Robust Broadband Control of Acoustic Duct

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Abstract

Passivity-based robust controller design methodology is presented for a broad-band control of acoustic duct. Controller design is based on finite-dimensional approximation and is shown to be robust to unmodeled dynamics and parametric uncertainties. The acoustic duct model being inherently non-passive, passification techniques are used to render the system passive. The control design methodology exploits inherent robustness of passivity-based controllers and selective mode attenuation capability of resonant mode controllers. The resulting controller is low-order, robust, broadband, and has guaranteed stability.

1 Introduction

Control of acoustic systems is a challenging problem due to computational complexity arising as a result of very high-order models, non-minimum phase behavior introduced by finite dimensional approximations, uncertainties introduced by non-uniform boundary conditions, and acoustic interactions with the dynamics of enclosure structure. The system characteristics which make controller design very difficult include: no natural roll-off at high frequencies, high modal density, and resonant peaks which dominate the dynamics. For this reason any uncertainty in the model has a significant effect on the closed-loop stability.

Until recently, most of the noise control techniques focused on feedforward cancelation. It is only in the last few years the attempts have been made to use feedback control. Some of the applications of feedback control to active noise control can be found in [1, 2]. Much work continues towards the design and analysis of feedback control for active noise control. This paper presents robust feedback control design methodology based on the ideas given in [3] - [7] along with the experimental validation.

Analytical model of an acoustic duct is linear, time-invariant, and infinite-dimensional, however, for most control applications, finite-dimensional approximation is needed. A simple method using symbolic computation is presented in [8] to derive models for virtually any configuration of an acoustic duct. A finite-dimensional approximation is obtained using assumed modes approach. A detailed discussion on the modeling and identification can be found in [8].

Next section gives a brief review of the control design techniques used in this paper.

2 Passivity-based Control

A large class of physical systems, such as flexible space structures with collocated and compatible actuators and sensors, can be classified as being naturally passive. Robust stabilization and control of such systems has received considerable attention in the literature, and a number of stability results exist in that area. The least restrictive result for linear time-invariant (LTI) systems states that the feedback interconnection of a positive-real (PR) system and a marginally strict positive-real (MSPR) system is asymptotically stable. Some nonlinear extensions of these results are also obtained in [9]. Passivity-based controllers have proved to be highly effective in robustly controlling inherently passive linear and nonlinear systems.

Most physical systems, however, are not inherently passive, and passivity-based control techniques cannot extend directly for such systems. Acoustic system is one such example. One method of making non-passive systems amenable to passivity-based control is to *passify* such systems (i.e., rendering system passive) using suitable compensation. If the compensated system is *robustly passive* despite plant uncertainties, it can be robustly stabilized by any MSPR controller. In [5] various passification techniques were presented and some numerical

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examples were given for demonstrating the use of such techniques.

2.1 Passification Methods

Four different passification methods, series, feedback, feedforward, and hybrid passification were given in [5] for finite-dimensional linear time-invariant non-passive systems. In the case of open-loop unstable systems, the first step in passification is to stabilize the system using feedback compensation and then use, if necessary, additional compensation to render the stabilized plant positive-real. For nonminimum-phase systems, the first step in passification is to render the system minimum-phase by feedforward compensation and then use additional compensation if necessary to render the resulting minimum-phase plant positive-real. Once passified, the system can be controlled by any MSPR or weakly SPR (WSPR) controller [9]. One important thing to be noted here is that, in the case of inherently passive systems, the use of an MSPR controller gurantees stability robustness to unmodeled dynamics and parametric uncertainties; however, in the case of non-passive systems which are rendered passive using passifying compensation, the stability robustness depends on robustness of passification. That is, the problem of *robust stability* is transformed into the problem of *robust passification*. In [6], a number of sufficient conditions were derived to check the robustness of passification.

The acoustic plant under consideration is a 1-D duct. Analytical model for this can be obtained using finite dimensional approximation based on assumed modes approach [8]. This model has non-minimum phase zeros introduced by quartic symmetry needed for realizing zero dynamics of the plant to match the true frequency response of the system. The system identification algorithm also yields the plant with such undesirable non-minimum phase characteristics. Obviously, such a plant model is not amenable for the passivity-based control in its original form. However, the passification techniques in [5] can be used in conjunction with control techniques in [9] to control such system.

