|\ \Delta \in RH_\infty, \\ \left|\Delta(e^{-jwT})\right| < \left|W(e^{-jwT})\right|,\ \forall wT \in [0, p\,] \Big\}. \tag{5.1}$$

Then it is well known that the closed loop system of Fig.1 for all $\tilde{P} \in \mathcal{P}(P,W)$ is stable if and only if

$$\left|T(e^{-jwT})\right| \left|W(e^{-jwT})\right| \le 1,\ \forall wT \in [0, p\,], \tag{5.2}$$

where the $T(z)$ denotes the complimentary sensitivity function of the system. Equation (5.2) shows that, for any fixed $w \in \mathbf{R}$, the upper bound of $|\Delta|$ that can guarantee the stability of the system for all $\tilde{P} \in \mathcal{P}(P,W)$ is given by $1\Big/\left|T(e^{-jwT})\right|$. Consequently, as making the value of $\left|T(e^{-jwT})\right|$ smaller, the robust stability of the system can be guaranteed in the presence of larger class of plant perturbation. From Fig.1, the complimentary sensitivity function is given by

$$T(z^{-1}) = \frac{P(z^{-1})\tilde{C}(z^{-1})}{\prod_{i=1}^{h}(1 - z^{-L_i}) + P(z^{-1})\tilde{C}(z^{-1})}\ . \tag{5.3}$$

Note that the value of $\left|T(e^{-jwT})\right|$ is equal to unity at $w = 2p\,k / L_i\,T,\ k \in \mathbf{I}_+$, independent of the controller $\tilde{C}(z^{-1})$. This indicates that the upper bound of $|\Delta|$ that can guarantee the stability of the system is restricted to one at $w = 2p\,k / L_i\,T,\ k \in \mathbf{I}_+$.

Now, in order to improve the robustness of the system, the z^{-L_i} in the controller $C(z^{-1})$ is replaced with the $Q(z)z^{-L_i}$ [1,2,4], where the $Q(z)$ is a zero phase low pass FIR filter satisfying

$$0 \le Q(e^{jwT}) \le 1,\ \ \forall wT \in [0, p\,]\ . \tag{5.4}$$

Then the controllers of eqs.(2.4) and (3.3) are modified as follows:

$$C(z^{-1}) = \tilde{C}(z^{-1}) \prod_{i=1}^{h} \frac{1}{1 - Q(z)\,z^{-L_i}}\ , \tag{5.5}$$

where

$$\tilde{C}(z^{-1}) = \sum_{i=1}^{h} Q(z)\,z^{-L_i + d + m_{F_i} + m} \left\{ z^{-m_{F_i}} F_i(z) \right\} \left\{ z^{-m} n_u(z) \right\} \\ \times \prod_{j=1}^{i-1}(1 - Q(z)\,z^{-L_j}) \prod_{j=i+1}^{h} \left(1 - a(z,F_j)Q(z)\,z^{-L_j}\right) \frac{d(z^{-1})}{n_s(z^{-1})} \tag{5.6}$$

and the $F_i(z)$'s are given by eq.(4.6). The complimentary sensitivity function is represented as

$$T(z^{-1}) = 1 - \prod_{i=1}^{h} (1 - T_i(z^{-1}))\ , \tag{5.7}$$

where

$$T_i(z^{-1}) = \frac{Q(z)(1 - a(z,F_i))z^{-L_i}}{1 - Q(z)a(z,F_i)z^{-L_i}}\ . \tag{5.8}$$

Equations (5.7) and (5.8) show that if the value of the $Q(e^{jwT})$ at high frequencies is close to zero, then is so for both $\left|T_i(e^{-jwT})\right|$ and $\left|T(e^{-jwT})\right|$. Therefore the introduction of the zero phase low pass FIR filter $Q(z)$ not only can maintain the learning mechanism of the internal model of eq.(2.4) at low frequencies, but also can guarantee at high frequencies the robust stability in the presence of larger class of plant perturbation than that without $Q(z)$. Note that the value of $\left|T(e^{-jwT})\right|$ depends on w and $Q(e^{jwT})$. In order to stress its dependency, we write $T_i(e^{-jwT})$ as $T_i(w, Q)$. The following corollary shows how the value of $Q(e^{jwT})$ effects that of $\left|T(e^{-jwT})\right|$ and the radius of the dominant pole of the closed loop system.

