Automatic Tuning of Smith-predictor Design Using Optimal Parameter Mismatch

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Abstract

The Smith-predictor is well known for its delay-free characteristic and suitable for regulating systems with an excessively long time delay. Previous studies have found that by introducing appropriate parametric or temporal mismatch can significantly improve system performance. In this paper, the previous theoretical results are summarized to form an automatic tuning procedure based on optimal values obtained from closed form integration solutions. The proposed procedure is verified experimentally on a fluid temperature control system of Stanford's quiet hydraulic precision lathe. Test results show good practical feasibility and deserve more real world applications.

Introduction

The Smith-predictor proposed by O. J. M. Smith [1] has been used and cited frequently for solving long time-delay problems. Figure 1 shows the control scheme of the Smith-predictor. Its transfer function can be written as,

$$\frac{Y(s)}{X(s)}=\frac{C(s)G_p(s)e^{-sT_p}}{1+C(s)G_q(s)+C(s)\left[G_p(s)e^{-sT_p}-G_q(s)e^{-sT_q}\right]} \quad (1)$$

If the model parameters match the real plant, i.e. $G_p(s)=G_q(s)$ and $T_p=T_q$, the above transfer function can be simplified to

$$\frac{Y(s)}{X(s)}=\frac{C(s)G_p(s)}{1+C(s)G_p(s)}e^{-sT_p} \quad (2)$$

This simplification excludes the time-delay effect from the control loop, and converts the corresponding control design to a delay free problem. Due to this attractive delay-free feature, the Smith-predictor is famous for compensating systems with long time-delay.

Despite the controversial issue that Smith-predictor is sensitivity to its plant parameter variations based on some frequency domain studies [2-3], Marshall and his coworkers have investigated the problem theoretically by studying system characteristic equations and closed form solutions of performance index integration [5-7]. Marshall's studies have pointed out the possibility that system performance of a Smith-predictor control system could be improved by introducing appropriate parameter mismatch.

Huang and DeBra [8-10] have further extended Marshall's work, formed a complete solution, and tested successfully on systems with slow dynamics and an excessively long time delay.

This paper summarizes the previous results and proposes an automatic optimal Smith-predictor tuning procedure to facilitate the use of the elaborate theoretical works. The proposed tuning procedure is also verified on the temperature control system of Stanford's quiet hydraulic precision lathe to demonstrate its feasibility.

Analysis Method

The analytical method uses the infinite time integration of a quadratic error function as system performance index and the Parseval's theorem to carry out the integration in complex variable domain. The Parseval's theorem is expressed as

$$J=\int_0^{\infty}e^2(t)dt=\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty}E(s)E(-s)ds \quad (3)$$

For more involved integration procedures can be found in [6-7].

Parameter Normalization Processes

Before evaluating the integration, there are two parameter normalization procedures that have to be carried out. The first one is to normalize the parameter values of the original system relative to a base value. A convenient choice is the delay time of plant.

Assume the time-delay system discussed is a first order system plus a long dead time, and is regulated by a PI controller. Their transfer function can be expressed as

$$C(s)=k_p\left(1+\frac{1}{T_i s}\right) \quad (4.a)$$

$$G_p(s) = \frac{k}{as+1} e^{-sT} \quad (4.b)$$

where k_p is the proportional gain, T_i is the integration time constant of the PI regulator, k is the DC gain, a is the time constant, and T is the delay time of plant transfer function. Normalizing the parameters with respect to plant time-delay obtains the corresponding time-scaled controller and plant transfer function shown below,

$$\tilde{C}(\tilde{s}) = K_c \left(\frac{s+q_c}{s} \right) \quad (5.a)$$

$$\tilde{G}_p(\tilde{s}) = \frac{K_p}{\tilde{s}+p} e^{-\tilde{s}\tau} \quad (5.b)$$

where the symbol, ~, represents the time scaled Laplace operator, and $\tau = 1$. The corresponding transformation equations are written as

$$\tilde{s} = sT \quad (6.a)$$

$$K_c = k_p \quad (6.b)$$

$$p = \frac{T}{a} \quad (6.c)$$

$$q_c = \left(\frac{T}{T_i} \right) \quad (6.d)$$

$$K_p = k \cdot \left(\frac{T}{a} \right) \quad (6.e)$$

$$K = K_c K_p \quad (6.f)$$

The corresponding system control block diagram after parameter normalization process is shown in Figure 2.

