Control of the System Synchronous Generator-Exciter via VSC

Alexander G. Loukianov*, J. Cabrera-Vázquez*, Vadim I. Utkin **, José M. Cañedo*, Juan M. Ramírez*

*Department of Electrical Engineering, CINVESTAV IPN,
Apartado Postal 31-438, Plaza La Luna, Guadalajara, Jal. 44550, Mexico,
E-mail: louk@gdl.cinvestav.mx, jacava@prodigy.net.mx, canedoj[jramirez]@gdl.cinvestav.mx

**Department of Electrical Engineering, The Ohio-State University,
Columbus, Ohio 43210-1272, E-mail: utkin@ee.eng.ohio-state.edu

Abstract

The objective of this paper is to design nonlinear observer-based excitation controller for power system comprising a single synchronous generator connected to an infinite bus and an exciter system. The controller proposed is based on the using Variable Structure Systems and singular perturbation systems concepts, and Block Control Principle. A nonlinear observer for estimation of mechanical torque and rotor fluxes, is included. This combined approach enables to compensate the inherent nonlinearities of the generator and to reject external disturbances.

1 Introduction

The design of robust stabilising feedback controllers for power systems with modelling and unmodelling uncertainties and unknown disturbances remains one of the most important problems in control theory. A fruitful and relatively simple approach, is based on the use the concept of Variable Structure Systems and Sliding Mode Control [1]. This scheme provides, first and foremost, performance robustness, and secondly decomposition and simplicity of the control design procedure.

It is known that the model of power systems is highly nonlinear, and, therefore, during last decade a concept feedback linearization (FL) have been used [2,3,4,5] to design continuous nonlinear controller which overcomes the known limitations of traditional linear controllers. However the direct implementation of FL scheme results in a computationally expensive, and in addition, sensitive to the plant parameters variations control algorithm.

In this paper we shall resort to the block control [6] and sliding mode techniques [1] combined with singular perturbation methods, to obtain a simpler controller, computationally law demanding, that takes into account structural constraints. The main feature of the proposed control is robustness to disturbances and plant parameter variations. The advantage of singular perturbation methods in the control is the possibility of dealing with reduced order model instead of the full-order system. To simplify the design procedure, a switching function is formed using block control linearization technique and neglecting the fast dynamics of the exciter system. But discontinuities in the sliding mode control can excite the unmodeled exciter dynamics, leading to oscillations in the state vector. This phenomena is known as “chattering” [7]. To prevent these oscillations a nonlinear observer is designed, and the ideal sliding mode in the auxiliary observer loop is produced.

The paper is organized as follows. Section 2 reviews the full order model of the power system. In Section 3 the block control and singular perturbation methods are applied to design an ideal sliding mode controller. Section 4 presents a nonlinear observer design. This observer estimates the excitation and rotor fluxes and the mechanical torque needed for designing the discontinuous surface. Section 5 discusses simulation results.

2 Power System Dynamics

The complete mathematical model of the single machine infinite-bus system employing the currents as the state variables, comprises stator dynamics [8]

$$\mathbf{L}_{ss}\frac{d\mathbf{i}_s}{dt}+\mathbf{L}_{sr}\frac{d\mathbf{i}_r}{dt}=-\mathbf{G}_{ss}(\omega)\mathbf{i}_s-\mathbf{G}_{sr}(\omega)\mathbf{i}_r+\mathbf{V}_s \qquad (1)$$

rotor damper windings dynamics

$$\mathbf{L}_{rs}\frac{d\mathbf{i}_s}{dt}+\mathbf{L}_{rr}\frac{d\mathbf{i}_r}{dt}=-\mathbf{G}_{rs}(\omega)\mathbf{i}_s-\mathbf{G}_{rs}(\omega)\mathbf{i}_r+\mathbf{b}_r V_f \qquad (2)$$

exciter system dynamics

$$\tau_f\frac{dV_f}{dt}=-V_f+b_f u \qquad (3)$$

mechanical dynamics

$$\frac{d\delta}{dt}=\omega-\omega_s \qquad (4)$$

$$\frac{d\omega}{dt}=\frac{\omega_s}{2H}(T_m-T_e) \qquad (5)$$

and load constraints

$$\mathbf{V}_s=L'_e\frac{d\mathbf{i}_s}{dt}+\mathbf{R}_L(\omega)\mathbf{i}_s+V^{\infty}\mathbf{Y} \qquad (6)$$

