Auxiliary Particle Filters for Tracking a Maneuvering Target

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Abstract

We consider the recursive state estimation of a highly maneuverable target. In contrast to standard target tracking literature we do not rely on linearized motion models and measurement relations, or on any Gaussian assumptions. Instead, we apply optimal recursive Bayesian filters directly to the nonlinear target model. We present novel sequential simulation based algorithms developed explicitly for the maneuvering target tracking problem. These Monte Carlo filters perform optimal inference by simulating a large number of tracks, or particles. Each particle is assigned a probability weight determined by its likelihood. The main advantage of our approach is that linearizations and Gaussian assumptions need not be considered. Instead, a nonlinear model is directly used during the prediction and likelihood update. Detailed nonlinear dynamics models and non-Gaussian sensors can therefore be utilized in an optimal manner resulting in high performance gains. In a simulation comparison with current state-of-the-art tracking algorithms we show that our approach yields performance improvements. Moreover, incorporation of physical constraints with sustained optimal performance is straightforward, which is virtually impossible to incorporate for linear Gaussian filters. With the particle filtering approach we advocate these constraints are easily introduced and improve the results.

1 Introduction

Traditionally, target tracking problems are solved using linearized tracking filters, mainly extended Kalman filters (EKF) [1]. For highly maneuvering targets or for low observation rates different maneuvering modes are used to describe the motion. Therefore, target maneuvers are often described by multiple linearized models. A common method is the *Interacting Multiple Model* filter (IMM) [6]. State-of-the-art estimation and tracking literature [2], present state estimation and prediction performed by switching between models or by mixing them. If the filter update is slow or the target maneuver large the linearized solution may not always be good. Non-linear models in state equation and measurement relation and a non-Gaussian noise assumption may also lead to non optimal solutions. Moreover, the incorporation of constraints to the system parameters is complicated using linearized techniques.

The sequential Monte Carlo methods, or particle filters, provide general solutions to many problems where linearizations and Gaussian approximations are intractable or would yield too low performance. The maneuvering target tracking problem is an application which has strong elements of nonlinearity. Non-Gaussian noise assumptions and incorporation of constraints on some of the system parameters can also be performed in a natural way by using these simulation based methods. The constraints are due to limitations in state variables but could also be induced by the terrain, such as land avoidance for tracking ships. Monte Carlo techniques have been a growing research area lately due to improved computer performance. Particularly some aspects of the bearings-only tracking application has been investigated [3].

In this paper we extend the *Auxiliary Particle Filters* of Pitt and Shepard [8] to the case of multiple nonlinear models, switching according to a Markov transition kernel. Each particle is split deterministically into a number of possible maneuver hypotheses and the likelihood is adjusted by the Markov transition probabilities.

The algorithm is implemented for the classical *Bayesian Bootstrap* method [5] using the Auxiliary Particle method. In a simulation study we compare this filter to a linearized method using an *Interacting Multiple Model* filter (IMM) [6] based on three extended Kalman filters (EKF) for an Air Traffic Control (ATC) track-while-scan (TWS) application. The problem under consideration incorporates nonlinear effects both in the prediction and measurement model and some constraints on the system states.

2 The target tracking model

A general target tracking problem consists of a non-linear state equation and a non-linear measurement relation of the form

$$\begin{aligned} x_{t+1} &= f(x_t, v_t) \\ y_t &= h(x_t, e_t) \end{aligned}$$

where the process noise v_t and measurement noise e_t are non-Gaussian, describing the target maneuver and measurement. Some states or combination thereof are in general constrained by some parameter dependent set $\Lambda_i(x)$. The constraints of states are due to target maneuvering capabilities or to terrain constraints. In this paper only constraint of the target speed is considered among a state dependent turn rate. The constraint is defined in discrete time, but a continuous constraint could also be possible, if a numerical solution to the state equation is used.