The other parameter normalization procedure is to extrude the quiet period due to time-delay from the infinite time integration, and multiply the performance index by the normalized gain, K, to eliminate the gain factor effect from the performance index. The corresponding performance is expressed as,

$$KJ_{sp} = KJ - K\tau \quad (7)$$

where the subscript *sp* represents for system controlled by Smith-predictor. The value of $K\tau$ is an importance index for determining the range of allowable parameter mismatch.

Performance Enhancement Using Parameter Mismatch

According to the block diagram shown in Figure 2, there are three time-normalized parameters in the Smith-predictor: the dc gain, K_q, the pole location, $-q$, and the delay time, τ_q. Detailed derivations and performance index integration results for different parameter mismatch can be found in [9-10]. The optimal mismatch values are therefore can be found by numerical search by use of the derived integration closed form solutions. The following sections will show only the resulting characteristic curves of individual parameter mismatched case, and the curve-fitted result of the optimal mismatch values.

Model pole location mismatch

Assume there is only a pole location mismatch in the Smith-predictor, the curves of normalized performance index for different $K\tau$ values are shown in Figure 3. Figure 4 shows the numerical search result of the optimal pole location. The plus symbols represent the numerically searched optimal values, while the solid line is the result generated by a curve-fitted empirical equation. The empirical equation is found as,

$$\left. \frac{q}{p} \right|_{opt} = x|_{opt} = \frac{2.136 \cdot K\tau + 1.0}{2.226 \cdot K\tau + 0.1} \quad (8)$$

The subscripts ***opt*** shown in Equation (8) represent the optimal values.

DC gain mismatch

Assume there is only a DC gain mismatch between the model and plant, the curves of normalized performance index at different $K\tau$ values are shown in Figure 5. Figure 6 shows numerical search results of the optimal DC gain mismatch. The curve-fitted empirical equation is found as,

$$\left. \frac{K_q}{K_p} \right|_{opt} = y|_{opt} = \frac{1.688 \cdot K\tau - 1.00}{1.566 \cdot K\tau - 0.05} \quad (9)$$

Temporal mismatch

When there is only a temporal mismatch in the Smith-predictor, the characteristic curves of normalized performance index at different $K\tau$ values are shown in Figure 7. Follow the previous approach, the optimal temporal ratios can be found numerically, and the optimal values are curve fitted by the following empirical equation.

$$\left. \frac{T_q}{T_p} \right|_{opt} = r|_{opt} = \frac{1.387 \cdot K\tau - 1.000}{1.135 \cdot K\tau - 0.358} \text{, for } K\tau > 0.74 \quad (10.a)$$

$$\left. \frac{T_q}{T_p} \right|_{opt} = r|_{opt} = 0 \text{, for } K\tau \le 0.74 \quad (10.b)$$

Figure 8 shows the comparison between the curve-fitted result and the optimal temporal data points. The reason why Equation (10) is separated into two parts is due to there is no r<0 in reality.

Plant Parameter Estimation

Before exploiting the empirical equations, plant parameters have to be estimated. The combination of open loop step response and relay excitation feedback methods are chosen for plant parameter estimation. The step response is used to determine the static gain (k) of the plant, while the relay excitation method is used to determine the plant parameters. Detailed analyses with regard to the use of relay feedback control are given in [11-12]. The ultimate gain of process is approximated as

$$K_u \cong \frac{4d}{\pi e} \quad (11)$$

where d is the relay output, and e is the error. By finding the process ultimate oscillation period (T_u) from the registered oscillation data, the process parameters can be derived as,

$$a = \frac{T_u}{2\pi}\sqrt{(kK_u)^2 - 1} \quad (12.a)$$

$$T = \frac{T_u}{2\pi}\left(\pi - \tan^{-1}\left(\frac{2\pi a}{T_u}\right)\right) \quad (12.b)$$

Automatic Smith-predictor Design Procedure

Summarizing the above results, an automatic Smith-predictor design procedure that uses the numerically searched optimal mismatch values could be proposed as follows:

1. Process model identification:
   - 1.1 Change the set point level and register the steady state open loop output response. Find the DC gain (k) of the process.
   - 1.2 Switch to relay control, register the oscillation data, and determine the resulting ultimate gain (K_u) and ultimate period (T_u).
   - 1.3 Based on the registered oscillation data, find the estimated process time constant a, and delay-time T by use of Equation (12).