where

$\mathbf{i}_s = (i_d, i_q)^T$, $\mathbf{i}_r = (i_f, i_g, i_{kd}, i_{kq})^T$, $\mathbf{V}_s = (V_d, V_q)^T$,

$$[\mathbf{L}] = \begin{bmatrix} -L_d & 0 & L_{md} & 0 & L_{md} & 0 \\ 0 & -L_q & 0 & L_{mq} & 0 & L_{mq} \\ -L_{md} & 0 & L_f & 0 & L_{md} & 0 \\ 0 & -L_{mq} & 0 & L_g & 0 & L_{mq} \\ -L_{md} & 0 & L_{md} & 0 & L_{kd} & 0 \\ 0 & -L_{mq} & 0 & L_{mq} & 0 & L_{kq} \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} \mathbf{L}_{ss} & \mathbf{L}_{sr} \\ \mathbf{L}_{rs} & \mathbf{L}_{rr} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_{ss} & \mathbf{G}_{sr} \\ \mathbf{G}_{rs} & \mathbf{G}_{rr} \end{bmatrix}, \quad \mathbf{b}_r = (1,0,0,0)^T,$$

$$\mathbf{G} = \begin{bmatrix} -R_s & \omega L_q & 0 & -\omega L_{mq} & 0 & -\omega L_{mq} \\ -\omega L_d & -R_s & \omega L_{md} & 0 & \omega L_{mq} & 0 \\ 0 & 0 & R_f & 0 & 0 & 0 \\ 0 & 0 & 0 & R_g & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{kd} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{kq} \end{bmatrix}$$

$$L'_e = \frac{L_e}{\omega_s}, \quad \mathbf{R}_L(\omega) = \begin{bmatrix} R_e & -\omega L'_e \\ \omega L'_e & R_e \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} sin\delta \\ \cos\delta \end{bmatrix},$$

i_d and i_q are the direct-axis and quadrature-axis stator currents; i_f is the field current; i_{kd}, i_{kq} and i_g are the direct-axis and quadrature-axis damper windings currents; ω is the angular velocity; V_d and V_q are the direct-axis and quadrature-axis terminal voltages; V_f is the excitation voltage; δ is the power angle of the generator; ω_s is the rated synchronous speed, u is the control input, R_s and R_f are the stator and field resistances; R_g, R_{kd} and R_{kq} are the damper windings resistances; L_d and L_q are the direct-axis and quadrature-axis self-inductances; L_f is the rotor self-inductance; L_{kd} and L_{kq} are the direct-axis and quadrature-axis damper windings self-inductances; L_{md} and L_{mq} are the direct-axis and quadrature-axis magnetizing inductances, H is the inertia constant; V^∞ is the value of the infinite-bus voltage; L_e and R_e are the transformer plus transmission line inductance and resistance; b_f is positive constant parameter, and τ_f is a small time constant of the exciter system fast dynamics. The electrical torque T_e is expressed in terms of the currents as follows:

$$T_e = (L_q - L_d) i_d i_q + L_{md} i_q (i_f + i_{kd} - L_{mq} i_d (i_g + i_{kq}) \quad (7)$$

The mechanical torque T_m applied to the shaft it is assumed to be a slowly varying function of time. Thus:

$$\dot{T}_m = 0$$

Since the sensitivity of the fluxes with respect to parameter variations is lower than that of the currents, it is more suitable the representation of the electrical dynamics in terms of the stator current $\mathbf{i}_s$ and the rotor flux ψ_r, $\psi_r^T = (\psi_f, \psi_g, \psi_{kd}, \psi_q)^T$, using the following transformation between fluxes and currents:

$$\psi_r = \mathbf{L}'_{rs}\mathbf{i}_s + \mathbf{L}'_{sr}\mathbf{i}_r \quad (8)$$

where

$$[\mathbf{L}'_{rs} \;\; \mathbf{L}'_{rr}] = \begin{bmatrix} -L_{md} & 0 & L_f & 0 & L_{md} & 0 \\ 0 & -L_{mq} & 0 & L_g & 0 & L_{mq} \\ -L_{md} & 0 & L_{md} & 0 & L_{kd} & 0 \\ 0 & -L_{mq} & 0 & L_{mq} & 0 & L_{kq} \end{bmatrix}$$

Combining equations (1) to (8), the complete model of the generator is presented in the nonlinear state-space form as:

$$\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{b}_1 V_f + \mathbf{d}_1 T_m \quad (9)$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{b}_2 V_f \quad (10)$$

$$\tau_f \dot{V}_f = -V_f + b_f u \quad (11)$$

where $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$, $\mathbf{x}_1 = (\delta, \omega, \psi_f)^T$,