Following standard target tracking literature [2], tracking an aircraft in nearly coordinated flight using a radar sensor yields a model with nonlinear elements both in the dynamical state equation and in the measurement relation. The model is a discretized continuous time nonlinear stochastic differential equation model where the turn rate state ω gives a strong nonlinear behavior. For highly maneuvering targets we extended it with a maneuvering model where the target maneuver is described by a Markovian switching structure. The amount of the turn rate is assumed velocity dependent due to the fact that the pilot will induce a moderate turn rate traveling at high speed. The turn rate is modeled so a change will be visible in the measurement directly following the onset of a turn, and modeled as a set of three discrete values for right turn, straight flying or left turn. The discrete system is given by

$$\mathrm{X}_{t+1} = A(\omega_{t+1})\mathrm{X}_t + [B_v B_\omega]v_t \qquad (1)$$

$$\mathrm{X}_t = \begin{pmatrix} x_t & \omega_{t+1} \end{pmatrix}^T, x_t = \begin{pmatrix} \xi & \dot{\xi} & \eta & \dot{\eta} \end{pmatrix}^T \qquad (2)$$

where ξ and η is the Cartesian position coordinates and $\dot{\xi}, \dot{\eta}$ the velocity components. The velocity is constrained to some set V, i.e., $\sqrt{\dot{\xi}^2 + \dot{\eta}^2} \in V$.

$$A(\omega) = \begin{pmatrix} 1 & \frac{\sin(\omega T)}{\omega} & 0 & -\frac{(1-\cos(\omega T))}{\omega} & 0 \\ 0 & \cos(\omega T) & 0 & -\sin(\omega T) & 0 \\ 0 & \frac{(1-\cos(\omega T))}{\omega} & 1 & \frac{\sin(\omega T)}{\omega} & 0 \\ 0 & \sin(\omega T) & 0 & \cos(\omega T) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B_v = \begin{pmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \\ 0 & 0 \end{pmatrix}, B_\omega = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The turn rate is assume velocity dependent according to the following model $\omega = a_{typ}/\sqrt{\dot{\xi}^2 + \dot{\eta}^2}$, where a_{typ} is the typical maneuvering acceleration which is modeled as a set of three discrete values, having a Markovian switching structure. The radar measurements are modeled as

$$y_t = h(x_t) + e_t = \begin{pmatrix} \sqrt{\xi^2 + \eta^2} \\ \arctan(\frac{\eta}{\xi}) \end{pmatrix} + e_t$$

where e_t is zero mean noise with covariance R_t. Independence in time and between the measurement and process noise is assumed.

In the Bayesian bootstrap algorithm presented in section 4 the prediction stage is performed by simply simulating equation (1) with the noise model. For systems with an explicit solution this yields a simple and efficient predictor.

3 Particle filters

Many engineering problems are by nature recursive and require on-line solutions. Common applications such as state estimation, recursive identification and adaptive filtering often require recursive solutions to problems having both nonlinear and non-Gaussian character. The seminal paper of Gordon et al. [5] marks the onset of a rebirth for algorithms based on Monte Carlo simulation techniques for solving this important class of problems in an optimal manner.

The sequential Monte Carlo methods, or particle filters provide an approximative Bayesian solution to discrete time recursive identification or filtering problems by updating an approximative description of the posterior filtering density. Let x_t denote the state of the observed system and $Y_t = \{y_i\}_{i=1}^t$ be the set of observed measurements until present time. The Monte Carlo filter approximate the density $p(x_t|Y_t)$ by a large set of N particles $\{x_t^i\}_{i=1}^N$, where each particle has an assigned relative weight, w_t^i, chosen such that all weights sum to unity. The location and weight of each particle reflect the value of the density in the region of the state space, $p(x_t|Y_t) \approx \frac{1}{N}\sum_{i=1}^N \delta(x_t - x_t^i)$. The particle filter updates the particle location and the corresponding weights recursively with each new observed measurement.

