A Design of Low-Order Active Noise Controllers in a Small Cavity

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Abstract

In this paper we designed low-order controllers for active control of acoustic noise in a small cavity. The conventional robust control synthesis methods usually give controller with the same or higher order than the generalized plants, and these controllers are difficult to handle and to be implemented. Any implementation error would make the performance of the system degraded, or even make the closed loop system unstable. Therefore, it is necessary, if possible, to find a lower order controller with a similar performance of a full order controller. We use a convex optimization algorithm with a nonlinear equality constraint to find a controller with predetermined order. To verify the performances, the designed controllers are implemented with TMS320C31 DSP, and their performances are compared with that of the full order controller. The results show that the low order controllers have almost same performances as the full order one. Therefore, the designed low-order controller could be used more efficiently in practice.

1 Introduction

This paper presents the design of an active noise controller in a small cavity. The idea on the active noise control in the small cavity system was suggested by Olsen [15] and his idea has been developed by Wheeler, Carme, Nelson and Elliott [14]. Their method was based on a classical compensator design. A phase lead-lag compensator was designed for better gain and phase margins. However, gain and phase margins do not guarantee the stability of the system when it has uncertainties[6, 18, 19]. In the view point of robustness, the method presented in [14] may be less practical since the sound characteristics in the small cavity are very nonlinear and easy to variate due to many reasons. Therefore, it is desirable to analyze these uncertainties in the small cavity and design a controller based on them. An attempts to apply the robust control design for the small cavity system were made in [3] by solving Linear Matrix Inequalities and in [2] by solving D-K iterations. In [3, 2], we solved the robust $\mathcal{H}_\infty$ control problems to design full-order controllers. The objective was to minimize the $\mathcal{H}_\infty$ norm of the closed loop transfer function, which means minimizing the energy of acoustic noise in the small cavity with guaranteed stability under uncertainties.

However, the conventional approaches for the robust $\mathcal{H}_\infty$ control problem would give us very higher order controllers which are hard to be implemented and tuned, even destabilizes the closed loop system with very small perturbations on the controllers' parameters. Unfortunately, there is few tool to analyze this drawback. The knowledge known by today is that low order controllers are more robust than high order controllers. The problems for reduced order control or fixed order control designs are formulated as double Linear Matrix Inequalities (LMI's), and we could use convex optimization techniques to solve the problems with some modifications [9, 16, 11, 7]. Another method is to formulate the problem as Biaffine Matrix Inequalities (BMI's), which are more general formulation than LMI's, but more difficult to solve [10, 13].

In this paper, we use the double LMI's approach to design low-order robust Active Noise Controllers (ANC's) in a small cavity system. The nonconvex constraint which is induced in the design procedure is relaxed as a convex one with additional nonlinear equality constraint, and a conventional Semidefinite Programming (SP) is used to solve the problem. The rest of this paper is organized as follows. Section 2 is devoted to introduce the ANC problem in the small cavity system, and formulate the design objectives. In section 3, a reduced-order controller design problem is considered. In section 4, using the proposed method, several fixed order controllers are designed, and their performances are compared to that of the full-order controller. Then, we will conclude in section 5.

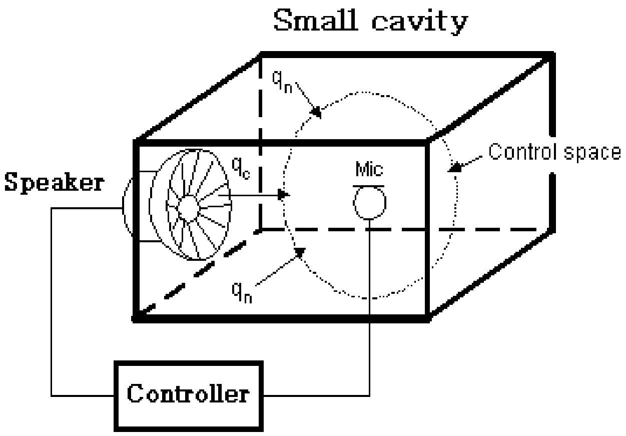

Figure 1: The small cavity system (Small cavity system) (q_n : noise sound, q_c : control sound)