2. $K\tau$ value determination:
   - 2.1 Use the proportional control only, and tune the gain to 0.5 K_u as suggested by Ziegler-Nichols [4].
   - 2.2 Normalize process parameters according to Equation (6) and determine the corresponding $K\tau$ value.

3. Optimal Smith-predictor tuning design:
   - 3.1 Based on the resulting $K\tau$ value, determine the appropriate control construction, and tune up system performance by introducing the optimal parameter mismatch of the Smith-predictor according to Equation (8), (9), or (10).
   - 3.2 Test step response of the process, wait for the output to reach steady state, and evaluate the performance index.
   - 3.3 Repeat step 3.2 and change the mismatched parameter in turn until the optimal Smith-predictor tuning is found.

Block diagram that shows the tuning procedure is shown in Figure 11. During the test, a period of wait time has to be added between each step to make sure that the process has smoothly migrated from one state to another steady state.

Experimental test plant – Stanford Quiet Hydraulic Precision Lathe

The Stanford Quiet Hydraulic (QH) precision lathe is used to test the theoretical parameter mismatch results and the automatic tuning procedure. Its simplified functional block diagram is shown in Figure 9.

The machine is mainly composed of three parts: the temperature control system, the pumping system, and the actuation system. The regulation of heat exchanger outlet temperature is defined as inner loop, and the regulation of shower point temperature at the actuation location is defined as outer loop. The estimated process parameter values of the temperature control system are found in Table 1.

PI regulator is suitable for processes with excessive long dead time as discussed in [11]. The controller for the temperature control system of QH precision lathe is therefore chosen as PI regulator. This choice is also consistent with the previous theoretical derivations, and is decent for testing the optimal tuning of parameter mismatched Smith-predictor. The block diagram of the overall implementation of the temperature control system is shown in Figure 10. For more detailed system description and servo loop control implementation please refer to [8-9].

Experimental results and discussion

Some time responses are shown in Figure 12 and 13. Figure 12 shows the inner loop responses by introducing DC gain mismatch. The results demonstrate that by intentionally tune down the value of model DC gain can appropriately improve system response. Figure 13 shows the outer loop time responses by introducing temporal mismatch. The responses demonstrate that the system performance can be improved by incorporating temporal mismatch due to the process has a lower value of $K\tau$. More informative comparisons of the theoretical and experimental results can be found in [9-10].

Figure 14 shows the test process of the proposed automatic tuning procedure applied to the inner loop temperature control system. It illustrates the benefit for tuning slow process based on the Smith-predictor with intentional parameter mismatch. A good response can be found automatically within a reasonable amount of time.

Conclusion

The theoretical results for parameter mismatch studies of Smith-predictor design are summarized to form an automatic tuning procedure to relieve the tedious work for manipulating complicated mathematical equations. The results are tested on the liquid temperature control system of Stanford's Quiet Hydraulic precision lathe, and great consistencies are observed. As the analytical solutions are derived based on the normalized parameters, they are general, useful, and can be applied to the other similar real world systems.

The study in this paper is limited to single parameter mismatch, and suitable only for systems that can be described by a simple first order model plus a long dead time. To further extend the scope of the research, studies for the combination of multiple parameter mismatches of the Smith-predictor and the derivations for higher order systems are interesting topics for further investigations.

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Table 1 Estimated model parameters of the liquid temperature control system of QH precision lathe

	Inner loop heat exchanger model	Outer loop process model
DC gain (k) (°C/volt)	0.068	0.695
time constant (a) (sec/rad)	23.0	490.6
time delay (T) (sec)	17.8	101