The non-linear prediction density $p(x_t|Y_{t-1})$ and filtering density $p(x_t|Y_t)$ for the Bayesian interference is given by

$$\begin{aligned} p(x_t|Y_{t-1}) &= \int p(x_t|x_{t-1})p(x_{t-1}|Y_{t-1})dx_{t-1} \quad (3) \\ p(x_t|Y_t) &\propto p(y_t|x_t)p(x_t|Y_{t-1}) \quad (4) \end{aligned}$$

The main idea is to approximate $p(x_t|Y_{t-1})$ with

$$p(x_t|Y_{t-1}) \approx \frac{1}{N}\sum_{i=1}^{N}\delta(x_t - x_t^i)$$

Inserting into (4) yields a density to sample from. This can be done by using the Bayesian bootstrap or Sampling Importance Resampling (SIR) algorithm from [5].

Bayesian bootstrap

1. Set $t = 0$, and generate N samples $\{x_0^i\}_{i=1}^N$ from $p(x_0)$.
2. Compute the likelihood weights $\mathrm{w}_t^i = p(y_t|x_t^i)$ and normalize, i.e., $\tilde{\mathrm{w}}_t^i = \mathrm{w}_t^i / \sum_{j=1}^N \mathrm{w}_t^j, i = 1, \ldots, N$.
3. Generate a new set $\{x_t^{i\star}\}_{i=1}^N$ by resampling with replacement N times from $\{x_t^i\}_{i=1}^N$, where $Pr(x_t^{i\star} = x_t^j) = \tilde{\mathrm{w}}_t^j$.
4. Predict (simulate) new particles, i.e., $x_{t+1}^i = f(x_t^{i\star}, v_t), i = 1, \ldots, N$ using different noise realizations for the particles.
5. Increase t and iterate to item 2.

4 Auxiliary Particle filter for maneuvering targets

In this paper we are interested in what performance one can achieve by using true nonlinear inference based on Monte Carlo algorithms, applied to a target tracking application. In contrast to what is standard procedure in target tracking we choose not to linearize the target models for each mode. Instead, we apply particle filters directly to the mode switching model. The maneuvers are modeled by a discrete parameter ω (turn rate) with a Markovian switching structure yielding a mode switching, or jumping, model. In [7] a similar idea is used for linear jump Markov models, applied to a bootstrap filter. By using particle filters, nonlinearities in the system and measurement models and constraints in system parameters are incorporated in a natural way.

For reliable handling of the mode hypotheses we extend the state-of-the-art particle filter of Pitt and Shepard [8]. These so called auxiliary particle filters use resampling on the predicted particles to select which particles to use in the prediction and measurement update. We introduce a deterministic splitting of each particle into several offspring, each one representing a different target maneuver. Each offspring is weighted by the Markov transition probability for the maneuver and its likelihood. In Figure 1 an example of three different maneuver assumptions in the deterministic particle splitting is visualized in the upper plot. The three predicted particle clouds conditioned on the turn rate are clearly distinguished in the figure. The resampled cloud using the predicted particles is viewed in the lower plot. In the lower plot in Figure 1 the resampled particles are predicted one step forward.

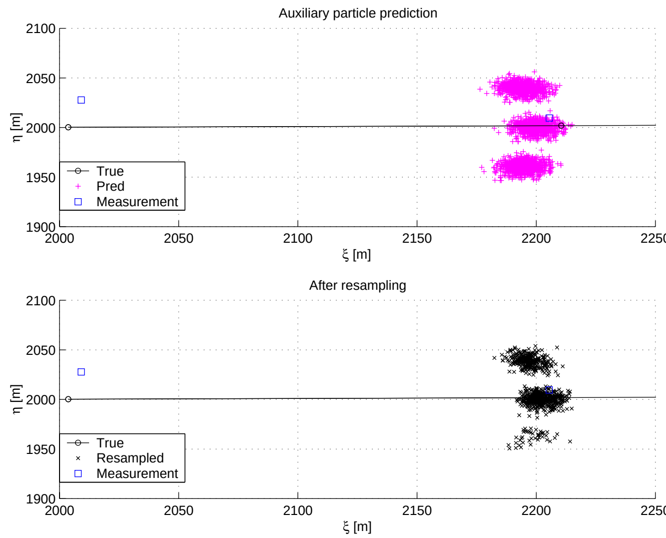

Figure 1: The three different maneuvers, Auxiliary particle filter

System constraints are also incorporated in the model, so that non feasible maneuvers are avoided using the particle filtering technique.