2 ANC Problem in the Small cavity System

Figure 1 shows the small cavity system. The system consists of small enclosed space (called a small cavity in acoustic term), a control speaker, and an error microphone which are connected via a feedback controller. External noise sound is transferred into the small cavity through vibrations of the cavity wall. The desired feedback controller must minimize energy of the acoustic noise in the small cavity. This problem could be cast on the $\mathcal{H}_\infty$ regulation problem by supposing the noise as external disturbances. Figure 2 shows the block diagram of the small cavity system. In Figure 2, K is a controller to be designed and G is the electroacoustic transfer function between the control speaker and the error microphone in the small cavity shown in Figure 1.

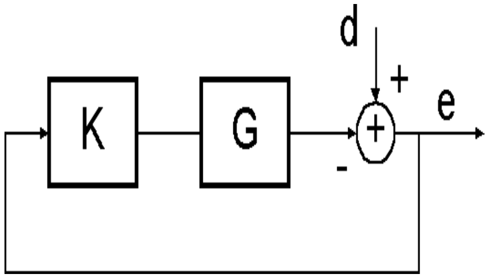

Figure 2: The block diagram of the cavity system (d:noise, e:error, K:controller, G:electroacoustic path from control speaker to error microphone)

The characteristic of the electroacoustic path is measured by Dynamic Signal Analyzer (HP35670A) and the transfer function of the plant G is obtained using curve fitting of the measured data [3, 2]. The identified poles and zeros, and gain of the model are represented as follows.

$$\text{Poles} \;:\; \left(\begin{array}{c} -2.0109e+2 \pm j4.0951e+2 \\ -3.5747e+1 \pm j6.5969e+1 \\ -1.1111e+1 \end{array} \right. \tag{1}$$

$$\text{Zeros} \;:\; \left(\begin{array}{c} -9.3186e+0 \pm j1.2090e+1 \\ 2.2632e+3 \\ 2.6877e+3 \\ 1.1111e+1 \end{array} \right. \tag{2}$$

$$\text{Gain} \;:\; -3.0008e-2 \tag{3}$$

Obviously, we could not avoid unmodelled dynamics. In the small cavity system, the unmodelled dynamics include nonlinear characteristics of the control speaker and the error microphone, time-delay, small variations of the cavity wall, etc. They might cause the closed system to be unstable. Therefore, they must be suitably considered in the design procedure to prevent the closed system tending to be unstable. In other words, we should design a controller which suppresses the effects of the unmodelled dynamics to guarantee the stability of the closed loop system.

At the summing junction in Figure 2, noise sound and control sound are added and the error microphone measures the error sound (residual sound pressure). To minimize the error sound yields to minimization of the noise sound of the small cavity. Therefore, our performance objective could be expressed with a prescribed performance level ϵ.

$$\sup_{\|d\|_2<1} \|S\|_2 < \epsilon \text{ over some frequency ranges}$$

or equivalently,

$$\|W_1 S\|_\infty < 1 \tag{4}$$

In (4), W_1 is a weight function which specifies the control performance and frequency ranges where the performance must be achieved, and S is the sensitivity function. In the small cavity, most noise exist only over low frequency ranges (below $700Hz$), and the desirable performance level we wanted was set to $20dB$ reduction. This specification is expressed as (5).

$$W_1 = \frac{10(s/37,699+1)^2}{(s/3,398+1)^2} \tag{5}$$

As mentioned above, the model must represent uncertainties suitably for robustness. We assume the model to be a multiplicative uncertainty model,

$$G = G_0(1+\Delta W_2) \tag{6}$$

where W_2 is a weight function which has uncertainty information, and $|\Delta| < 1$. We could select W_2 as (7), based on experimental experiences (see [3, 2] for more details), to express uncertainties.

$$W_2 = \frac{0.2(s/9,425+1)^3}{(s/125,664+1)^3} \tag{7}$$

For the multiplicative uncertain model, the closed loop system should satisfy the following constraint for robustness.

$$\|W_2 T\|_\infty < 1 \tag{8}$$

where T is the complementary sensitivity function.

