Robust Finite Horizon MPC without Terminal Constraints

Camile and Jan M. Maciejowski

Control Group, Department of Engineering, University of Cambridge
Trumpington Street, Cambridge CB2 1PZ, United Kingdom
Tel: +44-1223-339222, Fax: +44-1223-332662
http://www-control.eng.cam.ac.uk
car24,jmm@eng.cam.ac.uk

Abstract

In this paper a procedure for obtaining the parameters of a finite horizon model predictive controller to make it equivalent to an H_∞ normalised left coprime factorisation (NLCF) controller in the unconstrained case will be considered. The procedure will be based on a Linear Matrix Inequalities (LMI) approach to the solution of the discrete-time H_∞ control problem and will be solved by first considering the solution when the state is available for measurement.

Keywords: H_∞, NLCF, LMI, MPC.

1 Introduction

One of the major drawbacks of the finite horizon MPC formulation is that it does not give guaranteed stability even in the unconstrained cased. However, it can be shown that if a terminal constraint is placed on the state, then stability may be guaranteed if certain conditions are imposed on the terminal weight [7]. However, in this paper the MPC formulation without terminal weight will be considered and it will be shown that the parameters of the unconstrained controller may be selected in such a way that the MPC controller is equivalent to a Normalised Left Coprime Factorisation (NLCF) H_∞ controller which, when used with the loopshaping procedure proposed in [3], gives rise to a controller with very nice robust stability/performance guarantees.

A procedure for obtaining an MPC equivalent NLCF for the infinite horizon case was already outlined in [10] which, pointed out that the major advantage of implementing the NLCF controller as a predictive controller is that, a predictive controller with integral action, implemented with only input constraints, provides automatic anti-windup. Thus, implementing the NLCF controller as a predictive controller removes the need for the design of a separate anti-windup strategy. The formulation proposed in [10] has the disadvantage that the horizon length may be too long and this might lead to numerical problems during the MPC optimisation procedure. The finite horizon formulation proposed here thus gives an improvement in this regard as one can select smaller horizon lengths. The tuning procedure outlined is for the unconstrained controller.

The tuning procedure is based on an LMI approach to the solution of the discrete-time H_∞ control problem and the solution will be obtained by first considering the case when the states are available for measurement.

2 Problem Definition

Consider the discrete-time plant given by the following equation

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the state, input and output vectors respectively. (A, B) is stabilisable and (A, C) is detectable. It is assumed that this plant has already been augmented with the disturbance and reference model.

Now consider the MPC controller whose cost function is:

$$J(x,u) = \sum_{i=1}^{N_2} \|\hat{y}(k+i|k)\|^2_{Q(i)} + \sum_{i=0}^{N_u-1} \|\Delta u(k+i|k)\|^2_{R(i)} \tag{2}$$

where N_2 is the prediction horizon, N_u, the control horizon and $Q(i)$ and $R(i)$ are the weighting matrices.

The MPC problem is then to find a sequence of control inputs that minimises the cost function, that is,

$$\min_{\Delta\mathcal{U}} J(x,u) \tag{3}$$

where $\Delta\mathcal{U} = \Delta u(k|k), \Delta u(k+1|k), \cdots, \Delta u(k+N_u-1)$.

The predictions, $\hat{y}(k+i|k)$ $i = 1, 2, \cdots, N_2$ are obtained from the prediction model given by equation (1) and the estimate of the state at time k. The state estimate $\hat{x}(k|k-1)$ is obtained from a stable observer with observer gain L and then updated using the current measurement of the output at time k to obtain $\hat{x}(k|k)$, which is computed according to the corrector equation given below.

$$\hat{x}(k+1|k) = (A - LC)\,\hat{x}(k|k-1) + Bu(k) + Ly(k) \text{ Observer} \tag{4}$$

$$\hat{x}(k|k) = (I - LC)\,\hat{x}(k|k-1) + Ly(k) \text{ Corrector} \tag{5}$$

This corrector equation may only be used when there is adequate computation time otherwise the state estimates used would be $\hat{x}(k|k-1)$. Thus this leads to two form of controllers. One a strictly proper controller called the predicting form in which the predictions are based on $\hat{x}(k|k-1)$ and the other a non-strictly proper controller called the filtering form. For the current controller the output predictions are based on the estimate $\hat{x}(k|k)$ and this leads to a controller with improved performance.