3 Controller design

In this section, a robust control design methodology is presented which uses passivity-based design techniques discussed in Section 2.1. The open-loop system under consideration is a 46th order KSU acoustic-duct model including a speaker and a microphone. The physical dimensions of the KSU duct are: length 12ft and circular cross section of 11in diameter. The "truth" model of the plant is obtained experimentally, by system identification, using Standford Research 780 spectrum analyzer and sine-sweep excitation in the frequency band upto 1000 HZ. For control design purposes the frequency response data was used to obtain a finite dimensional model of the system which includes modes up to 500Hz. Both, discrete-time and continuous-time identification algorithms were used to identify the system. The results obtained using these two algorithms were found to be almost identical. Figure 1 shows the system identification results using the continuous-time algorithm. The finite dimensional model (nominal plant) is 22nd order. The modes above 500 Hz, which constitute the unmodeled dynamics of the plant, are considered as additive uncertainty. In addition, the plant is also assumed to have structured uncertainty due to modeling inaccuracies in the acoustic mode frequencies.

Let the open-loop system (i.e., the nominal plant model) be given by

$$P_o(s) \sim \begin{cases} \dot{x}_p(t) & = A_p x_p(t) + B_p u(t) \\ y_p(t) & = C_p x_p(t) + D_p u(t) \\ y_{\text{perf}}(t) & = C_{\text{perf}} x_p(t) + D_{\text{perf}} u(t) \end{cases}$$

where, $x_p(t)$ is $(n_p \times 1)$ state vector, $y_{\text{perf}}(t)$ is $(q \times 1)$ performance output vector, $y_p(t)$ is $(l \times 1)$ sensor output vector, and $u(t)$ is $(m \times 1)$ control input vector. For the system under consideration, it is assumed that $C_{\text{perf}} = C_p$ (i.e., $q = l$), $D_{\text{perf}} = D_p$, and the control speaker is same as the disturbance speaker. The open-loop Bode plot of the system ($P_o(s)$) is shown in Figure 2. The open-loop system is non-minimum phase, and therfore, not inherently passive. The characteristic of non-minimum phase behavior is representative of all finite-dimensional acoustic systems with non-colocated speakers and microphones. Since such a plant is not inherently passive, passivity-based control techniques cannot extend directly to this system. However, as stated in previous section, it is possible to exploit inherent stability robustness of passivity-based controllers even for non-passive systems if such a system can be rendered passive via suitable compensation. Once the system is rendered passive any WSPR [4] controller can be used to stabilize the closed-loop system. Moreover, as stated previously, if the passification is robust, stability is also robust. In [6], sufficient conditions for robust passification were given which can be used to test if the compensated system remains passive under perturbation given the upper bound on the uncertainty. Recently, in [7], LMI-based techniques were also given for robust passification. These techniques are especially useful for multi-input multi-output systems.

For the KSU acoustic duct system, the passification techniques presented in [5] were used to passify the system. The control system block diagram is shown in Figure 3. The controller design is accomplished in various steps. The first step in the design is to render the open-loop system passive via suitable passification technique. In this case, a simple feedforward constant-gain passifier ($C_{ff}(s) = D_{ff}$) was found to be adequate to render the open-loop system passive. The passified system is thus given by

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u(t) \\ y_1(t) &= C_p x_p(t) + D_1 u(t) \end{aligned} \quad (1)$$

where $D_1 = D_p + D_{ff}$ and D_{ff} is the constant-gain feedforward term. (The list of controller parameters is given in section 4). Please note that the passification is guaranteed for the truth model and not just for the nominal model. Having passified the system, as stated previously, any WSPR controller can now be designed to robustly stabilize the closed-loop system. Passification of the truth model guarantees closed-loop stability with any WSPR controller. Moreover, this stability is robust to unmodeled dynamics and parametric uncertainties as long as passivity of the system is maintained under perturbations. The parametric uncertainty considered here is the perturbations in all acoustic mode frequencies in the design model caused due to the change in the length of the acoustic duct. This uncertainty is realized in experimental set up by extending the telescopic duct by 1ft. It is to be noted that the perturbations occur simultaneously in all frequencies. The perturbed plant is checked again for passivity. If the perturbed plant is found to violate passivity, the passifying compensator is redesigned to ensure the robust passivity. In the present case, the perturbed plant was found to remain passive with the feedforward passifier designed for the nominal plant. Having ensured the passivity of the perturbed model, next step was to design a WSPR controller to meet the desired performance.

