Sliding Mode Control of an Underwater Robotic Manipulator

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Abstract

This paper deals with the application of the sliding mode control theory to the specific case of a manipulator for which mono–directional control actions only have to be considered. In particular the control of an underwater gripper is presented. Many even complex robotic structures, can be actuated by mono–directional control actions, for example the so called tendon–arms, the jet–actuated vehicles, underwater vehicles with mono–directional thrusters, etc.. The robotic system which is considered in the paper belongs to the above class, since it is actuated by voice coil motors which, acting on a hydraulic circuit, are able to generate mono–directional forces. In practical realizations actuators often show imprecise relationships between the electrical input signals and the mechanical outputs, that is joint forces or torques. Such a situation constitutes a source of uncertainties we have to deal with. A sliding mode control methodology based on the use of a simplex of constant control vectors is presented, which has revealed to be general enough to work with different applications too.

1 Introduction

Sliding mode control applied to mechanical systems has to be considered a very delicate task due to the so called chattering phenomenon[1]. The use of a continuous approximation of the control law, obtained by means of saturation, could appear a good methodology to counteract the chattering phenomenon. Unfortunately, when the behaviour of mechanical systems under the action of sliding mode controllers is examined, or simply when these systems are studied in simulation, a second important fact becomes apparent that is, due to the presence of non idealities the frequency of auto oscillations induced by discontinuous control can become unpredictably low so that hidden resonant modes can be excited.

This phenomenon becomes even worse if the discontinuous control laws are smoothed. On the basis of the previous considerations it is possible to identify a class of systems to which sliding mode control approach can be suitably applied. Examples of this kind are those mechanical systems which are directly driven by high bandwidth actuators and sensors, and for which the time delays, including the computing time, are such that the frequency which can be auto sustained is sufficiently high not to excite mechanical resonance of the system which can be experimentally evaluated. In such a situation the theoretical effectiveness of the sliding mode approach can be fully exploited.

In this paper it is presented a mechanical system, constituted by a three fingered gripper, designed to belong to the previously sketched class. In Section 2 the structural characteristics of the prototype of the gripper, designed to meet underwater manipulation specifications, are outlined and a dynamical model of the system is proposed. The design of the sliding mode control of the system is presented in Section 3; the control is based on the exploitation of the *simplex vector method* and suitably implemented by the mono–directional hydraulic actuators of the system. This method consists in identifying a collocation of the actuation system in the structure so that the control vector generated by the action of one actuator (or thruster) at a time form a simplex in a suitable vector space. In accordance with the sliding mode control theory, the control objective is represented by the zeroing of a suitable output vector thus the problem consists in identifying a switching logic causing the finite reaching of the control objective in which a single vector of the control simplex is univocally associated to a single region among the ones in which the output space is partitioned. Finally in Section 4 experimental results are presented.

2 The Gripper

The considered gripper is an underwater robotic manipulator exploiting the *elephant trunk* principle[2]. Due to the control design based on the sliding mode approach, this robotic hand (Fig. 1), can be regarded as one of the few successful application of the sliding mode control theory to a mechanical device. The control system has been investigated and developed together with the design and realization of the gripper itself so that they result to be well suited each other. Three fingers within the gripper are arranged by a symmetric geometry, so to allow the manipulation of objects within the finger–tips. having dimensions up to about 15 cm. During an EC Project,[1] the fingers and an *ad hoc* hydraulic actuation system electrically driven by linear motors, have been designed and developed. Both in the finger (Fig. 2) and in the actuation system, the basic element is a nickel bellow with a low wall–thickness and elastic–bodies characteristics also showing low axial rigidity, high flexibility, and high resistance to bending torques and pressure. Each finger is constituted by three bellows located at the vertices of an equilateral triangle. Then the three bellows are connected each other by means of two plates fixed respectively at the top and bottom of the finger system, whose centers are connected through a rigid link articulated by a cardan joint. In such a way the finger–tip is allowed to span a portion of a spherical surface. The movements of the tip are driven by a hydraulic system, which pumps oil within the bellows. A set of linear motors operates on a relevant set of control bellows which are connected one to one to the finger bellows by means of oil–filled pipelines. The whole hydraulic circuit is closed and hence characterized by constant volume (Fig. 3).

Each bellow acts as a hydraulic cylinder, it is friction–free, and it can operate at high frequency due to the absence of any mechanical coupling. By connecting a control bellow to a finger bellow in the constant volume closed circuit, it was possible to realize a force transmission mechanism.