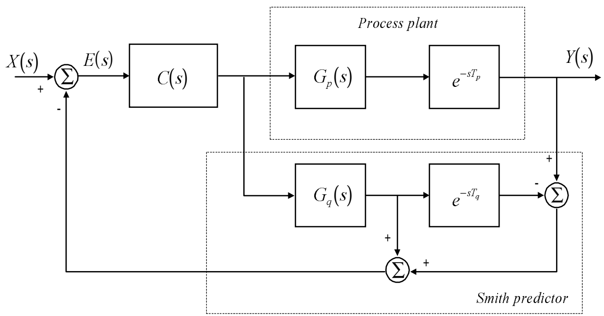

Figure 1 Control structure utilizing a Smith-predictor

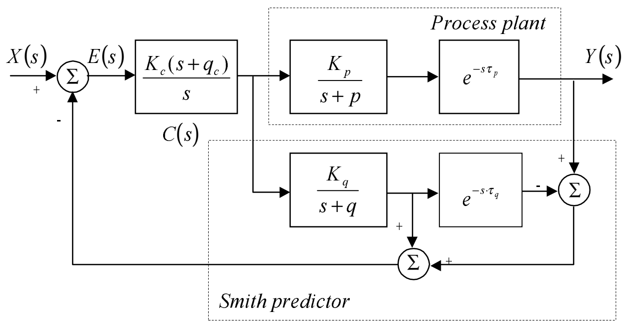

Figure 2 Smith-predictor in the format using normalized parameters

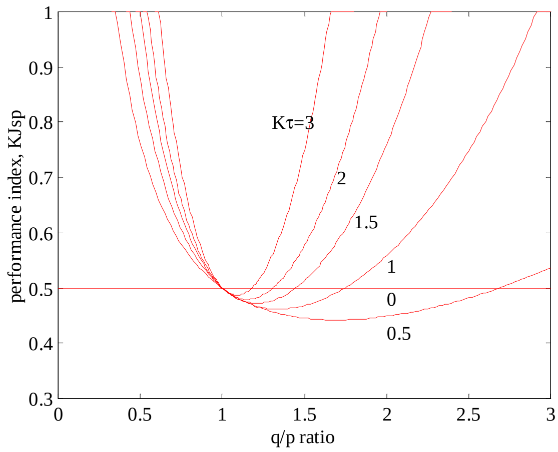

Figure 3 Normalized performance index plot for pole location mismatched case

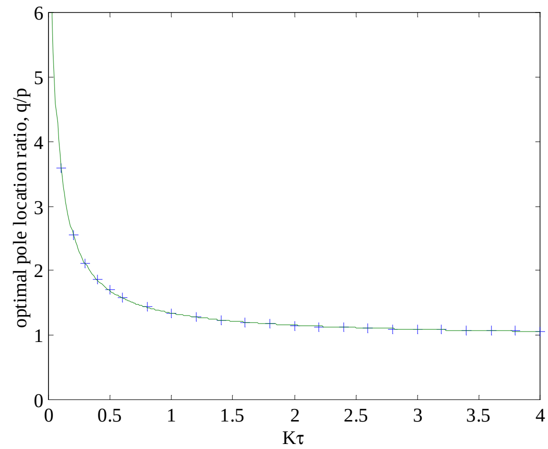

Figure 4 Optimal pole location ratios and curve-fit result for pole location mismatched case

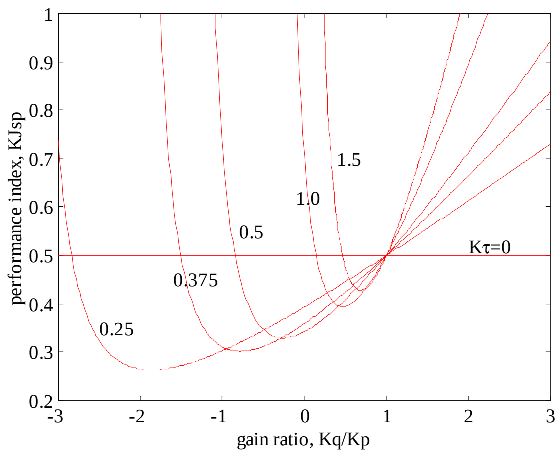

Figure 5 Normalized performance index plot for DC gain mismatched case

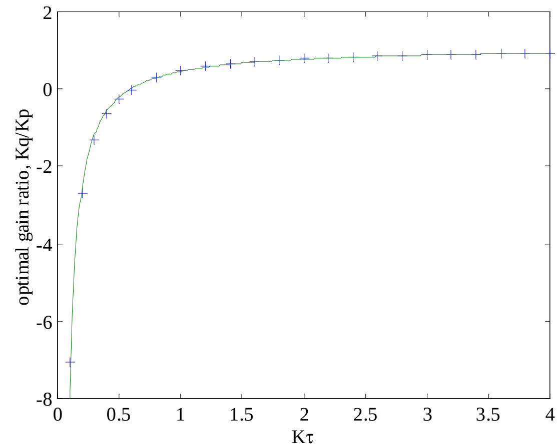

Figure 6 Optimal gain ratios and curve fitting result for DC gain mismatched case