$\mathbf{x}_2 = (\psi_g, \psi_{kd}, \psi_{kq}, i_d, i_q)^T$,

$\mathbf{f}(\mathbf{x}) = (\mathbf{f}_1(\mathbf{x}), \mathbf{f}_2(\mathbf{x}))^T$, $\mathbf{f}_1(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}))^T$,

$\mathbf{f}_2(\mathbf{x}) = (f_4(\mathbf{x}), f_5(\mathbf{x}), f_6(\mathbf{x}), f_7(\mathbf{x}), f_8(\mathbf{x}))^T$,

$f_1 = x_2 - \omega_s \quad f_2 = -a_{23}x_8x_3 + f_{22}(\mathbf{x}_2)$,

$f_{22} = a_{24}x_7x_4 - a_{25}x_8x_5 + a_{26}x_7x_6 + a_{28}x_7x_8$,

$f_3 = -a_{33}x_3 + a_{35}x_5 - a_{37}x_7$,

$f_4 = -a_{44}x_4 + a_{46}x_6 - a_{48}x_8$,

$f_5 = a_{53}x_3 - a_{55}x_5 - a_{57}x_7$,

$f_6 = a_{64}x_4 - a_{66}x_6 - a_{68}x_8$,

$$\begin{aligned} f_7 &= -a_{71}\cos x_1 + a_{73}x_3 + a_{75}x_5 - a_{77}x_7 \\ &\quad + x_2(-a_{74}x_4 - a_{76}x_6 + a_{78}x_8), \end{aligned}$$

$$\begin{aligned} f_8 &= -a_{81}\sin x_1 + a_{84}x_4 + a_{86}x_6 - a_{88}x_8 \\ &\quad + x_2(a_{83}x_3 + a_{85}x_5 - a_{87}x_7), \end{aligned}$$

$\mathbf{b}_1 = (0,0,b_3)^T$, $\mathbf{d}_1 = (0, d_m, 0)^T$, $\mathbf{b}_2 = (0,0,0,b_7,0)^T$,

a_{ij}, $(i,\ j = 2,...,8)$, a_f, b_f, b_3, b_7 and d_m are positive constant parameters depending on R_s, R_f, R_g, R_{kd}, R_{kq}, R_e, L_d, L_q, L_{kd}, L_{kq}, L_{md}, L_{mq}, L_e and V^∞.

It is assumed that the power angle x_1, the terminal voltage V_g, the speed x_2 and the stator currents x_7 and x_8 are available for measurement, and that the control input $u(t)$ should be bounded by

$$|u(t)| \le U_{max}\ ,\ U_{max} > 0 \quad (12)$$

3 The Ideal Sliding Mode Control

The control objectives first are to make the angle power x_1 be equal to a constant reference signal δ_{ref} , and the speed x_2 be equal to the rated synchronous speed ω_s . Secondly, the voltage regulation is important as well.

3.1 The Mechanical Dynamics Control

To simplify the control law, let us neglect the fast exciter system dynamics, setting $\tau_f = 0$ in (11). Then, in (9)

$$\dot{x}_3 = -a_{33}x_3 + a_{35}x_5 - a_{37}x_7 + \bar{b}_3 u \tag{13}$$

where $\bar{b}_3 = b_3 b_f$.

In accordance with the block control technique [6], set

$$\begin{aligned} x_1 &= z_1 + \delta_{ref} \\ x_2 &= x_2^c - k_1 z_1 + z_2 \\ x_3 &= x_3^c - \frac{1}{a_{23}x_8}(-k_2 z_2 + z_3) \end{aligned} \tag{14}$$

where x_2^c and x_3^c are calculated from $\dot{z}_1 = 0$ and $\dot{z}_2 = 0$ as $x_2^c = \omega_s$ and $x_3^c = \frac{1}{a_{23}x_8}\left(-k_1^2 z_1 + k_1 z_2 + f_{22}(\mathbf{x}_2) + d_m T_m\right)$ with $k_1 > 0$ and $k_2 > 0$. Using (13), the first two equations of (9) and the equation (13) in terms of new variables z_1, z_2 and z_3 become