2.1 The Mathematical Model of the Gripper

For each element of the gripper, two coordinate frames can be defined and it is possible to depict the finger–tip position in terms of a rotation matrix between these two frames. Let us consider the notation of Fig. 4 for the reference frame in the definition of the finger dynamic

[1] European Commission Directorate General XII MAST III Contract: *MAS3-CT95-0024*

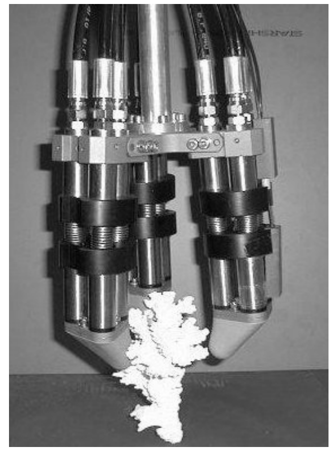

Figure 1: the gripper.

equations. The dynamic equation of the considered system can be expressed as

$$A\,\dot{\omega} + \omega \times p = \tau + M \qquad (1)$$

where A is the inertia matrix of the system, p is the angular momentum, ω is the angular velocity, τ is the vector of the torques applied to the finger, and M is the vector of the elastic torques applied to the finger by each bellow. Forces F_i, with $i = 1, 2, 3$, are the ones exerted by the voice coil motors, E_i, $i = 1, 2, 3$, are the elastic forces deriving from any bellow due to the axial displacement, and M_i, with $i = 1, 2, 3$, are the elastic torques depending on the angular deformation of each bellow. The F_i and E_i can be assumed proportional to quantities depending on the angular deformation of each bellow. The forces F_i exerted by the voice coil motors and the elastic forces E_i deriving from any bellow are all directed along the z axis of the reference frame, and their application points form a simplex of vectors in the frame of the finger, therefore the vectors τ and M of the torques applied to the system result to belong to

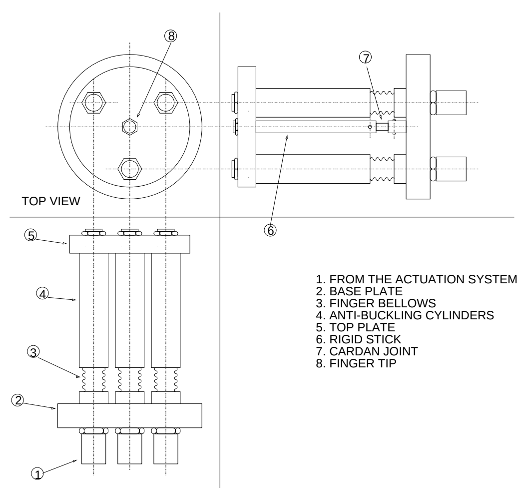

Figure 2: the finger.

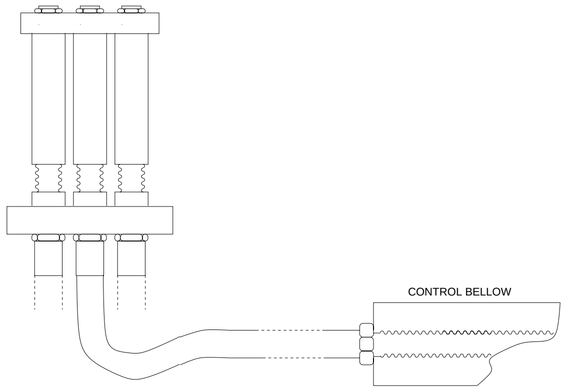

Figure 3: the actuation system.

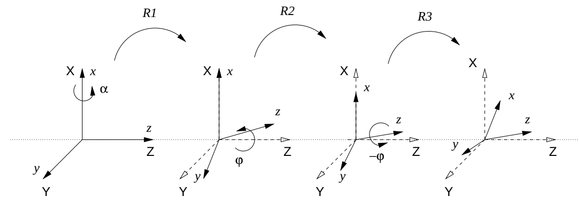

Figure 5: the coordinate frames during the rotation.

the (x, y)–plane of the reference frame where they form a simplex of vectors. The rotation matrix between the inertial frame and the reference one is given by

$$R(\alpha,\varphi) = \begin{bmatrix} 1-(1-\cos\alpha)\sin^2\varphi & (1-\cos\alpha)\sin\varphi\cos\varphi & \sin\alpha\sin\varphi \\ (1-\cos\alpha)\sin\varphi\cos\varphi & 1-(1-\cos\alpha)\cos^2\varphi & -\sin\alpha\cos\varphi \\ -\sin\alpha\sin\varphi & \sin\alpha\cos\varphi & \cos\alpha \end{bmatrix} \tag{2}$$

