Adaptive Control based on a Parametric Affine Model for tail-controlled Missiles

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Abstract

This paper presents an adaptive control against uncertainties in tail-controlled STT (Skid-to-Turn) missiles. First, we derive an analytic uncertainty model from a parametric affine missile model developed by the authors. Based on this analytic model, an adaptive feedback linearizing control law accompanied by a sliding mode control law is proposed. We provide analyses of stability and output tracking performance of the overall adaptive missile system. The performance and validity of the proposed adaptive control scheme is demonstrated by simulation.

NOMENCLATURE

(X, Y, Z) : Missile body coordinate system

U, V, W : $X-, Y-, Z-$ components of the linear velocity vector

p, q, r : $X-, Y-, Z-$ components of the angular velocity vector

I_x, I_y, I_z : $X-, Y-, Z-$ components of the moment of inertia vector

I_{xy}, I_{yz}, I_{zx} : Products of inertia

α, β, ϕ_A : Angle of attack, sideslip angle, and bank angle

($\alpha = \tan^{-1}(W/U)$, $\beta = \tan^{-1}(V/U)$, $\phi_A = \tan^{-1}(V/W)$)

V_S , V_M : Velocity of sound and Total velocity of missile

M_m : Mach number $(= V_M / V_S)$

m , ρ , Q : Missile mass, air density, dynamics pressure

S , D :Aerodynamic reference area and length of the missile

$\delta_r (\delta_q)$: Deflection of Yaw (Pitch) control fin

$\delta_r^c (\delta_q^c)$: Yaw (Pitch) control fin command

$A_y (A_z)$: Yaw (Pitch) achieved acceleration

$A_{yc} (A_{zc})$: Yaw (Pitch) commanded acceleration

$\dot{x}$: Derivative of x with respect to time

sgn(x): signum function defined by

sgn(x)=+1 if $x > 0$, sgn(x)=-1 if $x < 0$

sat(x): saturation function defined by

sat(x)=sgn(x) if $|x| \geq 1$, otherwise sat(x)= x

I. INTRODUCTION

There have been researches on autopilot designs for missiles with highly nonlinear characteristics using nonlinear control techniques [1-11]. Feedback linearization, one of the well-known nonlinear control approaches, is basically based on the inversion of nominal known functions and requires accurate knowledge of system dynamics. In actual situations, uncertain terms inevitably exist in missile dynamics since aerodynamic coefficients are obtained through wind-tunnel experiments, and the resulting inversion error causes performance degradation. This, in turn, necessitates the compensation for the uncertainties in missile dynamics.

As a result, many existing results on various robust and adaptive schemes were proposed to account for this uncertainty model in feedback linearization approach [12-17], and also applied to autopilot design [5, 6, 8]. However, the existing adaptive techniques for nonlinear systems generally require a linear parameterization of the uncertainties of the plant dynamics. That is, parametric uncertainty is usually required to be expressed linearly in terms of a set of unknown parameters [12-16] although there are results where the uncertainties enter nonlinearly in the dynamic equation [17]. On the other hand, since an analytic uncertainty model for the missile is hard to obtain, uncertainties are usually assumed to take an intuitive form. Accordingly, the design of adaptive controllers for nonlinear missile systems [8, 18, 19] is not easy to apply compared with that of neural-adaptive controllers [20, 21]. As a solution for this, noting that the nonlinear and adaptive control approaches are relatively well developed for an affine model, we developed a new modeling approach for missile systems [22]. Since the parametric affine missile model has a linear parametric form, we can easily apply the existing adaptive control technique [23] and design a practical and realistic adaptive controller of missile systems.

We derive here a parametric affine uncertainty model of STT missiles. The uncertainty model can be divided into two parts: parametric and non-parametric parts. Each part is compensated by an adaptive feedback linearizing control and a sliding mode control method, respectively. In addition, it is shown through rigorous analysis that the proposed method can make the overall system achieve satisfactory performance even when uncertainties exist in dynamic equations of missiles. Simulation results also demonstrate that our scheme can effectively reduce the influences of uncertainties.

II. PARAMETRIC AFFINE UNCERTAINTY MODEL

The adaptive control theory for the nonlinear system is developed for relatively restrictive class of systems [24, 25]. It usually requires that: i) the uncertainty of the nonlinear plant dynamics is linearly parameterized; ii) the full state is

This research was supported by the Agency for Defense Development, the Automatic Control Research Center of Seoul National University, and the Brain Korea 21 Project.

measurable; and iii) the nonlinearities can be canceled without causing unstable internal dynamics by the control input if the parameters are known. For the above requirements, we derive an analytic form of parametric affine uncertainty model based on parametric affine missile dynamics.

First, as usually made in modeling STT missiles, we make the following assumptions.

Assumption 2.1: The variations of m, I_y, and I_z are negligible ($\dot{m} = \dot{I} = 0$).
Assumption 2.2: The missile has Y and Z symmetry ($I_y = I_z = I_M$, $I_{xy} = I_{yz} = I_{zx} = 0$).
Assumption 2.3: The missile is roll-stabilized ($p = 0$).
Assumption 2.4: $U \cong V_M = \text{constant}$.