We seek to obtain the parameters, N_2, N_u, $Q(i)$ $(i = 1, 2, \cdots, N_2)$, $R(i)$ $(i = 0, 1, 2, \cdots, N_u)$ and the observer gain L such that finite horizon unconstrained MPC controller will be a NLCF controller [5], that is such that

$$\left\| \begin{bmatrix} S\tilde{M}^{-1} \\ KS\tilde{M}^{-1} \end{bmatrix} \right\|_\infty < \gamma \quad S := (I - GK)^{-1} \tag{6}$$

where $\tilde{M}^{-1}$ is the normalised left coprime factor of the plant $G(z) = C(zI - A)^{-1}B$ and where $K(z)$ is the transfer function of the MPC controller. We consider the solution for the predicting form of the controller.

3 The Predicting MPC Controller

Before outlining the proposed tuning procedure, the controller equations for the predictive controller with cost function given by equation (2) and prediction model given by (1) will be derived.

The cost function in equation (2) may be rewritten in the following form,

$$J(x, u) = \|\mathcal{Y}(k)\|^2_{\mathcal{Q}} + \|\Delta\mathcal{U}(k)\|^2_{\mathcal{R}} \tag{7}$$

where the matrices are defined as follows.

$$\mathcal{Y}(k) = \begin{bmatrix} \hat{y}(k+1|k) \\ \vdots \\ \hat{y}(k+N_2|k) \end{bmatrix} \qquad \Delta\mathcal{U}(k) = \begin{bmatrix} \Delta u(k|k) \\ \vdots \\ \Delta u(k+N_u-1|k) \end{bmatrix} \tag{8}$$

and

$$\mathcal{Q} = \begin{bmatrix} Q(1) & 0 & \cdots & 0 \\ 0 & Q(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q(N_2) \end{bmatrix} \tag{9}$$

$$\mathcal{R} = \begin{bmatrix} R(0) & 0 & \cdots & 0 \\ 0 & R(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R(N_u-1) \end{bmatrix} \tag{10}$$

It can be shown that $\mathcal{Y}(k)$ may be given by the following equation

$$\mathcal{Y}(k) = \Psi\hat{x}(k|k-1) + \Upsilon u(k-1) + \Theta\Delta\mathcal{U}(k) \tag{11}$$

where the matrices Ψ, Υ and Θ are defined as follows

$$\Psi = \begin{bmatrix} CA \\ \vdots \\ CA^{N_u} \\ CA^{N_u+1} \\ \vdots \\ CA^{N_2} \end{bmatrix}, \quad \Upsilon = \begin{bmatrix} CB \\ \vdots \\ \sum_{i=0}^{N_u-1} CA^iB \\ \sum_{i=0}^{N_u} CA^iB \\ \vdots \\ \sum_{i=0}^{N_2-1} CA^iB \end{bmatrix} \tag{12}$$

$$\Theta = \overline{C} \begin{bmatrix} B & \cdots & 0 \\ AB+B & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{N_u-1} A^iB & \cdots & B \\ \sum_{i=0}^{N_u} A^iB & \cdots & AB+B \\ \vdots & \vdots & \vdots \\ \sum_{i=0}^{N_2-1} A^iB & \cdots & \sum_{i=0}^{N_2-N_u} A^iB \end{bmatrix} \tag{13}$$

where

$$\overline{C} = \begin{bmatrix} C & 0 & \cdots & 0 \\ 0 & C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C \end{bmatrix} \tag{14}$$

The error between the future target response of the system and the free response (that is, when $\Delta\mathcal{U} = 0$) is given by $\mathcal{E}(k)$ and is defined in the following equation.

$$\mathcal{E}(k) = -\Psi\hat{x}(k) - \Upsilon u(k-1) \tag{15}$$

In (15) the target is set to zero because of the assumption that the plant included a model of the reference.