Therefore, we could rewrite the ANC problem expressed as (4) and (8) in terms of the control design problem in (9).

$$\begin{array}{cc} \text{Find} & K \\ \text{subject to} & \left\| \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \right\|_\infty < 1 \end{array} \tag{9}$$

The most well-known methods to solve this problem are the DGKF method, the μ synthesis using D-K iterations, and the LMI's formulation using convex optimization. However, their controllers are at least as high order as the order of generalized plant which includes orders of all weight functions and/or scaling functions. Sometimes, these controllers are too high order to be implemented for practical use. Recently, to overcome this drawback, reduced-order design methods have been actively studied. In [7, 11, 16, 9], LMI's formulations with a nonconvex constraint for reduced-order controllers were presented. As the most promising approach, BMI formulation also could be applied, but still being studied. In this paper we use LMI's formulations to solve the low-order controller design problem.

3 LMI approach for the reduced-order controller design

In this section, we introduce a general framework used for the low-order control design and propose a similar but different approach. Let's assume a generalized plant $P(s)$ is expressed as state space realization

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \end{aligned}$$

where their dimensions are

$$A \in \mathbb{R}^{n_x \times n_x}, \quad D_{11} \in \mathbb{R}^{n_z \times n_w}, \quad D_{22} \in \mathbb{R}^{n_y \times n_u}$$

and suppose the following two assumptions are satisfied.

(A1) (A, B_2, C_2) is stabilizable and detectable,

(A2) $D_{22} = 0$.

Given any proper real-rational controller,

$$K(s) = D_K + C_K(sI - A_K)^{-1} B_K, \quad A_K \in \mathbb{R}^{k \times k}$$

a realization of the closed-loop transfer function from w to z is obtained as

$$F(G,K)(s) = D_{cl} + C_{cl}(sI - A_{cl})^{-1} B_{cl} \tag{10}$$

where

$$\begin{aligned} A_{cl} &= \begin{pmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{pmatrix} \\ B_{cl} &= \begin{pmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{pmatrix} \\ C_{cl} &= \begin{pmatrix} C_1 + D_1 2 D_K C_2 & D_1 2 C_K \end{pmatrix} \\ D_{cl} &= D_{11} + D_{12} D_K D_{21} \end{aligned}$$

Gathering all controller parameters into the simple variable Θ

$$\Theta := \begin{pmatrix} A_K & B_K \\ C_K & D_K \end{pmatrix}$$

and introducing the following shorthand.

$$\begin{aligned} &\mathscr{A} = \begin{pmatrix} A & 0 \\ 0 & 0_k \end{pmatrix} \\ &\mathscr{B}_1 = \begin{pmatrix} B_2 \\ 0 \end{pmatrix} \qquad \mathscr{B}_2 = \begin{pmatrix} 0 & B_2 \\ I_k & 0 \end{pmatrix} \\ &\mathscr{C}_1 = \begin{pmatrix} C_1 & 0 \end{pmatrix} \qquad \mathscr{C}_2 = \begin{pmatrix} 0 & I_k \\ C_2 & 0 \end{pmatrix} \\ &\mathscr{D}_{12} = \begin{pmatrix} 0 & D_{12} \end{pmatrix} \quad \mathscr{D}_{21} = \begin{pmatrix} 0 \\ D_{21} \end{pmatrix} \end{aligned}$$

the closed-loop matrices A_{cl}, B_{cl}, C_{cl}, D_{cl} can be written as functions of the controller's parameters.

$$\begin{aligned} A_{cl} &= \mathscr{A} + \mathscr{B}_2 \Theta \mathscr{C}_2 \\ B_{cl} &= \mathscr{B}_1 + \mathscr{B}_2 \Theta \mathscr{D}_{21} \\ C_{cl} &= \mathscr{C}_1 + \mathscr{D}_{12} \Theta \mathscr{C}_2 \\ D_{cl} &= D_{11} + \mathscr{D}_{12} \Theta \mathscr{D}_{21} \end{aligned}$$