In the second step of the design, a series compensator is first designed which essentially consists of a parallel combinations of resonant mode controllers [3]. Each of these resonant mode controllers is a second order controller designed specifically to suppress a particular resonant mode of the system. The series compensator is given by

$$C(s) \sim \begin{cases} \dot{x}_s(t) &= A_s x_s(t) + B_s y_1(t) \\ y_2(t) &= C_s x_s(t) + D_s + y_1(t) \end{cases} \quad (2)$$

where the controller matrices $A_s,\ B_s,\ C_s,\ D_s$ have the form

$$\begin{aligned} A_s &= [\text{diag}\{A_i\}] \quad (i = 1, \cdots, r), \quad B_s = [B_1, \cdots, B_r]^T \\ C_s &= [C_1, \cdots, C_r], \quad D_s = D_1 + \cdots + D_r \end{aligned} \quad (3)$$

As seen in Equation (3), the series compensator is simply the combination of r resonant-mode controllers ($A_i,\ B_i,\ C_i,\ D_i,\ i = 1,\ \cdots,\ r$) in parallel. The zeros of such controller are designed to ensure that the combination of the plant and series compensator remains passive in the presence of structured as well as unstructured plant uncertainty. A combined system comprising passified plant and resonant compensator is given by

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}u(t) \\ y_2(t) &= \tilde{C}\tilde{x} + \tilde{D}u(t) \end{aligned} \quad (4)$$

where

$$\begin{aligned} \tilde{x} &= \begin{Bmatrix} x_p(t) \\ x_s(t) \end{Bmatrix}, \quad \tilde{A} = \begin{bmatrix} A_p & 0 \\ B_s C_p & A_s \end{bmatrix} \\ \tilde{B} &= [B_p \quad B_s D_1]^T, \ \tilde{C} = [D_s C_p \ C_s], \ \tilde{D} = D_s D_1 \end{aligned} \quad (5)$$

The final step of the design is to choose a feedback controller $C_{fb}(s)$ such that it satifies WSPR conditions [4]. Let the controller equations be given by

$$C_{fb}(s) \sim \begin{cases} \dot{x}_{fb}(t) &= A_{fb} x_{fb}(t) + B_{fb} y_2(t) \\ y_{fb}(t) &= C_{fb} x_{fb}(t) + D_{fb} y_2(t) \\ u(t) &= u_{\text{ref}}(t) - y_{fb}(t) \end{cases}$$

Then the closed-loop system becomes

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}y_{fb}(t) + \hat{B}u_{\text{ref}}(t) \\ y_2(t) &= \hat{C}\hat{x}(t) + \hat{D}u_{\text{ref}}(t) \\ y_{\text{perf}} &= C_{\text{perf}}\hat{x}(t) + \overline{D}_{\text{perf}} u_{\text{ref}} \end{aligned} \quad (6)$$

where

$$\begin{aligned} \hat{x} &= \begin{Bmatrix} \tilde{x} \\ x_{fb} \end{Bmatrix}, \ \hat{A} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} \\ \hat{B} &= \begin{bmatrix} \hat{B}_{11} & \hat{B}_{12} \end{bmatrix}^T, \ \hat{C} = \begin{bmatrix} \hat{C}_{11} & \hat{C}_{12} \end{bmatrix}, \ \hat{D} = \tilde{D}\overline{D}^{-1}, \\ C_{\text{perf}} &= \begin{bmatrix} C_{\text{perf}11} & C_{\text{perf}12} \end{bmatrix}, \ \overline{D}_{\text{perf}} = D_{\text{perf}}\overline{D}^{-1} \end{aligned} \quad (7)$$

and various terms in Eq. 7 are given by: $\hat{A}_{11} = \tilde{A} + \tilde{B}\overline{D}^{-1}D_{fb}\tilde{C}$, $\hat{A}_{12} = -\tilde{B}\overline{D}^{-1}C_{fb}$, $\hat{A}_{21} = B_{fb}(\tilde{C} + \tilde{D}\overline{D}^{-1}D_{fb}\tilde{C}$, $\hat{A}_{22} = A_{fb} - B_{fb}\tilde{D}\overline{D}^{-1}C_{fb}$, $\hat{B}_{11} = \tilde{B}\overline{D}^{-1}$, $\hat{B}_{12} = B_{fb}\tilde{D}\overline{D}^{-1}$ $\hat{C}_{11} = \tilde{C} + \tilde{D}\overline{D}^{-1}D_{fb}$, $\hat{C}_{12} = -\tilde{D}\overline{D}^{-1}C_{fb}$, $C_{\text{perf}11} = \tilde{C}_p + D_{\text{perf}}\overline{D}^{-1}D_{fb}\tilde{C}$, $C_{\text{perf}12} = -D_{\text{perf}}\overline{D}^{-1}C_{fb}$, $\overline{D} = (I + D_{fb}\tilde{D})$, and $\tilde{C}_p = \begin{bmatrix} C_p & 0_{l\times n_s} \end{bmatrix}$. The comparison of the open- and closed-loop bode plots for the design model is given in Fig. 4.