Since we can say the pitch dynamics in the same way, we proceed here only with the yaw dynamics:

$$\begin{cases} \dot{\beta} = -r + \dfrac{QS}{Um} C_y(M_m, \beta, \delta_r, \phi_A) \\ \dot{r} = -QC_a(\phi_A, \beta) - \dfrac{QS(l_f - l_g)}{I_M} C_y(M_m, \beta, \delta_r, \phi_A) \\ A_y = \dfrac{QS}{m} C_y(M_m, \beta, \delta_r, \phi_A), \end{cases} \tag{1}$$

where C_a is an aerodynamic function obtained from aerodynamic coefficients C_y and C_n [11] as

$$C_n(M_m, \beta, \delta_r, \phi_A) = -\frac{I_M}{SD} C_a(\phi_A, \beta) - \frac{l_f - l_g}{D} C_y(M_m, \beta, \delta_r, \phi_A) \,. \tag{2}$$

The authors have proposed a function approximation technique for the above $C_y(M_m, \beta, \delta_r, \phi_A)$ and $C_a(\phi_A, \beta)$, which is based on the combination of local parametric models through curve fitting using the corresponding influence functions [22]. The approximated functions are given by

$$\hat{C}_y(M_m, \beta, \delta_r, \phi_A) = \theta_f^T \phi_f + \theta_g^T \phi_g \delta_r, \tag{3a}$$

$$\hat{C}_a(\phi_A, \beta) = \theta_h^T \phi_h, \tag{3b}$$

where

$\mu_i(M_m) = \mu_i^0(M_m) \Big/ \sum_{j=1}^{N} \mu_j^0(M_m)$,

$\mu_i^0(M_m) = \exp(-(M_m - M_i)^2 / \sigma_i^2)$, M_i is Mach index,

$\theta_f^T = [\theta_{f1}^T, \ \cdots, \ \theta_{fN}^T]$,

$\theta_{fi}^T = [\theta_{fi1} \ \ \theta_{fi2} \ \ \theta_{fi3} \ \ \theta_{fi4}] = [c_{i1}^{f1} \ \ c_{i1}^{f2} \ \ c_{i2}^{f1} \ \ c_{i2}^{f2}]$,

$\theta_g^T = [\theta_{g1}^T, \ \cdots, \ \theta_{gN}^T]$, $\theta_{gi}^T = [\theta_{gi1} \ \ \theta_{gi2}] = [c_{i3}^{f1} \ \ c_{i3}^{f2}]$,

$\theta_h^T = [\theta_{h1} \ \ \theta_{h2} \ \ \theta_{h3}] = [c_{a1} \ \ c_{a2} \ \ c_{a3}]$,

$\phi_f^T = [\phi_{f1}^T, \ \cdots, \ \phi_{fN}^T]$,

$\phi_{fi}^T = [\phi_{fi1} \ \ \phi_{fi2} \ \ \phi_{fi3} \ \ \phi_{fi4}]$
$\quad = \mu_i(M_m)[\beta \ \ \beta \sin^2(2\phi_A) \ \ \beta^3 \ \ \beta^3 \sin^2(2\phi_A)]$,

$\phi_g^T = [\phi_{g1}^T, \ \cdots, \ \phi_{gN}^T]$,

$\phi_{gi}^T = [\phi_{gi1} \ \ \phi_{gi2}] = \mu_i(M_m)[1 \ \ \sin^2(2\phi_A)]$,

$\phi_h^T = [\phi_{h1} \ \ \phi_{h2} \ \ \phi_{h3}] = [\beta \ \ |\phi_A|\beta \ \ \beta^3]$, $i = 1, \cdots, N$.

Then, we have

$$\begin{cases} \dot{\beta} = -r + \dfrac{QS}{Um}(\theta_f^T \phi_f + \theta_g^T \phi_g \delta_r + \Delta_y) \\ \dot{r} = -Q(\theta_h^T \phi_h + \Delta_a) - \dfrac{QS(l_f - l_g)}{I_M}(\theta_f^T \phi_f + \theta_g^T \phi_g \delta_r + \Delta_y) \\ A_y = \dfrac{QS}{m}(\theta_f^T \phi_f + \theta_g^T \phi_g \delta_r + \Delta_y), \end{cases} \tag{4}$$

where $\Delta_y := C_y - \hat{C}_y$ and $\Delta_a := C_a - \hat{C}_a$ are approximation errors.

For notational convenience, we introduce the variables

$$x = \beta \ , \ \delta = \delta_r \ , \ y = A_y \tag{5}$$

to have

$$\begin{cases} \dot{x} = -r + \theta_f^T \psi_f + \theta_g^T \psi_g \delta + \Delta_{xa} \\ \dot{r} = h_v(\theta_h^T \psi_h + \Delta_{ha}) - h_v U(\theta_f^T \psi_f + \theta_g^T \psi_g \delta + \Delta_{xa}) \\ y = U(\theta_f^T \psi_f + \theta_g^T \psi_g \delta + \Delta_{xa}), \end{cases} \tag{6}$$

where

$\psi_f^T = [\psi_{f1}, \ \cdots, \ \psi_{fN}]$,

$\psi_{fi} = [\psi_{fi1} \ \ \psi_{fi2} \ \ \psi_{fi3} \ \ \psi_{fi4}]$
$\quad = \dfrac{QS}{Um} \mu_i(M_m)[\beta \ \ \beta \sin^2(2\phi_A) \ \ \beta^3 \ \ \beta^3 \sin^2(2\phi_A)]$,

$\psi_g^T = [\psi_{g1}, \ \cdots, \ \psi_{gN}]$,

$\psi_{gi} = [\psi_{gi1} \ \ \psi_{gi2}] = \dfrac{QS}{Um} \mu_i(M_m)[1 \ \ \sin^2(2\phi_A)]$,

$\psi_h^T = [\psi_{h1} \ \ \psi_{h2} \ \ \psi_{h3}] = -\dfrac{Q}{h_v}[\beta \ \ |\phi_A|\beta \ \ \beta^3]$, $\Delta_{xa} = \dfrac{QS}{Um}\Delta_y$,

$\Delta_{ha} = -\dfrac{Q}{h_v}\Delta_a$, $h_v = \dfrac{(l_f - l_g)m}{I_M}$.