The Auxiliary Particle Filter [8] extends the state x_t by predicting the state conditional upon particle k. At time t, the particle set $\{x_t^k\}_{k=1}^N$ and the corresponding weights π_t^k form the following approximations to the prediction and filter densities for the state of the target

$$\begin{aligned} p(x_{t+1}|Y_t) &= \sum_{j=1}^{N} p(x_{t+1}|x_t^j)\pi_t^j \\ p(x_{t+1}|Y_{t+1}) &\propto p(y_{t+1}|x_{t+1})\sum_{j=1}^{N} p(x_{t+1}|x_t^j)\pi_t^j \end{aligned}$$

By defining

$$p(x_{t+1}, k|Y_{t+1}) \propto p(y_{t+1}|x_{t+1})p(x_{t+1}|x_t^k)\pi_t^k, k = 1, \ldots, N$$

we can draw from this joint density and then discard the index, to produce a sample from the empirical filtering density as required. The index k is referred to as an auxiliary variable. For the target tracking application we consider the joint density of particle k at time t and the target state at time $t+1$. The turn rate ω_{t+1} indicates that the state is effected during the integration of the state equation from t to $t+1$, i.e., it will effect the states at time $t+1$, when applied at time t.

Using Bayes rule gives

$$p(x_{t+1}, \omega_{t+1}, k|Y_{t+1}) \propto$$
$$p(y_{t+1}|x_{t+1}, \omega_{t+1}, k)p(x_{t+1}, \omega_{t+1}, k|Y_t) =$$
$$p(y_{t+1}|x_{t+1})p(x_{t+1}|\omega_{t+1}, k, Y_t)p(\omega_{t+1}, k|Y_t) =$$
$$p(y_{t+1}|x_{t+1})p(x_{t+1}|x_t^k, \omega_{t+1})p(\omega_{t+1}|k, Y_t)p(k|Y_t) =$$
$$p(y_{t+1}|x_{t+1})p(x_{t+1}|x_t^k, \omega_{t+1})p(\omega_{t+1}|\omega_t^k)\pi_t^k \quad (5)$$

Approximating this expression by replacing x_{t+1} with the expected mean

$$\mu_{t+1}^k(\omega_{t+1}) = E(x_{t+1}|\omega_{t+1}, x_t^k)$$

in the first factor gives

$$p(x_{t+1}, \omega_{t+1}, k|Y_{t+1}) \approx$$
$$Cp(y_{t+1}|\mu_{t+1}^k(\omega_{t+1}))p(x_{t+1}|x_t^k, \omega_{t+1})p(\omega_{t+1}|\omega_t^k)\pi_t^k$$

Marginalization over x_{t+1} yields

$$p(\omega_{t+1}, k|Y_{t+1}) \approx Cp(y_{t+1}|\mu_{t+1}^k(\omega_{t+1}))p(\omega_{t+1}|\omega_t^k)\pi_t^k \quad (6)$$

where C is a normalization factor. Sampling from the density (5) can now be performed by resampling with replacement from the set $\{x_t^k\}_{k=1}^N$, where the index is chosen proportional to (6). The resampled candidates are then predicted using the system model. In summary, the algorithm is given by

Auxiliary Particle Filter for maneuvering target tracking (APF)