Thus the cost function may now be reformulated as:

$$\begin{aligned} J(x,u) =& \mathcal{E}(k)^T\mathcal{Q}\mathcal{E}(k) - 2\Delta\mathcal{U}(k)^T\Theta^T\mathcal{Q}\mathcal{E}(k) \\ &+ \Delta\mathcal{U}(k)^T\left[\Theta^T\mathcal{Q}\Theta + \mathcal{R}\right]\Delta\mathcal{U}(k)] \end{aligned} \tag{16}$$

In the unconstrained case, the future set of control moves is given by,

$$\Delta\mathcal{U}(k)_{opt} = \left(\Theta^T\mathcal{Q}\Theta + \mathcal{R}\right)^{-1}\Theta^T\mathcal{Q}\mathcal{E}(k) \tag{17}$$

However, in the receding horizon strategy only the solution corresponding to the first is used. Thus, the calculated control input for the unconstrained case is given by:

$$\Delta u(k)_{opt} = \begin{bmatrix} I_m, & 0_m, & \cdots, & 0_m \end{bmatrix} \Delta\mathcal{U}(k)_{opt} \tag{18}$$

$$\Delta u(k)_{opt} = K_m(\mathcal{Q}, \mathcal{R}, N_2, N_u)\mathcal{E}(k)_{opt} \tag{19}$$

where K_m is a gain which is dependent on $\mathcal{Q}$, $\mathcal{R}$, N_2 and N_u. In future we will just write K_m for short but it should be borne in mind that this is a parameter dependent gain.

It is simple to show that the controller equations for this form of the unconstrained MPC controller [8] is given by,

$$\begin{aligned}\begin{bmatrix}\hat{x}(k+1|k)\\ u(k)\end{bmatrix} &= \begin{bmatrix}A-LC-BK_m\Psi & B(I-K_m\Upsilon)\\ -K_m\Psi & I-K_m\Upsilon\end{bmatrix}\\ &\quad\times\begin{bmatrix}\hat{x}(k|k-1)\\ u(k-1)\end{bmatrix}+\begin{bmatrix}L\\0\end{bmatrix}y(k)\\ \Delta u(k) &= \begin{bmatrix}-K_m\Psi & -K_m\Upsilon\end{bmatrix}\begin{bmatrix}\hat{x}(k|k-1)\\ u(k-1)\end{bmatrix}\\ &:= \left[\begin{array}{c|c}A_K & B_K\\ \hline C_K & 0\end{array}\right]\end{aligned} \tag{20}$$

It can be observed that this is a strictly proper control structure.

4 Obtaining K_m and L

The tuning procedure for the predicting form of the MPC controller will be considered in this section.

The plant to be considered is described in (1). This plant is augmented with the input vector as shown below,

$$\begin{aligned}\begin{bmatrix}x(k+1)\\ u(k)\end{bmatrix} &= \begin{bmatrix}A & B\\ 0 & I\end{bmatrix}\begin{bmatrix}x(k)\\ u(k-1)\end{bmatrix}+\begin{bmatrix}B\\I\end{bmatrix}\Delta u(k)\\ y(k) &= \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}x(k)\\ u(k-1)\end{bmatrix}\end{aligned} \tag{21}$$

The generalised plant for the NLCF control problem is then given by [11]:

$$\begin{aligned}P &:= \left[\begin{array}{c|cc}\begin{bmatrix}A & B\\ 0 & I\end{bmatrix} & \begin{bmatrix}B_1\\0\end{bmatrix} & \begin{bmatrix}B_2\\I\end{bmatrix}\\ \hline \begin{bmatrix}C_1 & D_{12}\end{bmatrix} & D_{11} & D_{12}\\ \begin{bmatrix}C_2 & 0\end{bmatrix} & D_{21} & 0\end{array}\right]\\ &= \left[\begin{array}{c|cc}\tilde{A} & \tilde{B}_1 & \tilde{B}_2\\ \hline \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12}\\ \tilde{C}_2 & \tilde{D}_{21} & 0\end{array}\right]\end{aligned} \tag{22}$$

where

$$\begin{aligned}&B_1=-H_{KF};\quad B_2=B;\quad C_1=\begin{bmatrix}C^T & 0\end{bmatrix}^T\\ &C_2=C;\quad D_{11}=\begin{bmatrix}I\\0\end{bmatrix};\quad D_{12}=\begin{bmatrix}0\\I\end{bmatrix};\quad D_{21}=I\end{aligned} \tag{23}$$

and

$$H_{KF}=-AZC^T(I+CZC^T)^{-1} \tag{24}$$

is the observer gain associated with the Kalman Filter with the following Riccati equation,

$$Z=AZA^T-AZC^T(I+CZC^T)^{-1}CZA^T+BB^T \tag{25}$$

The tuning procedure will now be outlined by first considering the LMI solution to the state feedback problem.