Letting θ_f^0, θ_g^0, θ_h^0, fitted values from aerodynamic look-up table, are known portion of θ_f, θ_g, θ_h, whereas ϑ_x, ϑ_y are unknown portion of θ_f, θ_h, respectively, we can express the dynamics (6) as

$$\begin{cases} \dot{x} = \hat{f}(\theta_f^0, x) + \hat{g}(\theta_g^0, x)\delta + \vartheta_x^T \psi_x + \Delta_x \\ \dot{r} = h_v(\hat{h}(\theta_h^0, x) + \vartheta_y^T \psi_y + \Delta_h) \\ \quad - h_v U\{r + \hat{f}(\theta_f^0, x) + \hat{g}(\theta_g^0, x)\delta + \vartheta_x^T \psi_x + \Delta_x\} \\ y = U\{r + \hat{f}(\theta_f^0, x) + \hat{g}(\theta_g^0, x)\delta + \vartheta_x^T \psi_x + \Delta_x\}, \end{cases} \tag{7}$$

where

$\hat{f}(\theta_f, x) = -r + \theta_f^{0T} \psi_f$, $\hat{g}(\theta_g, x) = \theta_g^{0T} \psi_g$, $\hat{h}^T(\theta_h, x) = \theta_h^{0T} \psi_h$,

$\vartheta_x^T = [\vartheta_{x1}^T, \ \cdots, \ \vartheta_{xN}^T]$, $\vartheta_{xi}^T = [\vartheta_{xi1} \ \ \vartheta_{xi2}]$,

$\vartheta_y^T = [\vartheta_{y1}^T, \ \cdots, \ \vartheta_{yN}^T]$, $\vartheta_{yi}^T = [\vartheta_{yi1} \ \ \vartheta_{yi2}]$,

$\psi_x^T = [\psi_{x1}^T, \ \cdots, \ \psi_{xN}^T]$,

$\psi_{xi}^T = [\psi_{xi1}^T \ \ \psi_{xi2}^T] = \dfrac{QS}{Um} \mu_i(M_m)[\beta \ \ \beta^3]$,

$\psi_y^T = [\psi_{y1}^T, \ \cdots, \ \psi_{yN}^T]$,

$\psi_{yi}^T = [\psi_{yi1}^T \ \ \psi_{yi2}^T] = \dfrac{QS}{m} \mu_i(M_m)[\beta \ \ \beta^3]$,

In addition, Δ_x and Δ_h are actually composed of non-parametric terms causing negligible influence on the performance and arising from fitting errors.

Remark 2.1: As simulation results showed that uncertainties in $C_y(M_m,\beta,\delta_r,\phi_A)$, not $C_n(M_m,\beta,\delta_r,\phi_A)$ cause noticeable performance degradation, we must consider $\Delta C_y(M_m,\beta,\delta_r,\phi_A)$ and $\Delta C_a(\phi_A,\beta)$. As the relation

$$\vartheta_x^T\psi_x = \frac{QS}{Um}\Delta C_y(M_m,\beta,\delta_r,\phi_A), \tag{8}$$

$$\vartheta_y^T\psi_y = -\frac{Q}{h_v}\Delta C_a(\phi_A,\beta) \tag{9}$$

and also

$$\Delta C_a(\phi_A,\beta) = -\frac{h_v S}{m}\Delta C_y(M_m,\beta,\delta_r,\phi_A) \tag{10}$$

holds due to eqn. (2), we can analyze the output uncertainty $\vartheta_y^T\psi_y$ in terms of dynamic uncertainty $\vartheta_x^T\psi_x$.

Once we choose the control input satisfying

$$\hat{f}(\theta_f^0,x) + \hat{g}(\theta_g^0,x)\delta + \vartheta_x^T\psi_x = \beta_{der.} \tag{11}$$

in order to make x - dynamics in (7) almost linear, then r - dynamics becomes

$$\dot{r} = h_v\{\hat{h}(\theta_h^0,x) + \vartheta_y^T\psi_y + \Delta_h\} - h_v U\{r + \beta_{der.} + \Delta_x\}. \tag{12}$$

This shows that r converges to a steady state value relatively quickly as $h_v U$ is physically a very large value. In other words, we can assume that $\dot{r}$ can be equated to zero and r is actually a steady state value as

$$\begin{aligned} r = &-\{\hat{f}(\theta_f^0,x) + \hat{g}(\theta_g^0,x)\delta + \vartheta_x^T\psi_x + \Delta_x\} \\ &+ \frac{1}{U}\{\hat{h}(\theta_h^0,x) + \vartheta_y^T\psi_y + \Delta_h\} \end{aligned} \tag{13}$$

as in [22]. Simplifying further the third row of (7), we can obtain the final form of the missile dynamics as

$$\begin{cases} \dot{x} = \hat{f}(\theta_f^0,x) + \hat{g}(\theta_g^0,x)\delta + \vartheta_x^T\psi_x + \Delta_x \\ y = \hat{h}(\theta_h^0,x) + \vartheta_y^T\psi_y + \Delta_h. \end{cases} \tag{14}$$

III. Adaptive Control against Uncertainties in Affine Missile Dynamics

In this section, we propose an adaptive control law against uncertainties in a parametric affine missile model.