Theorem 1 (Theorem 4.2, [7]) *Consider a proper plant $P(s)$ and assume (A1)-(A2). Define*

$$\begin{aligned} \mathscr{P} &:= \begin{pmatrix} \mathscr{B}_2^T, 0_{(k+n_u)\times n_w}, \mathscr{D}_{12}^T \end{pmatrix} \\ \mathscr{Q} &:= \begin{pmatrix} \mathscr{C}_2, \mathscr{D}_{21}, 0_{(k+n_y)\times n_z} \end{pmatrix} \end{aligned}$$

and let $W_{\mathscr{P}}$ and $W_{\mathscr{Q}}$ be two matrices whose columns span the null spaces of $\mathscr{P}$ and $\mathscr{Q}$, respectively. Then the set of γ-suboptimal controllers of order k is nonempty if and only if there exists some positive definite matrix X_{cl} such that:

$$W_{\mathscr{P}}^T \Phi_{X_{cl}} W_{\mathscr{P}} < 0 \tag{11}$$
$$W_{\mathscr{Q}}^T \Psi_{X_{cl}} W_{\mathscr{Q}} < 0 \tag{12}$$

where

$$\Phi_{X_{cl}} := \begin{pmatrix} \mathscr{A}X_{cl}^{-1} + X_{cl}\mathscr{A} & \mathscr{B}_1 & X_{cl}^{-1}\mathscr{C}_1^T \\ \mathscr{B}_1^T & -\gamma I & D_{11}^T \\ \mathscr{C}_1 X_{cl}^{-1} & D_{11} & -\gamma I \end{pmatrix}$$
$$\Psi_{X_{cl}} := \begin{pmatrix} \mathscr{A}^T X_{cl} + X_{cl}\mathscr{A} & X_{cl}\mathscr{B}_1 & \mathscr{C}_1^T \\ \mathscr{B}_1^T X_{cl} & -\gamma I & D_{11}^T \\ \mathscr{C}_1 & D_{11} & -\gamma I \end{pmatrix}$$

This theorem gives a sufficient and necessary condition for a kth order controller for the plant $P(s)$, and the condition is expressed as LMI's. Note that the conditions are independent on the controller parameters, so it seems easy to solve using one of convex optimization algorithms. However, the variables of the LMI's are X_{cl} and its inverse X_{cl}^{-1} which are closely coupled. This constraint makes the problem difficult to solve.

To use efficient convex optimization techniques, let's use variables S and X instead of X_{cl}^{-1} and X_{cl} in (11) and (12). Then we can obtain two LMI's with variable X and S (note: the variable X is not related to the variable X in [7]) and one nonlinear equation $XS = 1$ or $X = S^{-1}$. To find a feasible solution, [11] use a XY centering algorithm and [9] use a cone complementary problem formulation. In this paper, we propose a similar but different approach.

The feasible X and S should be positive definite matrices. Therefore, using Schur complements we could derive a sufficient condition for $X = S^{-1} > 0$ as (13) and (14).

$$\begin{pmatrix} X & I \\ I & S \end{pmatrix} \geq 0 \tag{13}$$
$$\mathrm{Trace}(X - S^{-1}) = 0 \tag{14}$$

Finally, if there exists a optimal solution '0' of the optimization problem (15), we could say we have solved the original problem.

$$\begin{array}{ll} \text{Minimize} & \mathrm{Trace}(X - S^{-1}) \\ \text{subject to} & (11),\ (12),\ \text{and}\ (13). \end{array} \tag{15}$$

Once again, the objective is nonlinear, but this is easily approximated by utilizing the linearization method in [13]. The linearized approximation of $\mathrm{Trace}(X - S^{-1})$ is

$$\mathrm{Trace}(X - S^{-1}) \approx C_1 + \mathrm{Trace}(X + S0^{-1}SS0^{-1}) \tag{16}$$

and we could minimize this linear objective $\mathrm{Trace}(X + S0^{-1}SS0^{-1})$ instead of $\mathrm{Trace}(X - S^{-1})$.