The different orientation between the two reference frames is effect of three finite rotations as it is described in the following and shown in Fig. 5. The first rotation of an angle α, is performed about the x axis of the inertial frame. The second rotation of an angle φ is about the z axis of the inertial frame, and the rotation matrix depends on both φ, the rotation angle, and α due to the rotation versor. The third rotation is of an angle $-\varphi$ and is performed around the z axis of the frame fixed to the system. The last rotation takes into account the specific kinematic of the cardan joint. According to the Euler Theorem, it is possible to find the rotation angle $\theta(\alpha,\varphi)$ and the versor $n(\alpha,\varphi)$ of the rotation axis, so that in each instant a rotation of θ about n gives the orientation of the reference frame with respect to the inertial one

$$R(\alpha,\varphi) = R\left[(n(\alpha,\varphi), \theta(\alpha,\varphi)\right] \tag{3}$$

Once the versor n of the rotation axis and the rotation angle θ have been found, a rotation vector w can be defined according to

$$w = n \tan\left(\frac{\theta}{2}\right) \tag{4}$$

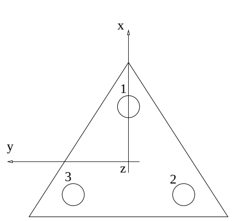

Figure 4: the coordinate frame.

The expression relating ω, the angular velocity, to w and $\dot{w}$ is

$$\omega = \frac{2}{1+\|w\|^2}\left(\dot{w} + w \times \dot{w}\right) \tag{5}$$

The obtained angle and versor for the considered system are $\theta(\alpha,\varphi) = \alpha$, and $n(\alpha,\varphi) = [\cos\varphi \ \sin\varphi \ 0]^T$. Therefore, according to Eq. (4) and Eq. (5), the following expressions for ω are obtained

$$\omega(\alpha,\varphi,\dot{\alpha},\dot{\varphi}) = \begin{bmatrix} \cos\varphi & -\sin\alpha\sin\varphi \\ \sin\varphi & \sin\alpha\cos\varphi \\ 0 & 1-\cos\alpha \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\varphi} \end{bmatrix} \tag{6}$$

$$\omega(\alpha,\varphi,\dot{\alpha},\dot{\varphi}) = J(\alpha,\varphi) \begin{bmatrix} \dot{\alpha} \\ \dot{\varphi} \end{bmatrix} \tag{7}$$

Since the finger can be regarded as a stick with mass m and length l, the inertia matrix has been chosen of the form

$$A = \mathrm{diag}\{I_i\}, \quad i = 1,2,3 \tag{8}$$

where

$$I_1 = \frac{1}{3}ml^2, \ I_2 = \frac{1}{3}ml^2, \ I_3 = 0 \tag{9}$$

Considering Eq. (1) and substituting the previously derived expressions, the dynamic equation for the finger has the form

$$J^T(\alpha,\varphi)AJ(\alpha,\varphi)\begin{bmatrix} \ddot{\alpha} \\ \ddot{\varphi} \end{bmatrix} = \Omega(\alpha,\varphi,\dot{\alpha},\dot{\varphi}) + J^T(\alpha,\varphi)\,\tau \tag{10}$$

where

$$\begin{aligned}\Omega(\alpha,\varphi,\dot{\alpha},\dot{\varphi}) &= -J^T(\alpha,\varphi)A\dot{J}(\alpha,\varphi,\dot{\alpha},\dot{\varphi})\begin{bmatrix} \dot{\alpha} \\ \dot{\varphi} \end{bmatrix} \\ &-J^T(\alpha,\varphi)\left[\omega(\alpha,\varphi,\dot{\alpha},\dot{\varphi}) \times A\,\omega(\alpha,\varphi,\dot{\alpha},\dot{\varphi})\right]\end{aligned} \tag{11}$$

3 Gripper Control

The design of the control system can be performed by considering a single finger, since the control of the entire manipulator in a free motion can be interpreted as three separate control problems. If the control bellows are properly actuated the end plate (the tip) is subject to three mono–directional actions and, as a consequence of the Caratheodory Theorem, the maximum number of degree of freedom which can be actuated is 2. In such a situation the bellows have the twofold role of transmitting the control torques to the tip plate and that of providing the structure with sufficient stiffness to the shear effects caused by the bending torque, allowing the computation of the actual finger position by means of the displacements of the three linear motors which can be accomplished by LVDT sensors. The complex modeling procedure together with the degree of uncertainty relevant in the determination of the inertia matrix and the nonlinear elasticity phenomena prevent the use of control methodology such as feedback linearization or adaptive control.