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2 \\ \dot{z}_2 &= -k_2 z_2 + z_3 \\ \dot{z}_3 &= f_0(\mathbf{x}, \delta_{ref}, T_m) + b_0(\mathbf{x}_2)u \end{aligned} \tag{15}$$

where

$$f_0 = -a_{23}x_8 f_3(\mathbf{x}) + \frac{\partial \bar{x}_3^c}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x}), \quad \bar{x}_3^c = a_{23}x_8 x_3^c,$$

$$b_0(\mathbf{x}_2) = -a_{23}x_8\bar{b}_3 + b_7(a_{24}x_4 + a_{26}x_6 - a_{28}x_8),$$

and $b_0(t) = b_0(\mathbf{x}_2(t))$ is a positive function for $t \geq 0$. Now, considering the bound (12), a control strategy can be proposed by

$$u = -U_{max}\operatorname{sign}(z_3) \tag{16}$$

Under the following condition

$$U_{max} > \left|u_{eq}(\mathbf{x}, \delta_{ref}, T_m)\right| \tag{17}$$

where u_{eq} is the *equivalent control* [1] calculated from $\dot{z}_3 = 0$, resulting

$$u_{eq} = \left(b_0(\mathbf{x}_2)\right)^{-1} f_0(\mathbf{x}, \delta_{ref}, T_m) \tag{18}$$

the time derivative of the Lyapunov candidate function $V_1 = \frac{1}{2}z_3^2$ along the trajectories of the closed-loop system (15) and (16)

$$\dot{V}_1 = f_0(\mathbf{x}, \delta_{ref}, T_m)z_3 - b_0(\mathbf{x}_2)U_{max}|z_3| \tag{19}$$

is negative. Therefore, the state vector reaches the sliding surface $z_3 = 0$ after a finite time interval. Once this is achieved, the sliding motion is governed by the following second order linear system

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2 \\ \dot{z}_2 &= -k_2 z_2 \end{aligned}$$

with desired eigenvalues $-k_1$ and $-k_2$. Note that this system corresponds to the linearized mechanical dynamics of the closed-loop system.

A crucial property of the sliding mode control (16) when applied to (9) and (10) is that, it yields the invariant subspace $\{\xi = (\delta_{ref}, 0, 0)^T,\ \mathbf{x}_2 \in R^5\}$, $\xi = (z_1, z_2, z_3)^T$. The dynamics of $\mathbf{x}_2$ on this invariant subspace is referred to as the *zero dynamics*. To derive this dynamics, the equivalent control u_{eq} (18) is substituted in (10), resulting:

$$\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{b}_2 u_{eq}(\mathbf{x}_1, \mathbf{x}_2, \delta_{ref}, T_m)$$

Then, the vector $\mathbf{x}_1$ is changed by $\hat{\mathbf{\imath}}$:

$$\dot{\mathbf{x}}_2 = \tilde{\mathbf{f}}_2(\xi, \mathbf{x}_2, \delta_{ref}, T_m),$$

$$\tilde{\mathbf{f}}_2 = \mathbf{f}_2(\mathbf{x})_{\mathbf{x}_1 = \varphi(\xi)} + \left(b_0(\mathbf{x}_2)\right)^{-1}\mathbf{b}_2 f_0(\mathbf{x}, \delta_{ref}, T_m)_{\mathbf{x}_1 = \varphi(\xi)}$$

where mapping φ is defined by (14). Finally, the vector ξ is zeroed, thus:

$$\dot{\mathbf{x}}_2 = \tilde{\mathbf{f}}_2(0, \mathbf{x}_2, \delta_{ref}, T_m)$$

An equilibrium point for this system is defined by δ_{ref} and the value of the mechanical torque T_m. Simulation results show that this equilibrium point is asymptotically stable (see Section 5).

3.2 The Terminal Voltage Control

In order to obtain the terminal voltage controller we need to derive the appropriate model. The equation (6) can be represented as

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} h_d(\mathbf{x}_1, \mathbf{x}_2) \\ h_q(\mathbf{x}_1, \mathbf{x}_2) \end{bmatrix} + \begin{bmatrix} b_d \\ 0 \end{bmatrix} V_f \tag{20}$$

where

$$h_d = V^{\infty} sin x_1 - L_e' a_{71}\cos x_1 + L_e'\{a_{73}x_3 + a_{75}x_5$$
$$-[a_{77} - (R_e/L_e')]x_7 - x_2[a_{74}x_4 + a_{76}x_6 + (1 - a_{78})x_8]\},$$

$$h_q = V^{\infty}\cos x_1 - L_e' a_{81}\sin x_1 + L_e'\{a_{84}x_4 + a_{86}x_6$$
$$-[a_{88} - (R_e/L_e')]x_8 + x_2[a_{83}x_3 + a_{85}x_5 + (1 - a_{87})x_7]\},$$

and $b_d = L_e' b_7$.