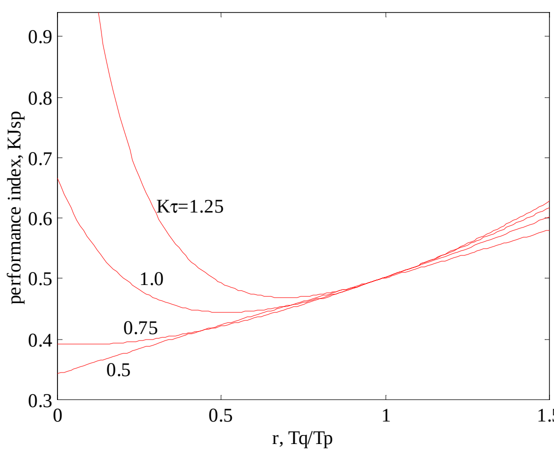

Figure 7 Normalized performance index plot for temporal mismatched case

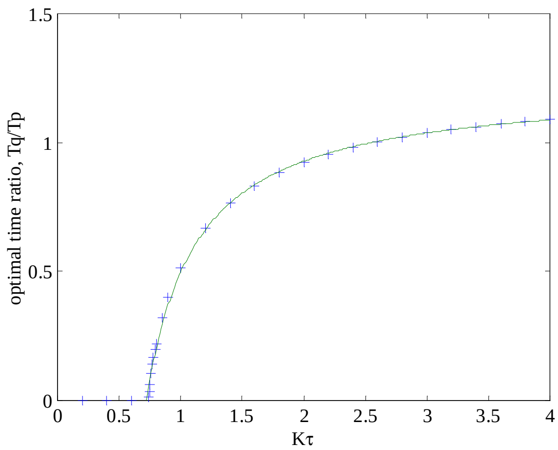

Figure 8 Optimal gain ratios and curve fitting result for temporal mismatched case

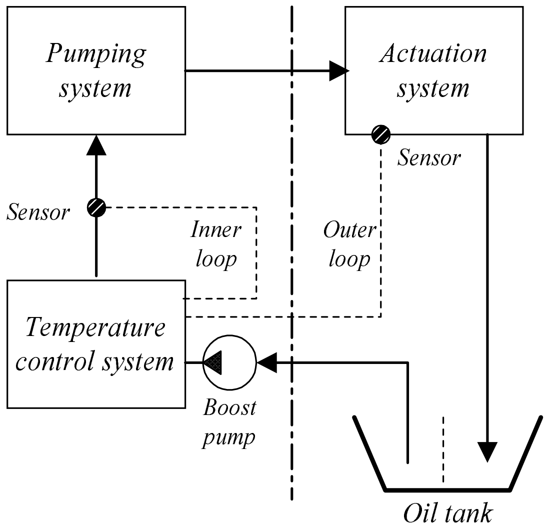

Figure 9 Simplified block diagram of the Quiet Hydraulic precision lathe

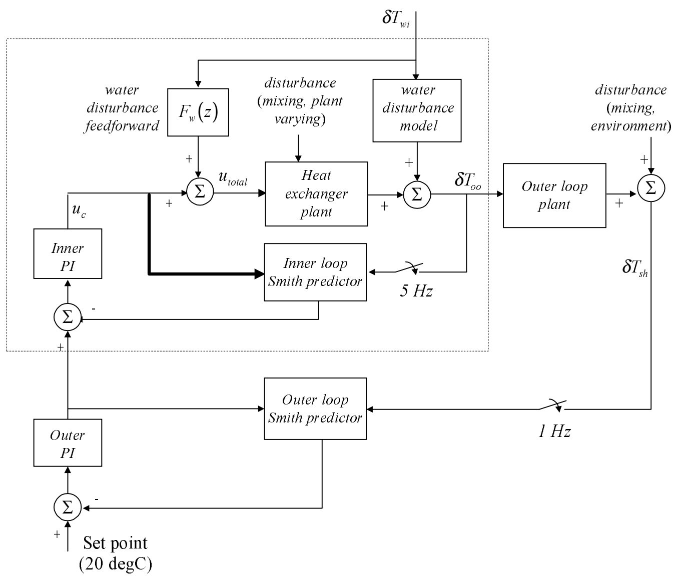

Figure 10 Block diagram of the overall implementation of oil temperature control system