4.1 The State Feedback Case

In the case that the states are available for measurement, the MPC predicting controller is simply given by

$$\Delta u(k)=\begin{bmatrix}-K_m\Psi & -K_m\Upsilon\end{bmatrix}\begin{bmatrix}x(k-1)\\ u(k-1)\end{bmatrix} \tag{26}$$

The generalised plant for this problem is

$$\begin{aligned}\tilde{x}(k+1) &= \tilde{A}\tilde{x}(k)+\tilde{B}_1w(k)+\tilde{B}_2\Delta u(k)\\ y(k) &= \tilde{x}(k)\\ z(k) &= \tilde{C}_1\tilde{x}(k)+\tilde{D}_{11}w(k)+\tilde{D}_{12}\Delta u(k)\end{aligned} \tag{27}$$

where

$$\tilde{x}(k)=\begin{bmatrix}x(k-1)^T & u(k-1)^T\end{bmatrix}^T \tag{28}$$

The following lemma states the condition under which a value of K_m may be determined to ensure that the gain from $w(k)$ to $z(k)$ does not exceed some pre-specified level γ.

Lemma 4.1 *Consider the system (27), given some desired level of performance $\gamma > \gamma_{min} > 0$, where γ_{min} is defined in [11]. If there exists a matrix $P = P^T > 0$ and Y that satisfy*

$$\begin{pmatrix}-P & \tilde{A}P+\tilde{B}_2Y & \tilde{B}_1 & 0\\ P\tilde{A}^T+Y^T\tilde{B}_2 & -P & 0 & P\tilde{C}_1^T+Y^T\tilde{D}_{12}^T\\ \tilde{B}_1^T & 0 & -\gamma I & \tilde{D}_{11}^T\\ 0 & \tilde{C}_1P+\tilde{D}_{12}Y & \tilde{D}_{11} & -\gamma I\end{pmatrix}<0 \tag{29}$$

and if

$$\begin{bmatrix}\Psi & \Upsilon\end{bmatrix} \tag{30}$$

has full row rank for all values of N_2 and N_u, then a K_m may be obtained to satisfy

$$K_m\begin{bmatrix}-\Psi & -\Upsilon\end{bmatrix}=YP^{-1} \tag{31}$$

if N_2-N_u+1 is selected to be greater than $n+m$. A K_m which satisfies (31) will lead to a closed loop system with L_2 gain from w to z of at most γ.

Proof: The condition given in (29) is a necessary and sufficient condition for the existence of a state feedback controller which guarantees quadratic stability with L_2 gain from w to z of at most γ [4]. The state feedback in this case is given by (31) and this equation has a solution for K_m if $[\Psi,\Upsilon]$ whose dimension is $(N_2-N_u+1)\times(n+m)$ has full column rank. Thus, $N_2-N_u+1\geq n+m$ is a necessary condition. This condition along with the condition that $[\Psi\ \Upsilon]$ has maximum row rank will ensure that (31) has a solution. ∎

4.2 The Full Solution

Remark 4.2 *In this section, it is assumed that a full state feedback controller has already been successfully designed so that a P and a Y has already been determined to satisfy (29)*

The MPC controller for the case when the states are not available for measurement is given by (20).

We assume that K_m was already determined as in Lemma 4.1 and that an observer gain L is to be determined so that the MPC controller guarantees an L_2 gain from w to z of γ. The following theorem shows how L may be determined.

Theorem 4.3 *For some desired level of performance $\gamma > \gamma_{min} > 0$ where γ_{min} is defined in [5], assume that there exist $P = P^T > 0$ and Y such that the LMI in Lemma 4.1 is satisfied. If there exists constant matrices $R = R^T > 0$ and $W = [\hat{W}^T\ 0]^T$ such that (32) is satisfied, then the MPC controller with observer gain $L = -R^{-1}\hat{W}$ guarantees L_2 gain from w to z of at most γ.*

$$\begin{pmatrix} -R & R\tilde{A}+W\tilde{C}_2 & 0 & R\tilde{B}_1+W\tilde{D}_{21} & 0 \\ \tilde{A}^T R+\tilde{C}_2^T W^T & -R & 0 & 0 & \tilde{C}_1^T \\ 0 & 0 & -P & 0 & P\tilde{C}_1^T+Y^T\tilde{D}_{12}^T \\ \tilde{B}_1^T R+\tilde{D}_{21}^T W^T & 0 & 0 & -\gamma I & \tilde{D}_{11}^T \\ 0 & \tilde{C}_1 & \tilde{C}_1 P+\tilde{D}_{12}Y & \tilde{D}_{11} & -\gamma I \end{pmatrix} < 0 \tag{32}$$