In order to test the controller under simultaneous perturbations in structured and unstructured parameters the controller was designed assuming 10% - 20% perturbation for the frequencies in the design model in addition to the unmodeled uncertainty which included modes above 500Hz. For validation purposes, the actual plant perturbations were obtained in experiments by increasing the acoustic duct length by 1 foot, as mentioned previously.

The overall controller, which is a combination of feedforward passifier, resonant compensator, and feedback compensator, is given by

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_{k1} y_p + B_{k2} u_{\text{ref}} \\ u(t) &= C_k x_k + D_{k1} y_p + D_{k2} u_{\text{ref}} \end{aligned} \quad (8)$$

If $(A_{fb},\ B_{fb},\ C_{fb},\ D_{fb})$ satisfy WSPR conditions, $(A_k,\ B_k,\ C_k,\ D_k)$ asymptotically stabilizes the plant. Moreover, the stability is robust to unmodeled dynamics and parametric uncertainties as long as $(\tilde{A},\ \tilde{B},\ \tilde{C},\ \tilde{D})$ remains passive under perturbed plant conditions. For the KSU duct problem, the feedback controller used is the simplest SPR controller, namely a constant-gain controller, i.e., $C_{fb}(s) = D_{fb}$. The important thing to be noted is that the overall controller, which is a combination of feedforward passifier, resonant compensator, and feedback compensator, is a low-order, output-feedback controller and therefore easy to implement. Moreover, the feedback gain of the controller D_{fb} can be increased as high as necessary without any concern for the closed-loop instability. This is a great advantage the proposed controller has over many other controllers presented in the literature which are prone to instability for higher values of gains.

4 Experimental validation

The controller design approach described in Section 3 was validated experimentally using KSU acoustic duct facility. A schematic of experimental set-up is given in Fig. 5. As shown in Fig. 5, the acoustic duct system with speaker A as actuator and the microphone located in the middle as the sensor is not inherently passive. However, it is rendered passive by a combination of feedforward controller $C_{ff}(s)$ and series compensator $C_s(s)$. This passified plant is shown in the dotted box in Fig. 5. The feedback controller is simply the constant-gain controller which satisfies SPR conditions. For the first set of experiments speaker B was kept inactive and the closed-loop system was tested with speaker A excited with sine sweep signal. In the second set of experiments it was assumed that the external disturbance $N(s)$ (generated by speaker B) can be sensed and was used for feedback. The controller parameters selected for the feedforward compensator ($C_{ff}(s)$), resonant controller ($C_s(s)$), and feedback controller (C_{fb}), are given below.

$$\begin{aligned} K_1(s) &= \frac{2.5 * (s/800+1)(s/850+1)}{s^2/863.94^2 + .05s/863.94 + 1} \\ K_2(s) &= \frac{1.5 * (s/1000+1)(s/1200+1)}{s^2/1110.1^2 + .03s/1110.1 + 1} \\ K_3(s) &= \frac{(s/1375+1)(s/1500+1)}{s^2/1410.3^2 + .03s/1410.3 + 1} \\ K_4(s) &= \frac{(s/1665+1)(s/1760+1)}{s^2/1685^2 + .05s/1685 + 1} \\ K_5(s) &= \frac{(s/1875+1)(s/2050+1)}{s^2/1998^2 + .03s/1998 + 1} \\ K_6(s) &= \frac{(s/1885+1)(s/2356+1)}{s^2/2295.2^2 + .03s/2295.2 + 1} \\ C_{fb}(s) &= 0.1; \quad C_{ff}(s) \ = \ 2.0 \end{aligned} \quad (9)$$