Theorem 5.1

For any zero phase FIR filters $Q(z)$ satisfying eq.(5.4), the repetitive control system with the controller of eqs.(5.5) and (5.6) is asymptotically stable and all poles of the closed loop system are on the region of $D[\tilde{r}_{\max}]$, where

$$\tilde{r}_{\max} = \max_{i=1\cdots h} r_i(F_i, Q), \tag{5.9}$$

$$r_i(F_i, Q) = \left(\frac{n_{\max} - n_{\min}}{n_{\max} + n_{\min}}\right)^{\frac{1}{\frac{L_i + m_Q}{N_i + 1} + m}} \tag{5.10}$$

and $m_Q \in \mathbf{I}_+$ denotes the highest degree of the $Q(z)$ in terms of z. Moreover for any fixed w and N_i, if

$$Q_1(e^{jwT}) \le Q_2(e^{jwT}), \tag{5.11}$$

then

$$\left|T_i(w, Q_1)\right| \le \left|T_i(w, Q_2)\right|, \quad i = 1 \cdots h. \tag{5.12}$$

Proof : omitted. Q.E.D.

Remark : This corollary provides important and interesting suggestions to decide the suitable low pass FIR filter $Q(z)$ as follows: As making the $Q(e^{jwT})$ smaller, the value of $\left|T_i(e^{-jwT})\right|$ becomes monotonically smaller and the robust stability can be guaranteed in the presence of larger class of perturbations. On the other hand, internal model is violated and the steady-state controlled error becomes larger. Consequently, the design of the $Q(z)$ requires the trade-off between the robustness and the tracking performance in the steady-state.

6. Simulation Result

In this section, the proposed schemes are applied to a positioning table using a DC servo motor [3]. The sampling period T is set to 10 (ms). The discrete-time characteristics of this plant is described as follows [8]:

$$G_p(z^{-1}) = \frac{z^{-1}(0.0082 + 0.031z^{-1})}{(1 - z^{-1})(1 - 0.799z^{-1})}, \tag{6.1}$$

By an appropriate feedback controller, this plant of eq.(6.1) is stabilized. This stabilized plant is given by

$$P(z^{-1}) = \frac{z^{-1}(0.0082 + 0.031z^{-1})}{(1 + 0.29z^{-1})(1 - 0.20z^{-1})(1 - 0.46z^{-1})(1 - 1.7z^{-1} + 0.73z^{-2})} \tag{6.2}$$

6.1 Pole of Closed Loop System

Two periods of the periodic desired output to be tracked are set as $\{L_1, L_2\} = \{299, 256\}$. Table 1 shows the $r_{\max}$ of eq.(4.5) and the dominate poles of the $G_e(z^{-1})$ of eq.(4.1) for some N_i's. It is confirmed that not only the FIR filters $F_i(z)$, $i = 1, 2$ given by eq.(4.6) guarantee the stability of the repetitive control system, but also as increasing N_i, the values of $r_{\max}$ become smaller and the dominate poles are closer to the origin.

6.2 Robustness

In order to improve the robustness of the system, the same zero phase low-pass filter $Q(z)$ as that of [1,4,7] is introduced to the controller of eqs.(5.5) and (5.6) :

$$Q(z) = \frac{z^{-1} + 2 + z}{4}. \tag{6.3}$$

The gains of the frequency complimentary sensitivity functions $T(e^{jwT})$ without and with the filter $Q(z)$ are plotted in Fig.3. It is illustrated that the introduction of the FIR filter $Q(z)$ not only can maintain the learning mechanism of the internal model of eq.(2.4) at low frequencies, but also can guarantee at high frequencies the robust stability in the presence of larger class of plant perturbation than that without $Q(z)$.