1. Set $t = 0$, and generate N samples $\{x_0^i\}_{i=1}^N$ from $p(x_0)$, set $\mu_t^k = x_t^k$, $k_j = j, j = 1, \ldots, N$.
2. Compute $\mu_{t+1}^k(\omega_{t+1}) = E\{x_{t+1}|x_t^k, \omega_{t+1}\}$ for every $\omega_{t+1} \in \mathcal{M}(x_t^k)$, where $\mathcal{M}(x_t^k)$ is the set of all feasible (state dependent) maneuvers. Number of particles after splitting: M.
3. Generate new indices k^j by sampling N times from $p(k|Y_{t+1}) \propto \pi_t^k p(y_{t+1}|\mu_{t+1}^k)p(\omega_{t+1}|\omega_t^k)$ and predict (simulate) the particles, i.e., $x_{t+1}^j = f(x_t^{k^j}, v_t), j =, \ldots, N$ with different noise realizations.
4. Compute the likelihood weights $\mathrm{w}_t^j = \frac{p(y_{t+1}|x_{t+1}^j)}{p(y_{t+1}|\mu_{t+1}^{k^j})}$ for $j = 1, \ldots, N$ and normalize, i.e., $\pi_t^j = \frac{\mathrm{w}_t^j}{\sum_{j=1}^M \mathrm{w}_t^j}$
5. Perform an optional resampling of the set $\{x_{t+1}^i\}_{i=1}^N$, using the probability weights. If resampling is chosen then reset $\pi_t^j = \frac{1}{N}, j =, \ldots, N$.
6. Increase t and iterate to item 2.

Implementing three different maneuvers for left turn, straight flying or right turn gives $M = 3N$ in the algorithm, which will only marginally increase the computational burden, since the Bayesian bootstrap method with/without auxiliary particle filters is linear in complexity $\mathcal{O}(M)$. This is because the resampling stage can be done in a fast way by using a classical algorithm for sampling M ordered independent identically distributed variables [4],[9].

5 Simulations

A simulation study using the nearly coordinated turn model from section 2 is performed. The maneuvering is done by setting up a flight path in accordance with the used Markov transition density. Simulations are performed by using the Auxiliary Particle Filter (APF) extended with the deterministic maneuver splitting discussed in section 4. A comparison to a traditional tracking method based on an IMM-filter consisting of three extended Kalman filters (EKF) with different turn assumptions is made. In the simulation study the only parameter constraint considered is a limitation on the target speed, to the interval $50 \leq |v| \leq 60$ m/s. The simulation step is re-run until a feasible speed is achieved. The distribution of the measurement noise is chosen to be Gaussian, with angular and distance standard deviations of 0.5^o respectively 20 m. The sampling period is chosen to $T = 4$ s to emulate a track-while-scan (TWS) behavior. For modern mono-pulse radars the update time and the angle standard deviation may be much smaller, but for non-monopulse system or for air-borne tracking systems much greater values should be used.

In Figure 2 one realization using Gaussian noise is viewed. The a posteriori probabilities for each coordinate is presented in Figure 3 for the predicted particles for one realization.