The design objective of the control law is to make the output of the system described by (14) follow the output of the reference model

$$\ddot{y}_d + a_m\dot{y}_d + b_m y_d = b_m y_c, \tag{15a}$$

or in state-space form

$$\frac{d}{dt}\begin{pmatrix} y_d \\ u_d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -b_m & -a_m \end{pmatrix}\begin{pmatrix} y_d \\ u_d \end{pmatrix} + \begin{pmatrix} 0 \\ b_m \end{pmatrix} y_c, \tag{15b}$$

where $a_m, b_m > 0$ are design parameters for an appropriate reference model, and $y_c = A_{yc}$.

Here, we define $\varepsilon = [\varepsilon_y \quad \varepsilon_u]^T$, where $\varepsilon_y = y - y_d$, $\varepsilon_u = u - u_d$.

1. *Adaptive Control Law*

In this subsection, we present an adaptive control law against uncertainties in missile dynamics described by (14) under the following assumptions.

Assumption 3.1: $|\phi_A|$, Δ_h are differentiable and $\frac{d}{dt}|\phi_A|$, $\frac{d}{dt}\Delta_h$ are bounded.

Assumption 3.2: ϑ_{fx}, ϑ_{fy}, Δ_x, and Δ_h are bounded.

The adaptive control law is given by

$$\delta = \delta_{ad} + \delta_{sl} + \delta_{nd}, \tag{16}$$

where δ_{ad}, δ_{sl}, and δ_{nd} are given by (17), combined with the compensator given by (18), and the parameter adaptation law given by (19).

$$\begin{aligned} \delta_{ad} = &\left[\left(\frac{\partial\hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T\frac{\partial\psi_{fy}}{\partial x}\right)\hat{g}(\theta_g^0,x)\right]^{-1} \cdot \\ &\left[u - \left(\frac{\partial\hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T\frac{\partial\psi_{fy}}{\partial x}\right)\hat{f}(\theta_f^0,x) + \frac{\partial\hat{h}}{\partial x}\hat{\vartheta}_{fx}^T\psi_{fx} - k_d\varepsilon_y\right], \end{aligned} \tag{17a}$$

$$\delta_{sl} = -\left[\left(\frac{\partial\hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T\frac{\partial\psi_{fy}}{\partial x}\right)\hat{g}(\theta_g^0,x)\right]^{-1} \operatorname{sgn}(\varepsilon_y)\cdot\hat{D}, \tag{17b}$$

$$\begin{aligned} \delta_{nd} = &-\left[\left(\frac{\partial\hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T\frac{\partial\psi_{fy}}{\partial x}\right)\hat{g}(\theta_g^0,x)\right]^{-1} \cdot \\ &\left((\kappa_1+\kappa_3)\frac{\partial\psi_{fy}}{\partial x}^T\frac{\partial\psi_{fy}}{\partial x} + \kappa_2\psi_{fx}^T\psi_{fx}\right)\operatorname{sgn}(\varepsilon_y), \end{aligned} \tag{17c}$$

$$\dot{u} = -a_m u + b_m(y_c - y). \tag{18}$$

$$\dot{\hat{\vartheta}}_{fx} = \gamma_{fx}\psi_{fx}\frac{\partial\hat{h}}{\partial x}\varepsilon_y, \tag{19a}$$

$$\dot{\hat{\vartheta}}_{fy} = \gamma_{fy}\left(\frac{\partial\psi_{fy}}{\partial x}\{\hat{f}(\theta_f^0,x) + \hat{g}(\theta_g^0,x)\delta\}\right)\varepsilon_y, \tag{19b}$$

$$\dot{\hat{D}}_x = \gamma_x\left|\frac{\partial\hat{h}}{\partial x}\right|\cdot|\varepsilon_y|,\ \dot{\hat{D}}_h = \gamma_h|\varepsilon_y|,\ \dot{\hat{D}}_p = \gamma_p\left|\frac{\partial\hat{h}}{\partial|\phi_A|}\right|\cdot|\varepsilon_y|. \tag{19c}$$

Here, k_d, and κ_i, $i = 1,2,3$, are positive constants, $\hat{\vartheta}_{fx}, \hat{\vartheta}_{fy}$ are estimates of $\vartheta_{fx}, \vartheta_{fy}$, $\hat{D} = \left|\frac{\partial\hat{h}}{\partial x}\right|\hat{D}_x + \hat{D}_h + \left|\frac{\partial\hat{h}}{\partial|\phi_A|}\right|\hat{D}_p$, and $\frac{\partial\hat{h}}{\partial|\phi_A|} = -\frac{Q}{h_v}c_{a2}\beta$. $\hat{D}_x$, $\hat{D}_h$, $\hat{D}_p$ are estimates of D_x, D_h, D_p, which are the upper bounds of Δ_x, $\frac{d}{dt}\Delta_h + \frac{1}{16\kappa_1}\vartheta_{fy}^T\vartheta_{fy} + \frac{1}{16\kappa_2}\vartheta_{fx}^T\vartheta_{fx} + \frac{1}{16\kappa_3}(\vartheta_{fy}\Delta_x)^T(\vartheta_{fy}\Delta_x)$,

$\frac{d}{dt}|\phi_A|$, respectively. $\gamma_{fx}, \gamma_{fy} \in R^{2N\times 2N}$, $\gamma_x \in R$, $\gamma_h \in R$, $\gamma_p \in R$.