The model for the terminal voltage V_g, $V_g = |\mathbf{V}_s| = \sqrt{V_d^2 + V_q^2}$ can be obtained by differentiating (20):

$$\frac{dV_g}{dt} = f_v(\mathbf{x}, V_f) + b_v u \tag{21}$$

where

$$f_v = \frac{1}{V_g}\left(V_d \frac{\partial h_d}{\partial \mathbf{x}} + V_q \frac{\partial h_q}{\partial \mathbf{x}} \right)\mathbf{f} - \frac{V_d}{V_g}\frac{b_d a_f}{\tau_f} V_f \,, \quad b_v = \frac{V_d}{V_g}\frac{b_d b_f}{\tau_f} \,.$$

The switching function for the system (21), z_v, is defined as

$$z_v = V_g - V_{ref}$$

where V_{ref} is the voltage reference. Then the control law

$$u = -U_{max}\mathrm{sign}(z_v) \tag{22}$$

under condition

$$U_{max} > \left| u_{eq}(\mathbf{x}, V_f, T_m) \right| \,, \; u_{eq} = b_v^{-1} f_v(\mathbf{x}, V_f) + b_v u$$

guaranties the sliding mode motion on the surface $z_v = 0$ after a finite time interval.

3.3 The Switching Logic Control

Having just one control input u for two switching surfaces $z_3 = 0$ and $z_v = 0$, a control strategy taking into account (16) and (22) can be finally proposed by

$$u = -U_{max}\mathrm{sign}(z) \tag{23}$$

where

$$z = \begin{cases} z_3 & if \quad |z_1| > e \\ z_v & if \quad |z_1| \le e \end{cases} \quad \text{and} \quad e = \begin{cases} e_1 & if \quad |z_v| > e_3 \\ e_2 & if \quad |z_v| \le e_3 \end{cases} \tag{24}$$

with $e_2 < e_1$.

Therefore, we are proposing a hierarchical control action through the proposed logic (24). Since the mechanical dynamic is slower than the electrical one, we spend the control resources, at first, to stabilize the angle x_1 and speed x_2. When the angle error z_1 reaches the layer with width e_1, the control resources will be spending to regulate terminal voltage V_g. After convergence of the error voltage such that $|z_v| \le e_3$, the control action reduces the power angle layer width from e_1 to e_2, ($e_1 > e_2$).

The proposed controller (23) provides the desired performance of the closed-loop system in the ideal case, i.e. in the absence of unmodelled dynamics, $\tau_f = 0$ (see Figure 1), and leads to oscillations in the state vector, "chattering" [7], when $\tau_f \ne 0$ because discontinuities in the control excite the unmodelled dynamics (11). In this case the derivative $\dot{V}_1$ (19) depends on V_f only, but not on the control input u, and $V_f(t) = u(t) = -U_{max}$ only for $|z_3| > \varepsilon$, $\varepsilon > 0$. Therefore, in this case the stability condition (17) satisfies outside the small vicinity $|z_3| > \varepsilon$ only.

4 Observer-Based Solution

In this section, to prevent the chattering and to estimate the unmeasured rotor fluxes, x_i, $i = 3,...,6$ and mechanical torque T_m, a nonlinear observer is proposed as

$$\begin{aligned}
\dot{\hat{x}}_2 &= -a_{23}x_8(t)\hat{x}_3 + a_{24}x_7(t)\hat{x}_4 - a_{25}x_8(t)\hat{x}_5 \\
&\quad + a_{26}x_7(t)\hat{x}_6 - a_{68}x_7(t)x_8(t) + d_m\hat{T}_m + l_1(x_2 - \hat{x}_2) \\
\dot{\hat{T}}_m &= l_2(x_2 - \hat{x}_2) \\
\dot{\hat{x}}_3 &= -a_{33}\hat{x}_3 + a_{35}\hat{x}_5 - a_{37}x_7(t) + b_3 u \\
\dot{\hat{x}}_4 &= -a_{44}\hat{x}_4 + a_{46}\hat{x}_6 - a_{48}x_8(t) \\
\dot{\hat{x}}_5 &= a_{53}\hat{x}_3 - a_{55}\hat{x}_5 - a_{57}x_7(t) \\
\dot{\hat{x}}_6 &= a_{64}\hat{x}_4 - a_{66}\hat{x}_6 - a_{68}x_8(t)
\end{aligned} \tag{25}$$