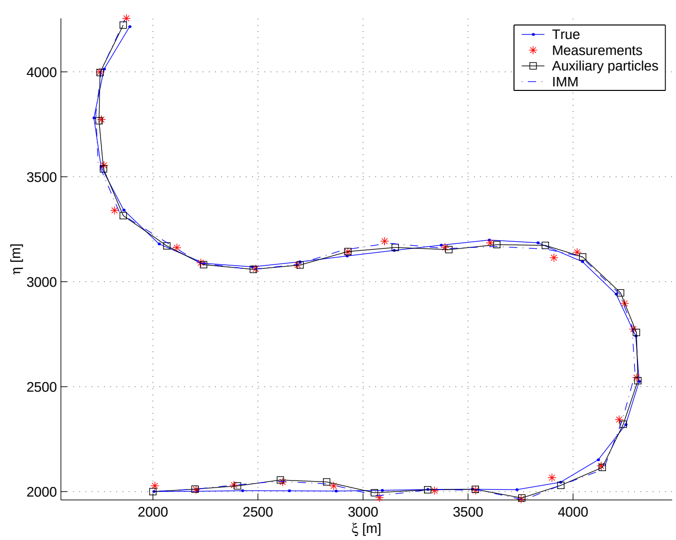

Figure 2: Simulation overview

In Table 1 the position *Root Mean Square Error*

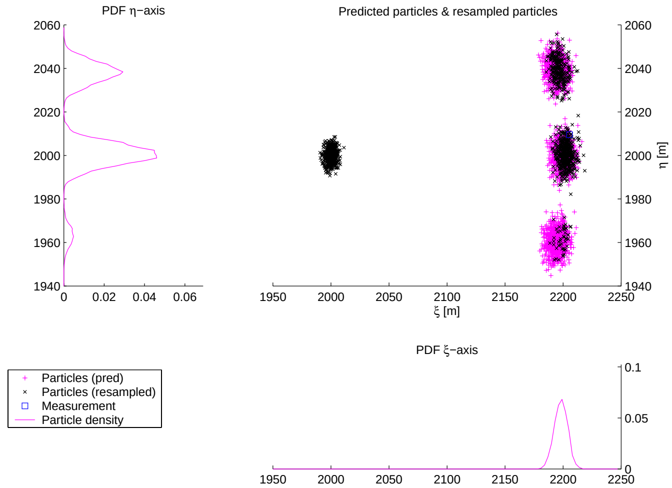

Figure 3: Auxiliary particles & posterior PDF

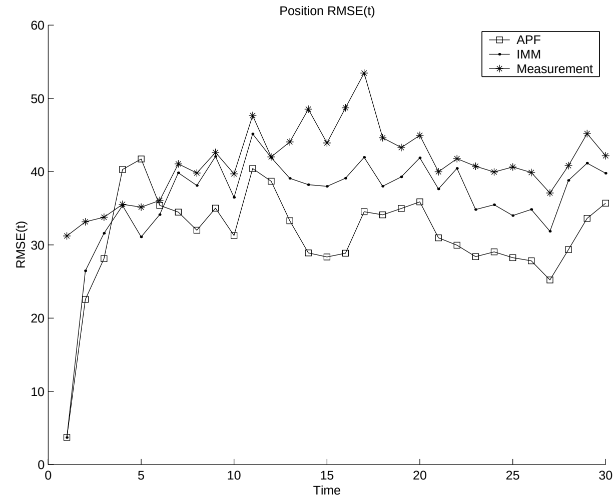

Figure 4: RMSE(t) for different tracking methods

(RMSE) for the Auxiliary Particle Filter (APF) with $N = 800$ particles is compared to the IMM-filter, using $N_{mc} = 100$ Monte Carlo simulations. The RMSE using measurements only is also presented in Table 1. The RMSE is defined by equation (7). As seen the Auxiliary Particle Filter method improves tracking performance.

$$\text{RMSE} = \sqrt{\frac{1}{L}\sum_{t=1}^{L}\frac{1}{N_{mc}}\sum_{i=1}^{N_{mc}}(\hat{\xi}_t^i - \xi_t^{\text{true}})^2 + (\hat{\eta}_t^i - \eta_t^{\text{true}})^2} \tag{7}$$

where $L = 30$ is the simulation path length and $\hat{\xi}_t^i, \hat{\eta}_t^i$, are the filter position estimates at time t in Monte Carlo run i.

	APF	IMM	Measurements
RMSE	32.09	37.05	41.5

Table 1: RMSE for 100 Monte Carlo simulations, using $N = 800$ particles

In Figure 4 the RMSE is presented for each time, i.e., according to equation (8) for the different methods

$$\text{RMSE(t)} = \sqrt{\frac{1}{N_{mc}}\sum_{i=1}^{N_{mc}}(\hat{\xi}_t^i - \xi_t^{\text{true}})^2 + (\hat{\eta}_t^i - \eta_t^{\text{true}})^2} \tag{8}$$

6 Conclusions

In the simulation study in section 5 the Auxiliary Particle Filter method improved tracking performance compared to the IMM-filter for the track-while-scan (TWS) Air Traffic Control (ATC) application. The particle filter method is also more flexible than traditional methods since it can also incorporate system constraints and non-Gaussian noise assumptions. However, simulation based methods such as particle filters can be time consuming if many particles are used. To improve the real-time execution performance the particle filter update could be run in parallel.

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