Each of the $K_i(s)$ compensators is a second-order controller which is a cascade of a lead block and a resonant controller. Resonant-mode controllers have a resonant mode frequency exactly at one of resonant modes of the system. This is the resonant mode which is intended to be damped. The experimental closed-loop frequency response of the system is given in Fig. 6. As seen form the figure, a reduction of up to 15dB is obtained in the freuquency range of interest. Also, the uncontrolled modes are not destabilized. The resonant mode controller with a lead compensator was designed to maintain the closed loop stability in the presence of modeling errors and parametric uncertainty. The parametric uncertainty was introduced in the system by changing the length of the duct as described previously. Please note that, under such plant perturbations, the resonant-mode controller poles do not match with the plant poles. However, this does not pose any problem for the closed-loop system stability since the passivity of the plant (truth model) under such perturbations is ensured by proper selection of the lead factors in the resonant controller. The worst effect that can be caused under such perturbations is the deterioration in the performance. Figure 7 shows the experimental closed-loop Bode plot of the system under such perturbation. As seen from this figure, although the performance has deteriorated compared to the nominal case, the reduction in the peaks is still satisfactory. Moreover, these plant perturbation do not cause controller to destabilize any uncontrolled modes. For the case when speaker B is used for persistent sinusoidal disturbance the experimental closed-loop response for the nominal case was obtained as shown in Figure 8. As seen from the figure, the performance for this case was slightly inferior to the case when speaker A was used for disturbance input. Nevertheless, in both cases, the noise level reduction was very satisfactory.

5 Conclusions

Robust broadband control of acoustic noise in a 1-D duct configuration using passivity-based controllers was presented. The acoustic duct system was rendered passive using passification techniques and it was ensured that the passification was robust to modeling errors and parametric uncertainties. The robust passification was used to ensure the closed-loop stability under plant perturbations. Simulation as well as experimental results were presented which show the effectiveness of such controllers.

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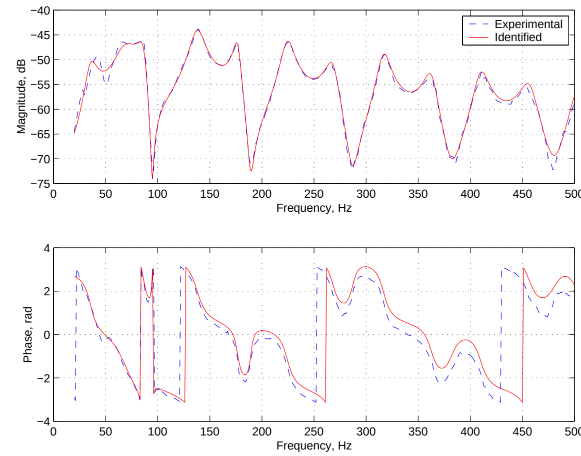

Figure 1: Experimental Vs identified model.