where $\hat{x}_i$, $i = 2,...,6$ and $\hat{T}_m$ are the estimated variables, l_1 and l_2 are observer gains. The stability of observer (25) may be analyzed by examining the following error dynamics equation:

$$\dot{\mathbf{e}} = \mathbf{A}(t)\mathrm{e} \tag{26}$$

where $\mathbf{e} = (e_2, e_m, e_3, e_4, e_5, e_6)^T$,

$$e_i = x_i - \hat{x}_i \,, \quad i = 2,...,6 \,, \quad e_m = T_m - \hat{T}_m \,,$$

$$\mathbf{A}(t) = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12}(t) \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix}, \quad \mathbf{A}_{11} = \begin{bmatrix} -l_1 & a_{29} \\ -l_2 & 0 \end{bmatrix},$$

$$\mathbf{A}_{12}(t) = \begin{bmatrix} -a_{23}(t) & a_{24}(t) & -a_{25}(t) & a_{26}(t) \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{A}_{22} = \begin{bmatrix} -a_{33} & 0 & a_{35} & 0 \\ 0 & -a_{44} & 0 & a_{46} \\ a_{53} & 0 & -a_{55} & 0 \\ 0 & a_{64} & 0 & -a_{66} \end{bmatrix}, \quad \text{and}$$

$$a_{23}(t) = a_{23}x_8(t) \,, \quad a_{24}(t) = a_{24}x_7(t) \,,$$
$$a_{25}(t) = a_{25}x_8(t) \,, \quad a_{26}(t) = a_{26}x_7(t) \,.$$

The nonlinear observer (25) can be seen as a linear system with time varying parameters when the variables $x_7(t)$ and $x_8(t)$ are assumed known functions. It is easy to see that the spectrum of the block matrix $\mathbf{A}(t)$ (26) consists of the eigenvalues of the block $\mathbf{A}_{11}$, which can be assigned by appropriate choice of the observer gains l_1 and l_2, and the eigenvalues of the block $\mathbf{A}_{22}$:

$$\lambda_{3,4} = -\tfrac{1}{2}(a_{33} + a_{55}) \pm \tfrac{1}{2}\sqrt{(a_{33} - a_{55})^2 + 4a_{35}a_{53}} \tag{27}$$

$$\lambda_{5,6} = -\tfrac{1}{2}(a_{44} + a_{66}) \pm \tfrac{1}{2}\sqrt{(a_{44} - a_{66})^2 + 4a_{46}a_{64}} \tag{28}$$

which are real and negative. The parameters of $\mathbf{A}_{12}(t)$ and its derivatives are bounded. Therefore, the linear system (26) with time varying parameters is asymptotically stable. The resulting estimates $\hat{x}_i$, $i = 2,...,6$ and $\hat{x}_m$ are employed in the control law (23), in particularly forming the estimation of the switching function z_3 (14) as

$$\hat{z}_3 = -a_{23}\hat{x}_8(\hat{x}_3^c + \hat{x}_3) + k_2\hat{z}_2$$

The equation for can be obtained using (25) as

$$\dot{\hat{z}}_3 = \hat{f}_0(\hat{\mathbf{x}}, \mathbf{e}, \delta_{ref}, \hat{T}_m,) + b_3 u$$

where $\hat{f}_0$ is a bounded function. With the observer-based control

$$u = -U_{max}\mathrm{sign}(\hat{z}_3) \qquad (29)$$

the derivative of the Lyapunov function candidate $\hat{V}_1 = \frac{1}{2}\hat{z}_3^2$

$$\dot{\hat{V}}_1 = \hat{f}_0(\hat{\mathbf{x}}, \mathbf{e}, \delta_{ref}, \hat{T}_m,)\hat{z}_3 - b_3 U_{max}\left|\hat{z}_3\right|$$

is negative if

$$\left|\hat{f}_0(\hat{\mathbf{x}}, \mathbf{e}, \delta_{ref}, \hat{T}_m,)\right| \le b_3 U_{max} - \eta$$

for some $\eta > 0$. In this domain we have that

$$\dot{\hat{V}}_1 \le -\eta\sqrt{V}$$

It means that the solution $V(t)$ vanishes after some

$$t_s < (2/\eta)\sqrt{V(0)}$$

and $\hat{z}(t) = 0$ holds exactly thereafter, for $t \ge t_s$.