Here, we define $\tilde{\vartheta}_{fx} = \vartheta_{fx} - \hat{\vartheta}_{fx}$, $\tilde{\vartheta}_{fy} = \vartheta_{fy} - \hat{\vartheta}_{fy}$, $\tilde{D}_x = D_x - \hat{D}_x$, $\tilde{D}_h = D_h - \hat{D}_h$, $\tilde{D}_p = D_p - \hat{D}_p$.

The stability of the overall adaptive missile system and the asymptotic convergence of the output tracking error is provided in the following theorem.

Theorem 3.2 (Adaptive control law):
The missile system described by (14) with the adaptive control law (16), combined with the compensator (18) and the parameter adaptation law (19) under Assumption 3.1 and 3.2 is stable in the sense that

1. $\tilde{\vartheta}_{fx}, \tilde{\vartheta}_{fy}, \hat{\vartheta}_{fx}, \hat{\vartheta}_{fy}, \tilde{D}_x, \tilde{D}_h, \tilde{D}_p, \hat{D}_x, \hat{D}_h, \hat{D}_p \in L_\infty$
2. $\varepsilon, \varepsilon_y, \varepsilon_u, \dot{\hat{\vartheta}}_{fx}, \dot{\hat{\vartheta}}_{fy}, \dot{\hat{D}}_x, \dot{\hat{D}}_h, \dot{\hat{D}}_p \in L_\infty \cap L_2$
3. $\dot{\varepsilon}, \dot{\varepsilon}_y, \dot{\varepsilon}_u \in L_\infty$
4. ε, ε_y, *and* ε_u *converges to zero asymptotically.*
5. $\dot{\hat{\vartheta}}_{fx}$, $\dot{\hat{\vartheta}}_{fy}$, $\dot{\hat{D}}_x$, $\dot{\hat{D}}_h$, *and* $\dot{\hat{D}}_p$ *converges to zero asymptotically.*

Proof: omitted here.

2. *Adaptive Control Law with Deadzone*

As parameter estimation errors are not guaranteed to be bounded only with adaptive control law in the presence of parameter variations and disturbances, we also employed a sliding mode controller in the previous subsection. The switching term, however, are not desirable for practical application, as it can cause chattering phenomenon. Accordingly, a saturation function is usually used instead of a switching function. However, the sliding surface, $\varepsilon_y = 0$, is not exactly guaranteed by just replacing a signum function with a saturation function, and furthermore $\hat{D}_x, \hat{D}_h, \hat{D}_p$ can grow always with time due to $\varepsilon_y \neq 0$. In this subsection, the above issue is solved using the adaptive control law with deadzone.

The deadzoned tracking error is defined as

$$\varepsilon_w = \varepsilon_y - d_w sat(\varepsilon_y / d_w), \tag{20}$$

where d_w is the width of deadzone and chosen here as a constant.

The modified adaptive control law is given by

$$\delta = \delta_{ad} + \delta_{sl} + \delta_{nd}, \tag{21}$$

where δ_{ad}, δ_{sl}, and δ_{nd} are given by (22), combined with the compensator given by (18), and the parameter adaptation law given by (23).

$$\delta_{ad} = \left[\left(\frac{\partial \hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T \frac{\partial \psi_{fy}}{\partial x}\right)\hat{g}(\theta_g^0, x)\right]^{-1} \cdot \left[u - \left(\frac{\partial \hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T \frac{\partial \psi_{fy}}{\partial x}\right)\hat{f}(\theta_f^0, x) + \frac{\partial \hat{h}}{\partial x}\hat{\vartheta}_{fx}^T \psi_{fx} - k_d \varepsilon_y\right], \tag{22a}$$

$$\delta_{sl} = -\left[\left(\frac{\partial \hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T \frac{\partial \psi_{fy}}{\partial x}\right)\hat{g}(\theta_g^0, x)\right]^{-1} sat(\varepsilon_y / d_w) \cdot \hat{D}, \tag{22b}$$

$$\delta_{nd} = -\left[\left(\frac{\partial \hat{h}}{\partial x} + \hat{\vartheta}_{fy}^T \frac{\partial \psi_{fy}}{\partial x}\right)\hat{g}(\theta_g^0, x)\right]^{-1} \cdot \left((\kappa_1 + \kappa_3)\frac{\partial \psi_{fy}}{\partial x}^T \frac{\partial \psi_{fy}}{\partial x} + \kappa_2 \psi_{fx}^T \psi_{fx}\right) sat(\varepsilon_y / d_w), \tag{22c}$$

$$\dot{\hat{\vartheta}}_{fx} = \gamma_{fx} \psi_{fx} \frac{\partial \hat{h}}{\partial x} \varepsilon_w, \tag{23a}$$

$$\dot{\hat{\vartheta}}_{fy} = \gamma_{fy}\left(\frac{\partial \psi_{fy}}{\partial x}\{\hat{f}(\theta_f^0, x) + \hat{g}(\theta_g^0, x)\delta\}\right)\varepsilon_w, \tag{23b}$$

$$\dot{\hat{D}}_x = \gamma_x \left|\frac{\partial \hat{h}}{\partial x}\right| \cdot |\varepsilon_w|, \ \dot{\hat{D}}_h = \gamma_h |\varepsilon_w|, \ \dot{\hat{D}}_p = \gamma_p \left|\frac{\partial \hat{h}}{\partial |\phi_A|}\right| \cdot |\varepsilon_w|. \tag{23c}$$

Here, we define $\varepsilon_d = [\varepsilon_w \ \ \varepsilon_u]^T$ and also introduce the following definition.