Proof: From the Bounded Real Lemma [4], the L_2 gain from w to z of at most $\gamma > \gamma_{min} > 0$, if there exists $X_{cl} = X_{cl}^T > 0$, such that,

$$\begin{pmatrix} -X_{cl}^{-1} & A_{cl} & B_{cl} & 0 \\ A_{cl}^T & -X_{cl} & 0 & C_{cl}^T \\ B_{cl}^T & 0 & -\gamma I & D_{cl}^T \\ 0 & C_{cl} & D_{cl} & -\gamma I \end{pmatrix} < 0 \tag{33}$$

Now if the states of the closed loop system is chosen to be $\begin{bmatrix} e(k)^T & \hat{\tilde{x}}(k)^T \end{bmatrix}$ where $e(k) = \tilde{x}(k) - \hat{\tilde{x}}(k)$ and $\hat{\tilde{x}}(k) = \begin{bmatrix} \hat{x}(k|k-1)^T & u(k-1) \end{bmatrix}$.

The closed loop matrices are then given by,

$$\begin{aligned} A_{cl} &= \begin{pmatrix} \tilde{A} - \hat{L}\tilde{C}_2 & 0 \\ \hat{L}\tilde{C}_2 & \tilde{A} + \tilde{B}_2 K_m Z \end{pmatrix} \\ B_{cl} &= \begin{pmatrix} \tilde{B}_1 - \hat{L}\tilde{D}_{21} \\ \hat{L}\tilde{D}_{21} \end{pmatrix} \\ C_{cl} &= \begin{pmatrix} \tilde{C}_1 & \tilde{C}_1 + \tilde{B}_2 K_m Z \end{pmatrix} \\ D_{cl} &= \tilde{D}_{11} \end{aligned} \tag{34}$$

where

$$\hat{L} = \begin{bmatrix} L \\ 0 \end{bmatrix}, \qquad Z = \begin{bmatrix} -\Psi & -\Upsilon \end{bmatrix} \tag{35}$$

Let X_{cl} be given by,

$$X_{cl} = \begin{pmatrix} R & 0 \\ 0 & P^{-1} \end{pmatrix} \tag{36}$$

Substituing these matrices in (33) gives

$$\begin{pmatrix} -R^{-1} & 0 & \tilde{A}-\hat{L}\tilde{C}_2 & 0 & \tilde{B}_1-\hat{L}\tilde{D}_{21} & 0 \\ 0 & -P & \hat{L}\tilde{C}_2 & \tilde{A}+\tilde{B}_2 K_m Z & \hat{L}\tilde{D}_{21} & 0 \\ \tilde{A}^T-\tilde{C}_2\hat{L}^T & \tilde{C}_2\hat{L}^T & -R & 0 & 0 & \tilde{C}_1^T \\ 0 & \tilde{A}^T+Z^T K_m^T \tilde{B}_2^T & 0 & -P^{-1} & 0 & \tilde{C}_1^T+Z^T K_m^T \tilde{D}_{12}^T \\ \tilde{B}_1^T-\tilde{D}_{21}^T\hat{L}^T & \tilde{D}_{21}^T\hat{L}^T & 0 & 0 & -\gamma I & \tilde{D}_{11}^T \\ 0 & 0 & \tilde{C}_1 & \tilde{C}_1+\tilde{D}_{12}K_m Z & \tilde{D}_{11} & -\gamma I \end{pmatrix} < 0 \tag{37}$$

After pre and post multiplying (37) by

$$\begin{pmatrix} I & & & & & \\ & I & & & & \\ & & I & & & \\ & & & P & & \\ & & & & I & \\ & & & & & I \end{pmatrix} \tag{38}$$