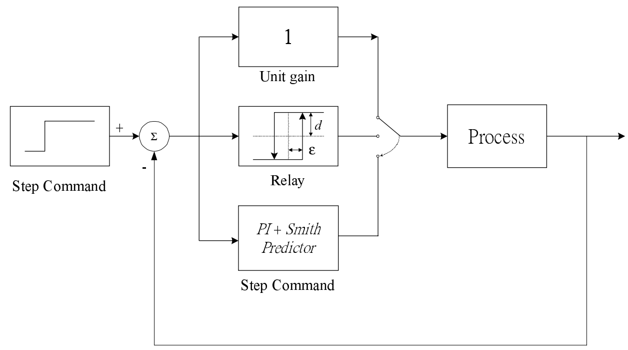

Figure 11 The block diagram representation of the automatic tuning procedure

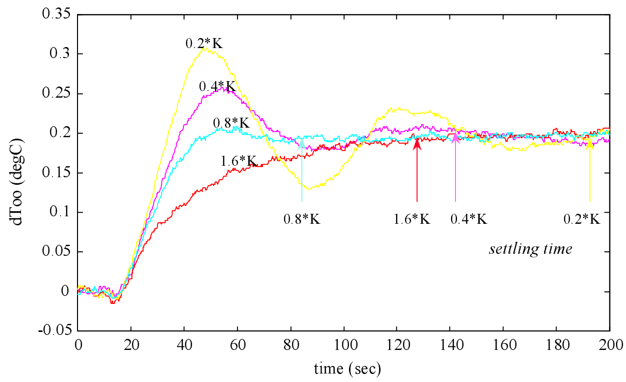

Figure 12 Time responses of the inner loop DC gain mismatched experimental results

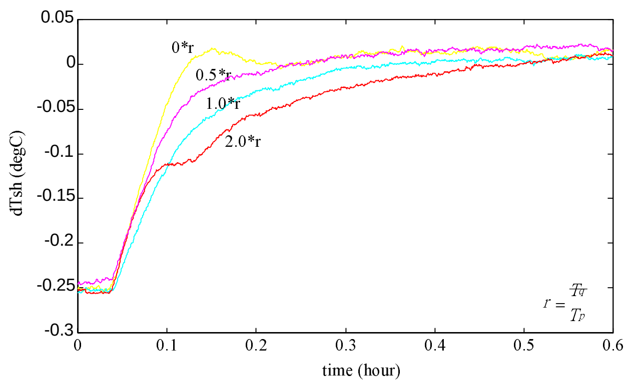

Figure 13 Time responses of the outer loop temporal mismatched experimental results

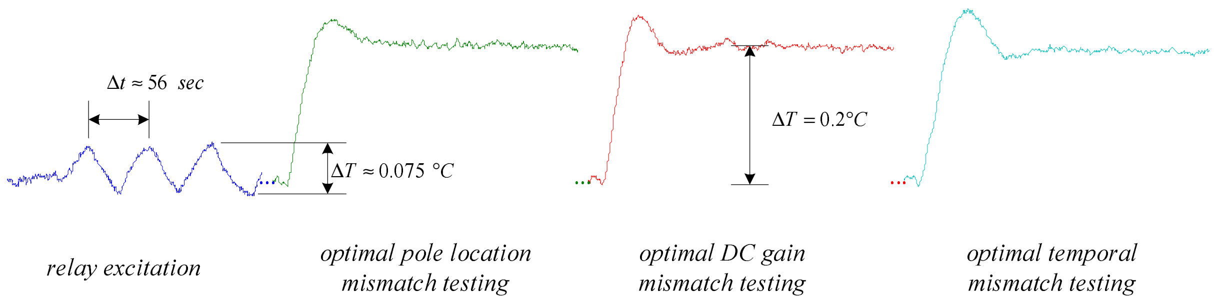

Figure 14 Time history of the automatic tuning process