When we have any optimal solution of the problem in (15), we can construct a controller by solving another LMI in (17) whose variable is only parameters of the controller.

$$\begin{pmatrix} A_{cl}^T X + XA_{cl} & XB_{cl} & C_{cl}^T \\ B_{cl}^T X & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{pmatrix} < 0 \tag{17}$$

Finally, we summarized the algorithm we described above.

[**Algorithm**]

Step 1. Find a feasible initial solution $(X0, S0)$ with initial γ. If it fails increase γ and try again. If found a feasible solution, set $j = 0$ and go to Step 2;

Step 2. Solve the following LMI's problem.

$$\begin{array}{cc} \min & \mathrm{Trace}(X + S0^{-1}SS0^{-1}) \\ \text{subject to} & (11),\ (12)\ \text{and}\ (13) \end{array}$$

Step 3. If the maximum eigenvalue of $X - S^{-1}$ is less then ε then go to Step 4. If X and S is a local minimum then to to Step 1, otherwise set $X0 = X$, $S0 = S$, $j = j + 1$, and go to Step 2.

Step 4. Construct a controller by solving the LMI feasibility problem in (17) and exit.

4 Reduced-order Active Noise Controller Design

The generalized plant of the small cavity system including weighting functions is 10th-order plant. To design a full-order robust controller, we used `$\mu$-Analysis and Synthesis Toolbox` of `Matlab`, and with the controller γ was 1.005.

Next, we designed reduce-order controllers satisfying the condition (9) and observed how low-order controller we could obtain. To use the proposed algorithm, we first set the initial γ to '1.005'. If the algorithm failed to get the initial feasible solution, we increased γ and tried again. In order to solve the LMI's problem, we used the semidefinite programming code **SP**[17] and its Matlab interface **LMITOOL**[8]. The **SP** parameters for absolute and relative convergence were set to 10^{-5}. To prevent unbounded solution we set $\mathbf{M}$ which was initially set to 1000. Table 2 shows the results.

Table 1: Designed Reduced-order Controllers using the proposed method

Order	γ	# of iter.
2	-	Fail
3	-	Fail
4	0.999	45
5	0.970	31
6	0.982	19
7	0.965	13

We could find 4 reduced-order controllers, and the least order controller was 4th-order one whose suboptimal γ was '0.999'. This means that we could use the 4th-order controller instead of the 10th order one with very little performance decrease. In the next section, to verity the performances of the designed controllers, we performed several noise rejection experiments in the small cavity system.

5 Experiments

Each controllers were implemented with TMS320C31 DSP. The sampling frequency was set to $10kHz$ and the continuous-time controllers were converted to discrete-time IIR format controllers using the bilinear transformation. For external noise, we used sweep sinusoidal signals which sweep from $20Hz$ to $800Hz$. The performance was measured as how much noise in the small cavity was reduced. To see performances, we measured power spectrum of noise signals when the controller was on and when the controller was off, and then compared their magnitude.

For the first experiment, we implemented the 10th-order controller. Figure 3 shows the experimental results. As we expected the noise was rejected to about -20dB in the desired frequency ranges. Next, we proceeded the same experiments with the low-order controllers. Figures 4 show their results. In these figures, the performances were not degraded so much. As the order of the controller reduced, the performance decreased little in the frequency ranges about $700Hz$ to $800Hz$, which are almost negligible. With these experimental results, we could see the performance are similar in each cases, and conclude that our design method was effective to design a low-order ANC in the small cavity for practical use.