(37) becomes

$$\begin{pmatrix} -R^{-1} & 0 & \tilde{A}-\hat{L}\tilde{C}_2 & 0 & \tilde{B}_1-\hat{L}\tilde{D}_{21} & 0 \\ 0 & -P & \hat{L}\tilde{C}_2 & \tilde{A}P+\tilde{B}_2 Y & \hat{L}\tilde{D}_{21} & 0 \\ \tilde{A}^T+\tilde{C}_2\hat{L}^T & \tilde{C}_2\hat{L}^T & -R & 0 & 0 & \tilde{C}_1^T \\ 0 & P\tilde{A}^T+Y^T\tilde{B}_2^T & 0 & -P & 0 & P\tilde{C}_1^T+Y^T\tilde{D}_{12}^T \\ \tilde{B}_1^T-\tilde{D}_{21}^T\hat{L}^T & \tilde{D}_{21}^T\hat{L}^T & 0 & 0 & -\gamma I & \tilde{D}_{11}^T \\ 0 & 0 & \tilde{C}_1 & \tilde{C}_1 P+\tilde{D}_{12}Y & \tilde{D}_{11} & -\gamma I \end{pmatrix} < 0 \tag{39}$$

where

$$K_m Z = YP^{-1} \tag{40}$$

Using the Matrix Projection Lemma [1], (39) is equivalent to,

$$\begin{pmatrix} -R^{-1} & \tilde{A}-\hat{L}\tilde{C}_2 & 0 & \tilde{B}_1-\hat{L}\tilde{D}_{21} & 0 \\ \tilde{A}^T+\tilde{C}_2\hat{L}^T & -R & 0 & 0 & \tilde{C}_1^T \\ 0 & 0 & -P & 0 & P\tilde{C}_1^T+Y^T\tilde{D}_{12}^T \\ \tilde{B}_1^T-\tilde{D}_{21}^T\hat{L}^T & 0 & 0 & -\gamma I & \tilde{D}_{11}^T \\ 0 & \tilde{C}_1 & \tilde{C}_1 P+\tilde{D}_{12}Y & \tilde{D}_{11} & -\gamma I \end{pmatrix} < 0 \tag{41}$$

After pre and post multiplying (41) by

$$\begin{pmatrix} R & & & & \\ & I & & & \\ & & I & & \\ & & & I & \\ & & & & I \end{pmatrix} \tag{42}$$

then equation (32) is obtained after setting

$$\hat{L} = -R^{-1} \begin{bmatrix} \hat{W} \\ 0 \end{bmatrix} \tag{43}$$

■

5 Determining $\mathcal{Q}$ and $\mathcal{R}$

Once K_m is determined it should now be possible to obtain $\mathcal{Q}$ and $\mathcal{R}$. However, it should be noted that since K_m is a truncated version of $(\Theta^T\mathcal{Q}\Theta+\mathcal{R})^{-1}\Theta^T\mathcal{Q}$ there is not enough data in K_m to calculate $\mathcal{Q}$ and $\mathcal{R}$ explicitly but a optimisation problem of the following form

$$\min_{\mathcal{Q},\,\mathcal{R}} \left\| K_m - \begin{bmatrix} I_m & 0_m & \cdots & 0_m \end{bmatrix} (\Theta^T\mathcal{Q}\Theta+\mathcal{R})^{-1}\Theta^T\mathcal{Q} \right\|_2 \tag{44}$$

could be set up to determine suitable values for these matrices. The problem may be simplified by assuming that $\mathcal{Q}$ and $\mathcal{R}$ are diagonal matrices.

In the literature on MPC [8] it as always been assumed that $\mathcal{Q}$ and $\mathcal{R}$ are positive definite matrices, this is to ensure that the Hessian given by

$$2(\Theta^T \mathcal{Q} \Theta + \mathcal{R}) \tag{45}$$

is positive definite, however other choices of these matrices can lead to a positive definite Hessian and therefore in the optimisation given in (44) the only restriction which will be placed on $\mathcal{Q}$ and $\mathcal{R}$ is that they lead to a positive definite Hessian. The intrepretation of the case when either $\mathcal{Q}$ or $\mathcal{R}$ is non-positive definite is still being investigated.

6 Summary of Tuning Procedure

The tuning procedure is now summarised below:

1. Augment the plant with the disturbance and reference model to get a representation of the form (1)

2. Perform H_∞ loopshaping to get good stability/performance properties

3. Select $N_2 - N_u + 1 \geq n + m$ and $N_u \leq N_2$ and form the MPC matrices given by (12)-(13).

4. For the specified performance $\gamma > \gamma_{min}$, determine Y and P to satisfy (29) in Lemma 4.1 and determine K_m according to (31).