Definition 3.1 (μ-small in the mean square sense (m.s.s.)) [23]:
Let $x : [0,\infty) \mapsto R^n$, *where* $x \in L_{2e}$, *and consider the set*

$$S(\mu) = \left\{x : [0,\infty) \mapsto R^n \mid \int_t^{t+T} x^T(\tau)x(\tau)d\tau \le c_0 \mu T + c_1, \ \ \forall t, T \ge 0\right\}$$

for a given constant $\mu \ge 0$, where $c_0, c_1 \ge 0$ are some finite constants, and c_0 is independent of μ. We say that x is μ-small in the m.s.s. if $x \in S(\mu)$.

The stability and the output tracking performance for the adaptive control law with deadzone is shown in the following theorem.

Theorem 3.3 (Adaptive control law with deadzone):
The missile system described by (14) with the adaptive control law (21), combined with the compensator (18) and the parameter adaptation law (23) under Assumption 3.1 and 3.2 is stable in the sense that

1. $\tilde{\vartheta}_{fx}, \tilde{\vartheta}_{fy}, \hat{\vartheta}_{fx}, \hat{\vartheta}_{fy}, \tilde{D}_x, \tilde{D}_h, \tilde{D}_p, \hat{D}_x, \hat{D}_h, \hat{D}_p \in L_\infty$
2. $\varepsilon_w, \dot{\hat{\vartheta}}_{fx}, \dot{\hat{\vartheta}}_{fy}, \dot{\hat{D}}_x, \dot{\hat{D}}_h, \dot{\hat{D}}_p \in L_\infty \cap L_2$
3. $\dot{\varepsilon}_d, \dot{\varepsilon}_w, \dot{\varepsilon}_u \in L_\infty$
4. ε_w, $\dot{\hat{\vartheta}}_{fx}$, $\dot{\hat{\vartheta}}_{fy}$, $\dot{\hat{D}}_x$, $\dot{\hat{D}}_h$, *and* $\dot{\hat{D}}_p$ *converge to zero asymptotically.*
5. $\lim_{t\to\infty} |\varepsilon_y(t)| \le d_w$
6. $\varepsilon_d, \varepsilon_u \in L_\infty \cap S((b_m d_w / a_m)^2)$
7. *In addition, when* $d_w = 0$, *6 can be replaced by*
   6′. $\varepsilon_d, \varepsilon_u \in L_\infty \cap L_2$
   and also ε_d *and* ε_u *converge to zero asymptotically.*

Proof: omitted here.

IV. Simulation Results

This section presents the simulation results for the adaptive controller. The performance degradation due to uncertainties and the recovered performance by the adaptive controller are shown through simulations. We employ an adaptive control law given by (21) with the compensator (18) and the parameter adaptation law (23) for each yaw and pitch dynamics.

1. Uncertainty Model

We assume that uncertainties exist in $C_y(M_m,\beta,\delta_r,\phi_A)$ and $C_z(M_m,\alpha,\delta_q,\phi_A)$ as

$$C_y(M_m,\beta,\delta_r,\phi_A)=C_{yn}(M_m,\beta,\delta_r,\phi_A)+(\frac{QS}{Um})^{-1}\vartheta_{fxy}^T\psi_{fx},$$

$$C_z(M_m,\alpha,\delta_q,\phi_A)=C_{zn}(M_m,\alpha,\delta_q,\phi_A)+(\frac{QS}{Um})^{-1}\vartheta_{fxz}^T\psi_{fx},$$

where $\vartheta_{fxy},\vartheta_{fxz}$ have the same form as ϑ_{fx}.

- Uncertainty Model 1:

$\vartheta_{fxyi}^T=\vartheta_{fxzi}^T=[4 \quad 10]$; $i=1,\cdots,3$, $\vartheta_{fxy5}^T=\vartheta_{fxz5}^T=[4 \quad 15]$,

$\vartheta_{fxy6}^T=\vartheta_{fxz6}^T=[3 \quad 10]$. (24a)

- Uncertainty Model 2:

$\vartheta_{fxy1}^T=\vartheta_{fxy2}^T=[4 \quad 60]$, $\vartheta_{fxy3}^T=[4 \quad 70]$, $\vartheta_{fxy4}^T=[4 \quad 80]$,

$\vartheta_{fxy5}^T=[4 \quad 100]$, $\vartheta_{fxy6}^T=[3 \quad 100]$

$\vartheta_{fxz1}^T=\vartheta_{fxz2}^T=[4 \quad 40]$, $\vartheta_{fxz3}^T=[4 \quad 50]$,

$\vartheta_{fxz4}^T=[4 \quad 60]$, $\vartheta_{fxz5}^T=[4 \quad 70]$, $\vartheta_{fxz6}^T=[3 \quad 70]$. (24b)

Here, Mach index number is $N=6$.