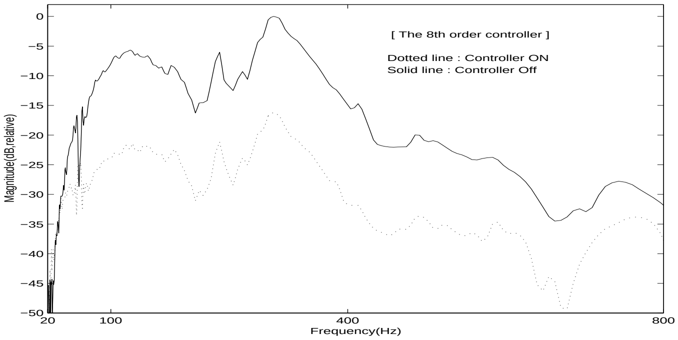

Figure 3: Noise Rejection Performance of the 8th order controller

6 Conclusion

In this paper, we used LMI formulation to design low-order robust ANC in the small cavity system. The algorithm was expressed in terms of LMI's with a nonlinear equality constraint, and we could use efficient semidefinite programming techniques to solve the problem. Using the algorithm, we designed 4 low-order controllers with little performance decrease. To verify their performances as an ANC, each controllers were implemented using TMS320C31 DSP and several noise rejection experiments were performed. As results, we could see all controllers had similar performances. Therefore, we could conclude that the designed low-order ANC could be used to with more efficiency for practical purposes.

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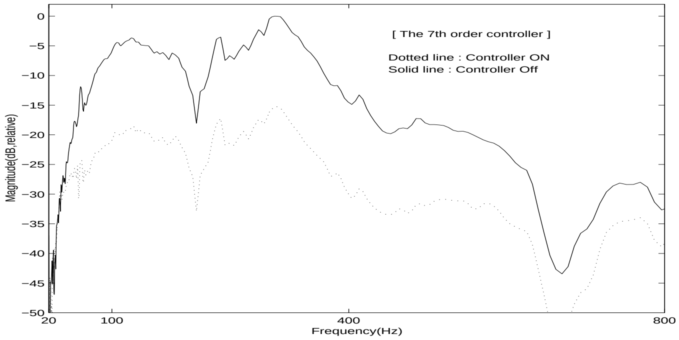
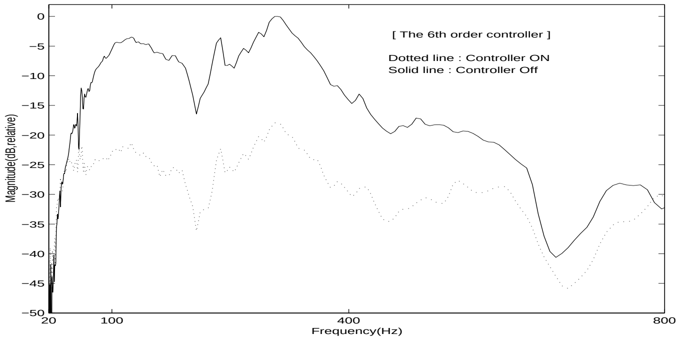
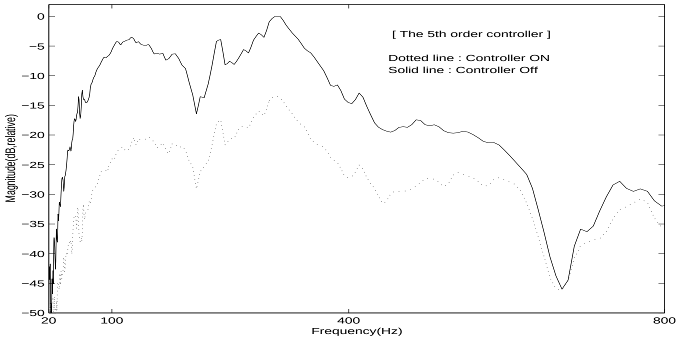


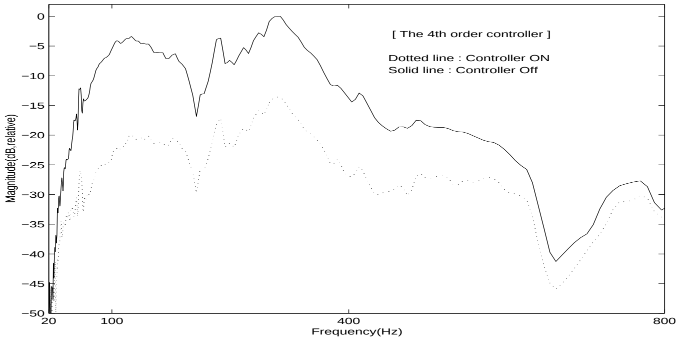
(a) 7th order controller



(b) 6th order controller



(c) 5th order controller



(d) 4th order controller

Figure 4: Noise Rejection Performance of Reduced order controllers