5 Simulation Results

The performance of the proposed controller was tested on first the 8th order model, and then on the complete 9th order model of the generator connected to an infinite bus through a transmission line.

The parameters of the synchronous machine, and transmission and exciter systems, all in p.u., except where indicated, are:

$R_s = 0.003$, $R_f = 0.021$, $R_g = 0.725$, $R_{kd} = 10.714$, $R_{kq} = 8.929$, $R_e = 0.05$, $L_d = 1.81$, $L_q = 1.76$, $L_{kd} = 1.831$, $L_{kq} = 1.735$, $L_{md} = 1.66$, $L_{mq} = 1.61$, $L_e = 0.3$, $H = 3.525$ sec., $b_f = 1$, $\tau_f = 0.015$ sec., $\delta_{ref} = 1.3314$, $\omega_s = 377\,rad\,s^{-1}$, $T_m = 0.9463$, $V^{\infty} = 1$.

The controller gains were adjusted to k_1=7 and k_2=15, and the observer gains were chosen as l_1=200 and l_2=187, resulting in the eigenvalues $\lambda_1 = \lambda_2 = 100$. The remaining observer eigenvalues were calculated using (27) and (28) as $\lambda_3 = -0.123$, $\lambda_4 = -33.922$, $\lambda_5 = -0.883$ and $\lambda_6 = -16.179$.

Figures 1, 2 and 3 depict results under three different events: *a)* simulation begins not from the equilibrium point; *b)* in t = 2 *sec*, T_m experienced a pulse for 0.5 *sec.*; and *c)* in t = 4 *sec.*, a three-phase short circuit for a period of 150 *ms* is simulated at the transformer terminals.

Fig. 1 shows the response of the unperturbed power system with the control (23), ($\tau_f = 0$, ideal sliding mode), and reveals some important aspects:

a) State variables hastily reach a steady state condition after small and large disturbances, exhibiting the stability of the closed-loop system.

b) The terminal voltage recovers their steady state value after the short circuit.

Fig. 2 depicts the same simulation as before but for the perturbed power system ($\tau_f \ne 0$). We can see that the unmodeled dynamics leads to the chattering.

Fig. 3 shows the response of the perturbed power system ($\tau_f \ne 0$) with the observer-based control, using (29) in (23). We can observe that the estimated signals are closely related to the actual ones, exhibiting a robust performance of the observer, and the performance almost the same as in the case of the ideal sliding mode.

6 Conclusions

A sliding mode controller is proposed exhibiting robust stability and performance when the plant experiences small and large disturbances. The inclusion of an external load torque and the simulation of a short circuit demonstrate the capability of the controller in rejecting bounded disturbances.

The design process, including analysis of stability, is discussed. The formulation employed makes easy to design a nonlinear observer that exhibits a good performanc for power systems with unmodeled dynamics and discontinuous control.

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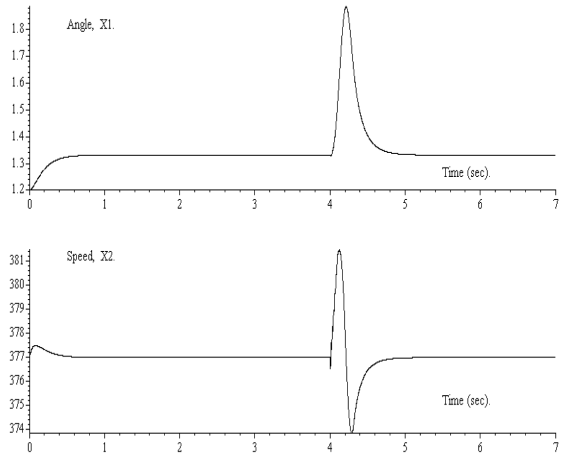