2. Simulation Environments and Design parameters

The full six-degree-of-freedom nonlinear equations have been simulated using the aerodynamic look-up tables. Although we observed that the performance can be improved for time-varying forward velority, we evaluate here the performance for tracking square wave commands with the velocity $U=884\,m/\text{sec}$ for more brief and clear comparison. The actual performance of the adaptive controller depends on several design parameters. Here, considering the adaptation speed and the transient tracking error, design parameters in the adaptive control law are chosen, respectively, as follows:

The gain k_d of the compensation term $k_d e_d$ in (22a) is $k_d=0.1$. $\gamma_{fx},\gamma_{fy},\gamma_x,\gamma_h,\gamma_p$ in (23) are, respectively, $\gamma_{fx}=diag(0.0225,0.45,\cdots,0.0225,0.45)$, $\gamma_{fy}=diag(0.00225,$ $0.045,\cdots,0.00225,0.045)$, $\gamma_x=10^{-8}$, $\gamma_h=10^{-2}$, $\gamma_p=10^{-5}$. The width of deadzone is chosen as $d_w=0.1$. Design parameters for a compensator in (18) are selected as $a_m=2\xi\omega_n$, $b_m=\omega_n^2$, where $\xi=0.7$, $\omega_n=15$. As an actuator model, we included the following low pass filter $\tau\dot{\delta}_r=-\delta_r+\delta_r^c$, $\tau\dot{\delta}_q=-\delta_q+\delta_q^c$ for each yaw and pitch channel, where the time constant $\tau=0.01\,\text{sec}$.

3. Performance of an adaptive controller

In this subsection, we evaluate the performance of the adaptive controller. The performance of a non-adaptive controller and an adaptive controller for uncertainty model 1 and 2 are shown in Fig. 1 (a) and (b), respectively. Here, we can see the influence of the uncertainty model and the performance of an adaptive controller, by which the tracking performance improves with adaptation. In addition, the evolution of the estimated parameters of uncertainties with time is given in Fig. 2 and 3 for uncertainty model 1 and 2, respectively. From the results of this section, we can see the validity of the proposed adaptive controller for a parameterized affine missile model.

V. Conclusion

We proposed here an adaptive control method to improve the performance of STT missiles against uncertainties. Due to its peculiar structure, a parametric affine missile model simplifies the analytic derivation of an uncertainty model. The simulation results as well as the stability and performance analysis confirm that adaptive control can be effectively used to improve the tracking performance in the presence of uncertainties. This is a desirable feature from both theoretical and practical viewpoints. This paper has a contribution in that the proposed adaptive control methods can be easily and practically applied to the missile system.

References

[1] Tahk, M., Briggs, M., and Menon, P.K.A., "Applications of Plant Inversion via State Feedback to Missile Autopilot Design," *Conference on Decision and Control*, Austin, Texas, pp. 730-735, Dec. 1988.

[2] Lian, K.-Y., Fu L.-C., Chuang, D.-M., and Kuo, T.-S., "Nonlinear Autopilot and Guidance for a Highly Maneuverable Missile," *Proceedings of the American Control Conference*, Baltimore, Maryland, pp. 2293-2297, Jun. 1994.

[3] Hull, R.A., Schumacher, D., and Qu, Z., "Design and Evaluation of Robust Nonlinear Missile Autopilots from a Performance Perspective," *Proc. of the American Control Conference*, Seattle, Washington, pp. 189-193, Jun. 1995.

[4] Hull, R.A., and Qu, Z., "Dynamic Robust Recursive Control Design and Its Application to a Nonlinear Missile Autopilot," *Proc. of the American Control Conference*, Albuquerque, New Mexico, pp. 833-837, Jun. 1997.

[5] Huang, J., and Lin, C.F., "Sliding Mode Control of HAVE DASH II Missile Systems," *Proceedings of American Control Conference*, San Francisco, California, pp. 183-187, Jun. 1993.

[6] Huang, J., Lin, C.F., Cloutier, J.R., Evers, J.H., and D'Souza, C., "Robust feedback linearization approach to autopilot design," *Proc. IEEE Conference on Control Applications*, Vol. 1, pp. 220-225, 1992.

[7] Romano, J.J., and Singh, S.N., "I-O map inversion, zero dynamics and flight control," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 26, No. 6, pp. 1022-1028, 1990.

[8] Singh, S.N., and Steinberg, M., "Adaptive Control of Feedback Linearizable Nonlinear Systems with Application to

Flight Control," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 4, pp. 871-877, 1996.

[9] Cheng, V.H.L., Njaka, C.E., and Menon, P.K., "Practical design methodologies for robust nonlinear flight control," *AIAA, Guidance Navigation and Control Conference,* 96-3785, San Diego, CA, Jul. 1996.

[10] Lee, J.I., Oh, J.H., Ha, I.J., Kim, E.G., and Cho, H.J., "A New Approach to Autopilot Design for Highly Nonlinear Missiles," *AIAA, Guidance Navigation and Control Conference,* 96-3915, San Diego, CA, Jul. 1996.

[11] Oh, J.H. and Ha, I.J., "Missile Autopilot Design Via Functional Inversion and Time-Scaled Transformation," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 33, No. 1, pp. 64-76, 1997.

[12] K. Nam and A. Arapostathis, "A model-reference adaptive control scheme for pure-feedback nonlinear systems, " *IEEE Trans. On Automatic Control*, vol.33, 1988, pp 803-811.

[13] D. Taylor and P.V. Kokotovic and R. Marino and I. Kanellakopoulos, "Adaptive regulation of nonlinear systems with unmodeled dynamics," *IEEE Trans. On Automatic Control*, vol.34, 1989, pp 405-412.

[14] S.S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. On Automatic Control*, vol.34, 1989, pp 1123-1131.

[15] I. Kanellakopoulos and P.V. Kokotovic and A.S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Trans. On Automatic Control*, vol.36, 1991, pp 1241-1253.

[16] I. Kanellakopoulos and P.V. Kokotovic and R. Marino, "An extended direct scheme for robust adaptive nonlinear control, " *Automatica*, vol.27, 1991, pp 247-255.