3.1 The sliding mode control approach

As already stated in Section 2, in the considered system the vector τ of the torques which can be applied to the finger results to belong to the (x, y)–plane of the reference frame (Fig. 4) where these external torques form a simplex of vectors. Therefore one has that

$$\tau = [\tau_x, \tau_y, 0]^T \tag{12}$$

and that due to the structure of the system

$$\begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} d \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{13}$$

The two state components α and φ are directly measurable. In order to solve the control problem according to sliding mode techniques, the first step to do is defining the sliding quantities

$$\begin{array}{l} s_1 = [\dot{\alpha}(t) - \dot{\alpha}^*(t)] + c_1[\alpha(t) - \alpha^*(t)] \\ s_2 = [\dot{\varphi}(t) - \dot{\varphi}^*(t)] + c_2[\varphi(t) - \varphi^*(t)] \end{array} \tag{14}$$

where $\alpha^*(t)$, $\varphi^*(t)$, $\dot{\alpha}^*(t)$, $\dot{\varphi}^*(t)$ are the reference signals to be tracked and their time derivatives. On $s_1 = 0$, $s_2 = 0$, the reduced system (the zero–dynamic) is arbitrarily exponentially stable. Since $\dot{\alpha}(t)$ and $\dot{\varphi}(t)$ are not available, the first problem consists in estimating these quantities starting from the available signals $\alpha(t)$, $\varphi(t)$.

This problem has been solved by means of second order sliding mode control technique[3][4]. Applied to the estimation/differentiation problem in question, this approach consists in building a simple observer

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = v \end{cases} \tag{15}$$

then, given $\alpha(t)$ the signal to be differentiated, define the observation error

$$\begin{cases} e_1 = z_1 - \alpha \\ \dot{e}_1 = z_2 - \dot{\alpha} \\ \dot{e}_2 = v - \ddot{\alpha} \end{cases} \tag{16}$$

assuming that the modulus of $\ddot{\alpha}(t)$ which depends on the dynamic equations, is bounded by a known constant $H > 0$, a control signal $v(t)$ designed according to the following second order sliding mode algorithm, is able to steer to zero both $e_1(t)$ and $e_2(t)$ in a finite time

$$\begin{array}{l} v(t) = -V(t) \ \text{sign}\left[e_1(t) - \frac{1}{2}e_{1_{max}}\right] \\ V(t) = \begin{cases} V_M & \text{if} \quad e_{1_{max}}\left[e_1(t) - \frac{1}{2}e_{1_{max}}\right] > 0 \\ \alpha V_M & \text{if} \quad e_{1_{max}}\left[e_1(t) - \frac{1}{2}e_{1_{max}}\right] < 0 \end{cases} \end{array} \tag{17}$$

where $V_M > H$, $\alpha > 1$ and $e_{1_{max}}$ is chosen according to the following

- When $t = 0$, set $e_{1_{max}} = e_1(0)$, $i = 0$, $t_{M_i} = 0$
- While $t \in [0, \infty)$, if $e_2(t) = 0$ then set $e_{1_{max}} = e_1(t)$, $i = i + 1$, and $t_{M_i} = t$;

after a short transient the signal $z_2(t)$ is equal to $\dot{\alpha}(t)$ and it is available to the control system. The proposed procedure is applied to estimate both $\dot{\alpha}(t)$ and $\dot{\varphi}(t)$.

The structure of the dynamic equation of the system is given in Eq. (10), and, in the working sub–space, the Jacobian matrix $J(\alpha, \varphi)$ is assumed to be known and the sub matrix $J_*(\alpha, \varphi) = \{J_{ij}(\alpha, \varphi)\}$ $i, j = 1, 2$ is assumed to be full–rank. It is apparent that the control matrix $\left[J^T(\alpha, \varphi) A J(\alpha, \varphi)\right]^{-1} J^T(\alpha, \varphi)$ is not symmetrical, and it cannot be ensured that its symmetrical part is positive definite. A possible sliding mode strategy could be: consider the transformed control vector

$$u(t) = J^T(\alpha, \varphi)\tau \tag{18}$$

and choose a Lyapunov function

$$V = \frac{1}{2} s^T \left[J^T(\alpha, \varphi) A J(\alpha, \varphi)\right] s \tag{19}$$