5. Determine R and W to satisy Theorem 4.3 and determine the value of L.

6. Perform the optimisation given by equation (44) to determine $\mathcal{Q}$ and $\mathcal{R}$.

7 Example

A simple numerical example will now be given to illustrate the tuning procedure. In this example it is assume that the reference is 0 and so it is not necessary to include the reference model in the plant.

Assume that the plant (including disturbance model) is described by

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.02 \\ 0.0002 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 10 \end{bmatrix} x(k) \end{aligned} \tag{46}$$

In this example we will select our shaping filters to be $W_1 = I$ and $W_2 = I$. N_2 is given the value 4 and $N_u = 2$. The matrices Ψ and Υ are then given by

$$\Psi = \begin{bmatrix} 1.2 & 10 \\ 1.4 & 10 \\ 1.6 & 10 \\ 1.8 & 10 \end{bmatrix}, \qquad \Upsilon = \begin{bmatrix} 0.022 \\ 0.048 \\ 0.078 \\ .112 \end{bmatrix} \tag{47}$$

The sample rate selected is $0.02s$. The minimum achievable gamma according to [5] is 2.34 so we may select any value of γ above this, let $\gamma = 2.6$. Feasible values of Y and P that satisfy (29) are then obtained and the calculated K_m leads to the following values of $\mathcal{Q}$ and $\mathcal{R}$

$$\begin{aligned} \mathcal{Q} &= diag\{-168, 209, -116, 26\} \\ \mathcal{R} &= diag\{0.0001, 1138.9\} \end{aligned} \tag{48}$$

The residual norm of the optimisation (44) is less than $1e^{-12}$. In this case $\mathcal{Q}$ is indefinite but the Hessian is positive definite.

A feasible value for L is calculated to be

$$L = \begin{bmatrix} 0.0191 \\ 0.0072 \end{bmatrix} \tag{49}$$

The following plots show simulation of the input and output response when these choice of parameters where chosen for the MPC controller. The initial state of the system is [0.1, 0.01];

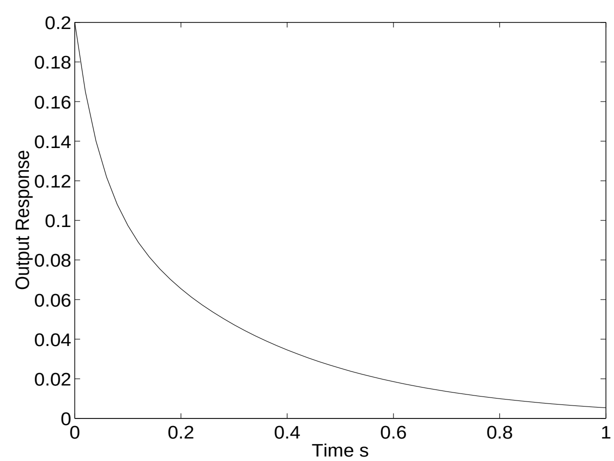

Figure 1: Output Response of MPC Controller

8 Conclusion

The NLCF controller has many desirable properties and it would be useful to obtain an MPC controller with these properties. Implementing the NLCF controller as an MPC controller has the advantage that automatic

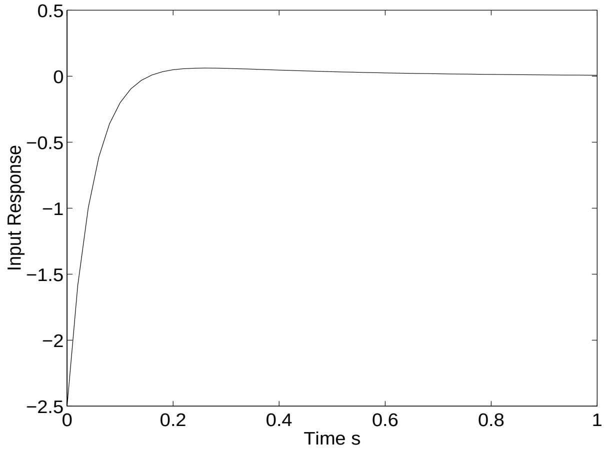

Figure 2: Input Response of MPC Controller

anti-windup is obtained when the MPC controller is implemented with integral action (as is typically the case), and implemented with constraints on the inputs only.

This paper has outlined a procedure for obtaining the parameters of a predicting form of an unconstrained model predictive controller to make it equivalent to the NLCF controller and for the first time provides a way of designing a robust unconstrained finite horizon model predictive controller without terminal constraints.

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