6.3 Tracking Performance

Figure 4 shows the repetitive signals r_1 and r_2 with the periods L_1 and L_2, respectively. The desired output consists of the sum of these two repetitive signals. Figure 5 shows the controlled error responses for the proposed controllers of $(N_1, N_2) = (0, 0)$ and $(1,1)$ respectively. It is confirmed that the proposed controllers provide faster convergence of the controlled error to zero as increasing N_i's.

6.4 Comparison with the previous methods

The proposed method is compared with the pervious ones, *e.g.,* [1,2,4-7], based on the periodic generator with the single period. First, we compare them from the viewpoint of the implementation of the controller. The pervious controllers need to involve the periodic generator of the $1/(1 - z^{-L})$ with the large period of $L = 76544$ from eq.(2.3). On the other hand, in the proposed controller, the dead-time length of the generator is reduced to the sum of two periods, *i.e.,*

$L_1 + L_2 = 555$. Therefore, we can see that the proposed controller can be implemented with much less memory elements than the previous ones. Secondly, we compare the convergence speed of the controlled error. In the previous controller, the settling time is at least larger than $LT = 765.44\,(s)$, since the previous controllers involve the dead-time of the LT. Figure 5 shows that the proposed controller provide much faster convergence of the controlled error to zero.

7. Conclusion

This paper has presented a new multiple repetitive controllers to regulate quickly any periodic signals with multiple periods in an explicit form.

Acknowledgments

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TABLE 1
The values of $r_{\max}$ and the dominate poles of the closed-loop system for some N_i's

(N_1, N_2)	$r_{\max}$	the dominate poles of the closed-loop system
(0, 0)	0.9977	- 0.9976
(1, 1)	0.9953	± 0.9953
(3, 3)	0.9907	± 0.9906

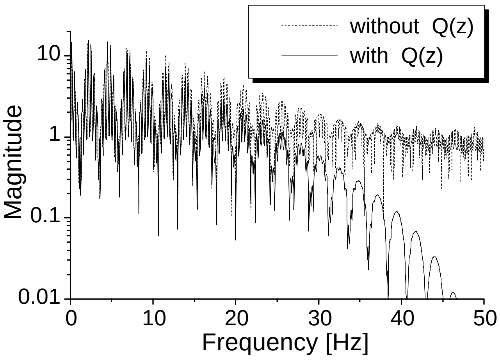

Fig. 3 Magnitude of frequency complementary sensitivity functions $T(e^{jwT})$

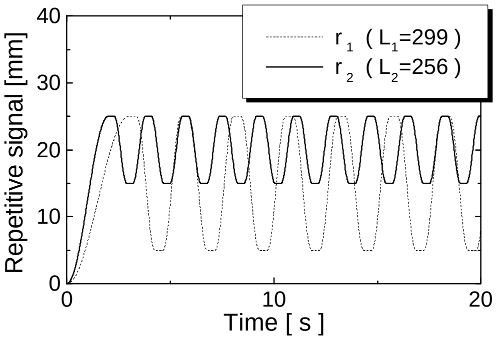

Fig. 4 Repetitive signals with two periods L_1 and L_2

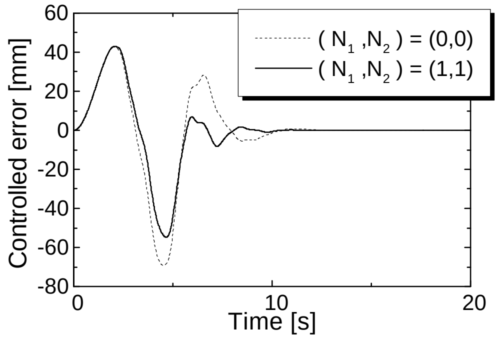

Fig. 5 Controlled error responses