the first time derivative is

$$\dot{V} = s^T \left[\psi(\alpha, \varphi, \dot{\alpha}, \dot{\varphi}) + u(t)\right] \tag{20}$$

then the components u_i, $i = 1, 2$ of the transformed control vector can be chosen component–wise discontinuous on $s_i = 0$

$$u_i(t) = -k_i(\alpha, \varphi, \dot{\alpha}, \dot{\varphi}) \ \text{sign}[s_i(t)] \quad i = 1, 2 \tag{21}$$

where $k_i(\alpha, \varphi, \dot{\alpha}, \dot{\varphi})$ are positive functions upper-bounding the modulus of $\psi_i(\alpha, \varphi, \dot{\alpha}, \dot{\varphi})$. The control objective is attained in a finite time[5]. According to the previously sketched control procedure the desired control signals $u_i(t)$ can be calculated by solving the following

$$\begin{bmatrix} -k_1(\alpha, \varphi, \dot{\alpha}, \dot{\varphi})\text{sign}[s_1(t)] \\ -k_2(\alpha, \varphi, \dot{\alpha}, \dot{\varphi})\text{sign}[s_2(t)] \end{bmatrix} = J^T(\alpha, \varphi)\tau \tag{22}$$

this means that the required control forces F_i which must be applied to the system, are discontinuous on both the sliding manifolds $s_1 = 0$, $s_2 = 0$, their amplitudes depend on α and φ even if the modulus of the drift terms $\psi_i(\alpha, \varphi, \dot{\alpha}, \dot{\varphi})$ can be upper–bounded by constant k_i, moreover the fact that only positive forces F_i can be applied is not taken into account.

3.2 Simplex–based controller

An alternative approach to traditional multi–input VSC techniques[6] can be the use of a set of constant control vectors, giving raise to a simplex of vectors[7]. This method appears to be consistent with the nature of the adopted actuation system, the control system is based on the simplex method and exploits only mono–directional control actions. On the basis of the assumptions that the matrix $J_*(\alpha, \varphi)$ is known, transform the sliding output s in a new sliding output

$$s_* = J_*(\alpha, \varphi)s \tag{23}$$

Consider again the Lyapunov function of Eq. (19), its first time derivative is

$$\begin{aligned} \dot{V} &= s^T J_*^T \left[J_*^{-T} \psi(\alpha, \varphi, \dot{\alpha}, \dot{\varphi}) + \tau_* \right] \\ &= {s_*}^T \left[\xi(\alpha, \varphi, \dot{\alpha}, \dot{\varphi}) + \tau_* \right] \end{aligned} \tag{24}$$

where $\tau_* = [\tau_x, \tau_y]^T$.

Choose three constant vectors

$$\tau_1 = \begin{bmatrix} \tau_{1x} \\ \tau_{1y} \end{bmatrix} \quad \tau_2 = \begin{bmatrix} \tau_{2x} \\ \tau_{2y} \end{bmatrix} \quad \tau_3 = \begin{bmatrix} \tau_{3x} \\ \tau_{3y} \end{bmatrix} \tag{25}$$

which form a simplex in $\Re^2$, containing the origin, that is

$$\sum_{i=1}^{3} \mu_i \tau_i = 0 \quad \text{and} \quad \sum_{i=1}^{3} \mu_i = 1 \quad \mu_i > 0 \tag{26}$$

They are realized so that $\forall i = 1, 2, 3$, $||\tau_i|| > ||\xi|| + k^2$, being k is an arbitrary constant, and the "obtuse angle" condition holds

$${\tau_i}^T \tau_j < -\varepsilon ||\tau_i|| \; ||\tau_j|| \quad \forall i, j = 1, 2, 3 \tag{27}$$

The following control switching logic guarantees the satisfaction of Lyapunov theorem and the achievement of the control goals.