Fig. 1(a,b) Angle x_1 *and speed* x_2

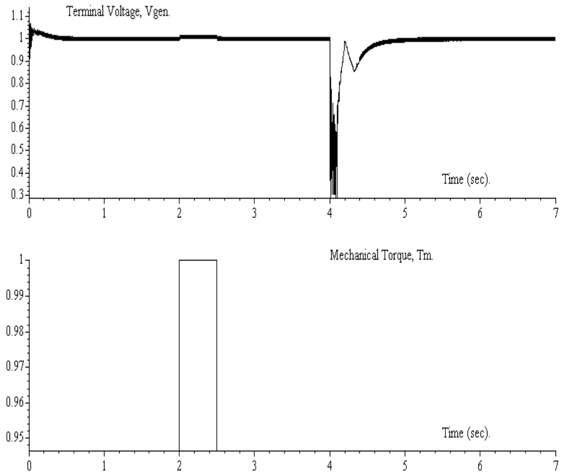

Fig.1(c,d) V_{gen} *and* T_m

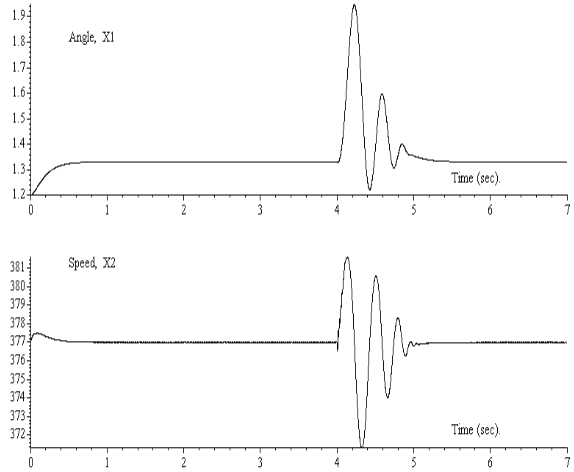

Fig. 2(a, b) Angle x_1 *and speed* x_2

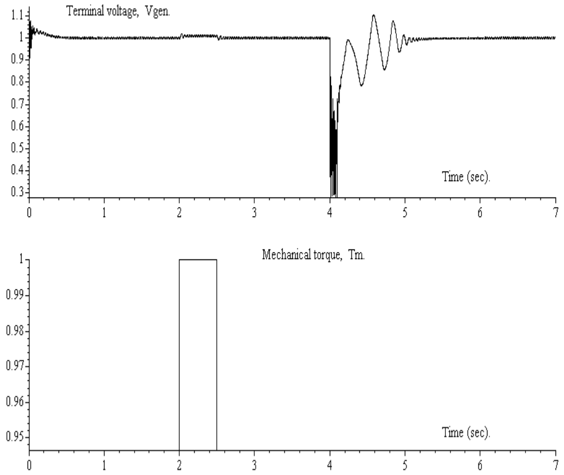

Fig. 2(c,d) V_{gen} *and* T_m

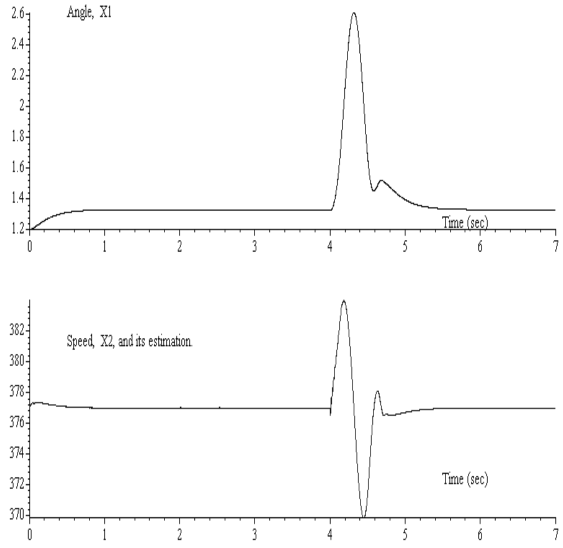

Fig. 3(a,b) Angle x_1, *speed* x_2 *and estimation* $\hat{x}_2$

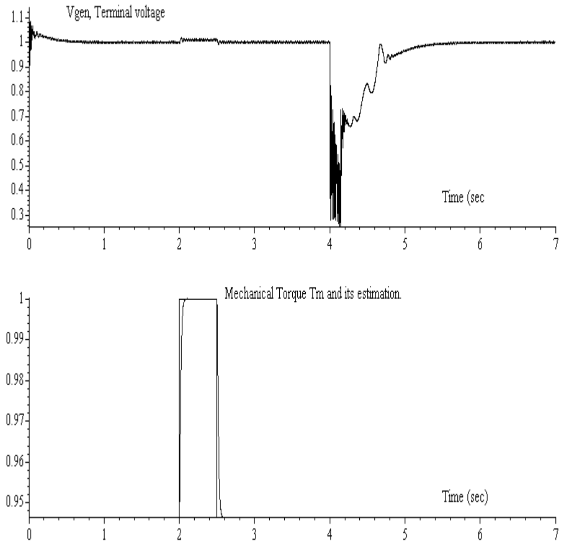

Fig. 3(c,d) V_{gen}, T_m *and estimation* $\hat{T}_m$