[17] R. Marino and P. Tomei, "Robust stabilization of feedback linearizable time-varying uncertain nonlinear systems," *Automatica*, vol.29, 1993, pp 181-189.

[18] E.W. Kamen, T.E. Bullock, and C.H. Song, "Adaptive control applied to missile autopilots," *Proc. Ameri. Contr. Conf.*, 1988, pp. 555-560.

[19] C.F. Lin and J.R. Cloutier *et al.*, "High performance, adaptive, robust bank-to-turn missile autopilot deign," *Proc. AIAA Guidance, Navigation, Control Conference*, vol.3, 1991, pp. 123-137.

[20] D.M. McDowell, G.W. Irwin, G. Lightbody, and G. McConnell, "Hybrid Neural Adaptive Control for Bank-to-Turn Missiles," *IEEE Trans. On Control Systems Technology*, vol.5, 1997, pp.297-308.

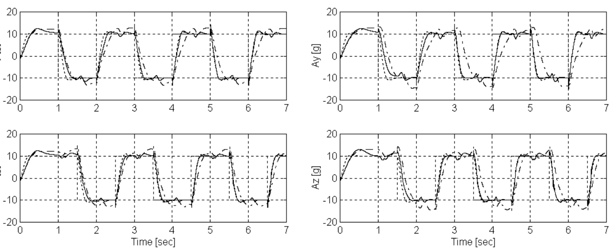
[21] M.B. McFarland and A.J. Calise, "Neural Networks for Stable Adaptive Control of Air-to-Air Missiles," Proceedings of the AIAA Guidance, Navigation, and Control Conference, Baltimore, MD, 1995, pp. 1280-1285.

[22] D. Chwa and J.Y. Choi, "New Parametric Affine Modeling and Control for Skid-to-Turn Missiles," *Proc. Ameri. Contr. Conf.*, Chicago, Illinois, USA, Jun. 2000, pp.1339-1343.

[23] P.A. Ioannou and J. Sun, *Stable and Robust Adaptive Control*, Englewood Cliffs, NJ: Prentice-Hall, 1995.

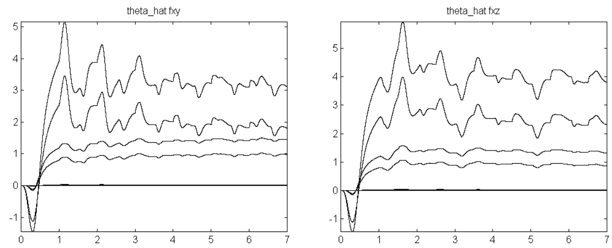
[24] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design,* John Wiley & Sons, Inc., 1995.

[25] J-J. E. Slotine and W. Li, *Applied Nonlinear Control,* Prentice Hall, 1991.

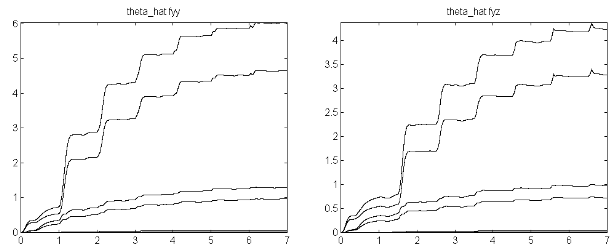


(a) (b)

Fig. 1. The performance of a non-adaptive and adaptive controller for the uncertainty model. (a) Uncertainty model 1 (b) Uncertainty model 2 (solid: adaptive control, dash-dotted: non-adaptive control, dotted: reference model)

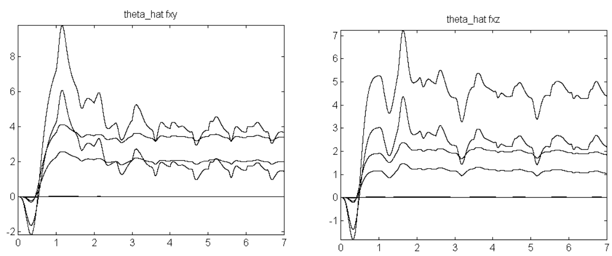


(a) $\hat{\vartheta}_{fxy}$ $(=\hat{\vartheta}_{fx}$; yaw) (b) $\hat{\vartheta}_{fxz}$ $(=\hat{\vartheta}_{fx}$; pitch)

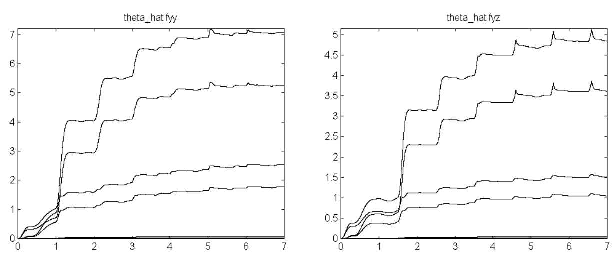


(c) $\hat{\vartheta}_{fy}$ (yaw) (d) $\hat{\vartheta}_{fy}$ (pitch)

Fig. 2. The estimated parameters for Uncertainty Model 1



(a) $\hat{\vartheta}_{fxy}$ $(=\hat{\vartheta}_{fx}$; yaw) (b) $\hat{\vartheta}_{fxz}$ $(=\hat{\vartheta}_{fx}$; pitch)



(c) $\hat{\vartheta}_{fy}$ (yaw) (d) $\hat{\vartheta}_{fy}$ (pitch)

Fig. 3. The estimated parameters for Uncertainty Model 2