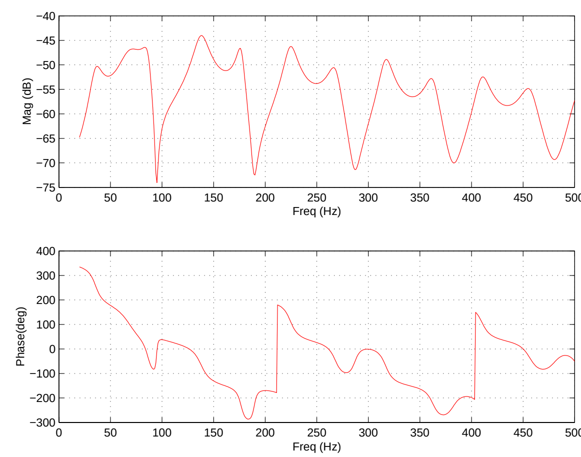

Figure 2: Open Loop Bode Plot

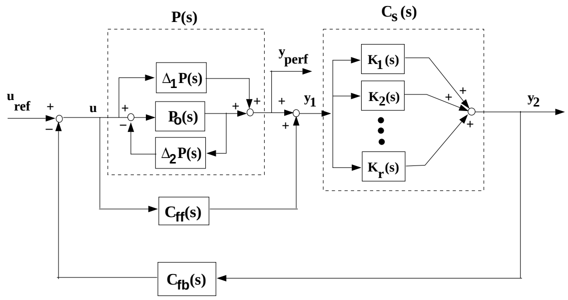

Figure 3: Control Scheme Block Diagram

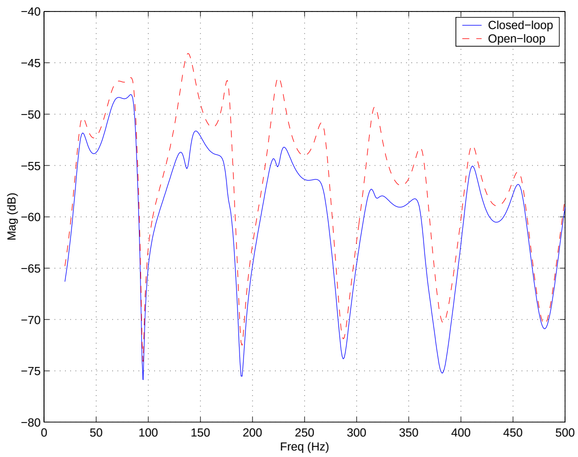

Figure 4: Simulated closed-loop frequency response for nominal case

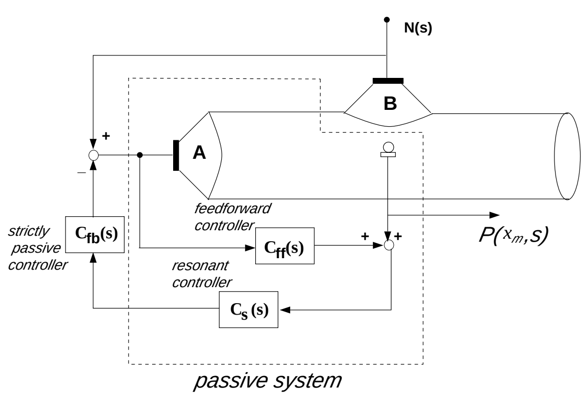

Figure 5: Schematic of experimental set-up

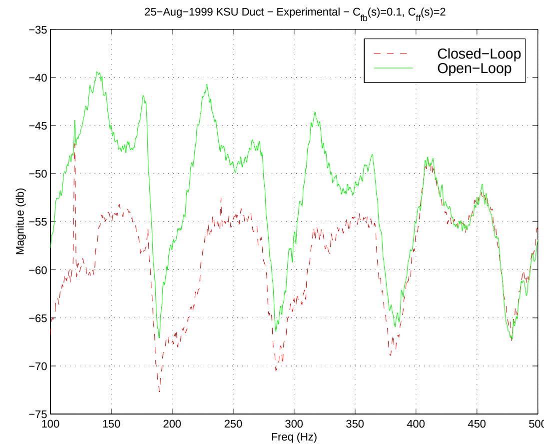

Figure 6: Experimental Bode plot of closed-loop system with nominal model

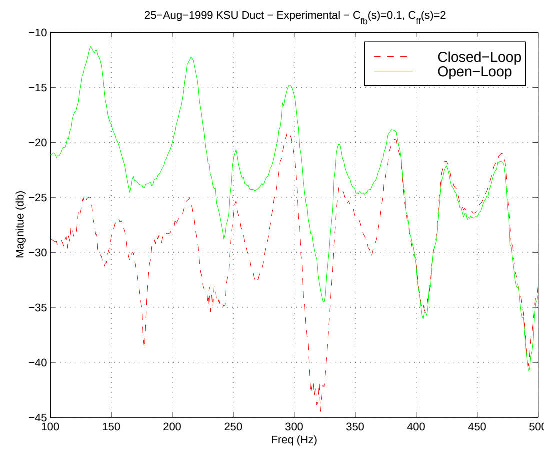

Figure 7: Experimental Bode plot of closed-loop system with perturbed model

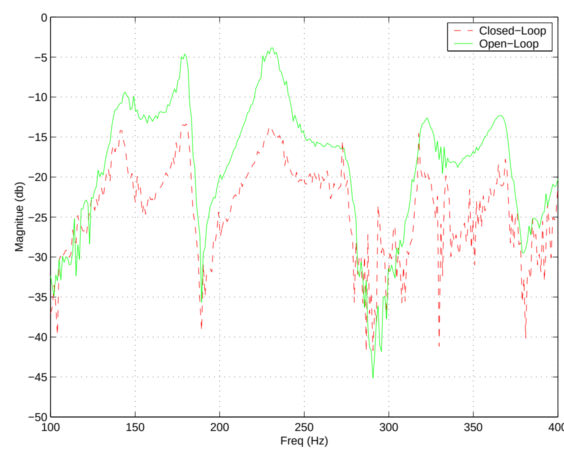

Figure 8: Experimental closed-loop response with persistent disturbance