First the plane s_* is divided in three non–overlapping regions Ω_i

$$\begin{array}{lll} s_* \in \Omega_1 & \text{if} \quad s_* = \lambda_2 \tau_2 + \lambda_3 \tau_3 & \lambda_2, \lambda_3 \geq 0 \\ s_* \in \Omega_2 & \text{if} \quad s_* = \lambda_1 \tau_1 + \lambda_3 \tau_3 & \lambda_1, \lambda_3 \geq 0 \\ s_* \in \Omega_3 & \text{if} \quad s_* = \lambda_1 \tau_1 + \lambda_2 \tau_2 & \lambda_1, \lambda_2 \geq 0 \end{array} \tag{28}$$

then the control vector is chosen according to the following switching logic

$$\text{if} \quad s_* \in \Omega_i \quad \text{set} \quad \tau_* = \tau_i \tag{29}$$

that is, if $s_*(t)$ belongs to a region Ω_i positively spanned by the two vectors τ_j and τ_k, with $j, k \neq i$, then τ_i is chosen as control action. The considered system has a symmetric structure and the forces exerted by each of the three control bellows have the same magnitude, thus the constant vectors of the simplex are chosen to be

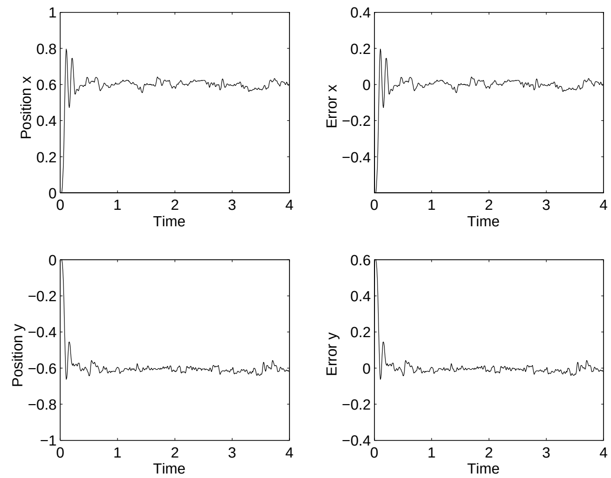

Figure 6: the step response without filtering.

$$\begin{array}{lll} \tau_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} dF & \text{if} \quad F_1 = F, & F_2 = F_3 = 0 \\ \tau_2 = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} dF & \text{if} \quad F_2 = F, & F_1 = F_3 = 0 \\ \tau_3 = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix} dF & \text{if} \quad F_3 = F, & F_1 = F_2 = 0 \end{array} \tag{30}$$

it follows that it is sufficient to activate just one voice coil motor at a time on the basis of the knowledge of the region to which $s_*(t)$ belongs.

4 Experimental Results

The proposed control strategy was applied to the gripper and several experiments were performed to demonstrate the effectiveness of the simplex–based sliding mode control technique. The experimental activity showed that a signal filtering was necessary, due to presence of electrical disturbances on some signals which affected the performances of the manipulator. In particular the sensor system has revealed to be quite sensitive to such electrical bias due to its magnetic nature. Diagrams in Fig. 6 show the step response of one finger of the gripper without signal filtering, while in Fig. 7 it is shown the result obtained by using a suited filter.

5 Conclusions

The research activity reported in this paper has been devoted to the solution of a specific control problems for a robotic device in which monodirectional control

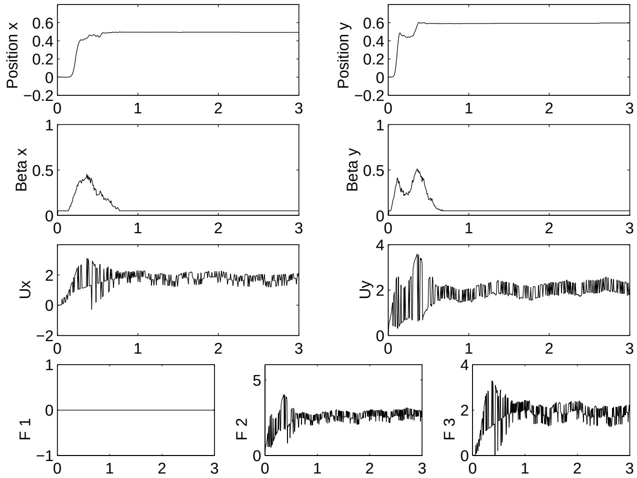

Figure 7: the step response with the filter.

actions are present. For such systems, a very promising approach is represented by a sliding mode control methodology based on the use of a simplex of constant control vectors. Due to its constructive features and to the high bandwidth of the chosen actuators, the presented prototype has allowed the implementation of almost–ideal sliding regimes, provided that the sampling frequency allowed by the digital implementation of the control algorithm is of the same order of the motors bandwidth. In order to fulfill the requirements of the proposed simplex methodology, a set of digital second order sliding mode observer/differentiator has been included in the control scheme. Extensive experimentations and simulation activity have confirmed the validity of the proposed approach.

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