Identification of Averaged Dynamics of a Controlled Combustion Instability[1]

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Abstract

Attenuation of thermoacoustic oscillations by active control has lately received considerable attention. In the present work, we identify the averaged dynamics or the dynamics of the amplitude of a controlled thermoacoustic instability. The identification is performed on a lean premixed combustor which has feedback control incorporated to attenuate the amplitude of the thermoacoustic oscillation. For the purpose of control, acoustic pressure in the combustor is sensed and fuel flow actuated by a phase-shifted version of the measured pressure signal. This control strategy, used by several authors in the literature, is motivated by the famous Rayleigh criterion for growth/attenuation of thermoacoustic oscillations. The averaged model we identify consists of a map of equilibrium amplitudes parametrized by the control phase shift, to which all the amplitudes settle exponentially fast from all initial conditions. An equilibrium map is obtained by fitting a curve through the equilibria found from steady state experiments, and decay rates at these equilibria are estimated from cumulative averaging of several step responses (the steps are in the controller parameter) to eliminate noise. The identified closed-loop model can be used to optimize feedback and for simulation studies.

1 Introduction

It is recognized that thermoacoustic instabilities pose several problems to the achievement of lean-burn low NO_x emission combustion. These problems include high-cycle fatigue, increased wall heat-transfer and possibilities of flame reversals [6, 12, 13]. The acuteness of the problem in the specific case of lean *premixed* combustion was observed in [5, 19]. That a good design can only ensure adequate performance at a single operating point only, is well recognized [13]. Further, passive methods sacrifice efficiency [7, 13]. Also, *ad hoc* design modifications to alter the fluid dynamic characteristics of the combustor are costly and time-consuming [14]. Thus, the dynamic nature of the problem presents an effective case for active control. The application of well-known control techniques requires sufficiently accurate reduced order models of the complex processes involved, capturing the essential dynamics. Though there exist several efforts in this direction, and even in the specific case of lean premixed combustion [17] which is dealt with here, there is no model yet, which inspires a confidence sufficient for its general acceptance. What is, however known from experimental efforts in several conditions is that the oscillation is dominated by a single frequency (for example, see Fig. 1),

[1] This work was supported by grants from AFOSR (to UTRC and UCSD), ONR, and NSF

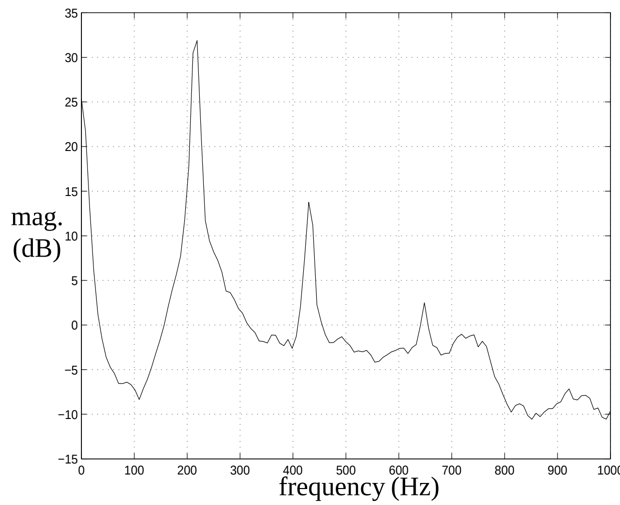

Figure 1: Typical power spectral density of acoustic pressure

$$p = p_0 + A\sin(\Omega t) + \text{h.o.t}, \qquad (1)$$

where p denotes the pressure, p_0 the static pressure, and $A\sin(\Omega t)$ denotes the dominant mode of oscillation. The generally adopted qualitative understanding of the instability is through the Rayleigh criterion [18], which states essentially that the oscillations grow/decay depending on whether the heat release and the acoustics are in/out of phase. Several control strategies have been applied to suppress thermoacoustic instabilities, some model-based (such as in [8, 11, 15, 20]), and some empirical (such as in [5, 12, 13, 14, 21]). All the control strategies sense the pressure fluctuation in the combustor, and feed it back through actuation by loudspeaker, by an auxiliary fuel source, or by modulating the main fuel supply. Most of them have been applied to the

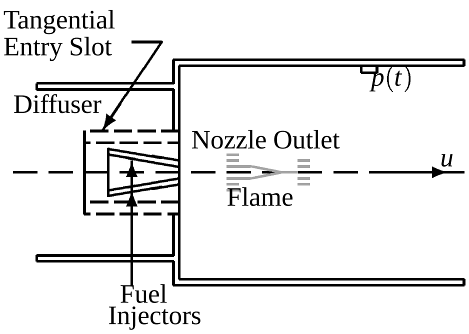

Figure 2: Schematic of a single nozzle combustor

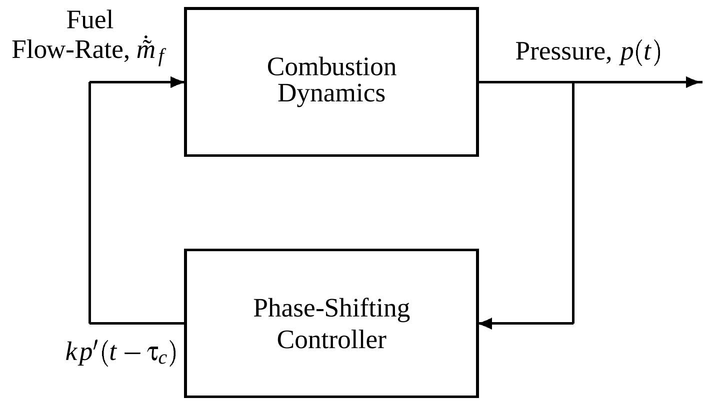

Figure 3: Phase shifting controller for thermoacoustic instabilities

case where the acoustics are dominated by a single harmonic, and the control objective is then to reduce the amplitude A. Many of the controls, that actuate the fuel flow with time-delayed/phase-shifted measured pressure fluctuation (such as in [2, 5, 13, 14, 16, 21] and in the present work) have been motivated by the Rayleigh criterion.

Section 2 describes the experimental setup of a combustor with a feedback controller to attenuate A, already reported in [2]. Section 3 presents our work on identification.

2 Feedback Control in an Experimental Combustor

Our work builds upon the experimental setup and a control design reported in [2]. Figure 2 shows a simplified schematic of a single nozzle combustor which has been fabricated by United Technologies Research Center (UTRC) for investigating combustion problems. Researchers at UTRC have investigated emissions and active attenuation of thermoacoustic oscillations on the combustor [2, 3]. The combustor is a cylindrical, single-chamber, gas-fueled, premixed combustor with an output of around 500 kW at full power. The fuel used was natural gas. The fuel and air are premixed in the tangential entry nozzle, and additional air enters from around the nozzle. The swirl in the fluid due to tangential entry helps to stabilize the flame. For the purpose of control, sensing is through the acoustic pressure $p(t)$ in the chamber, and actuation through the fuel-injectors. A static pressure sensor measures the mean pressure in the chamber, and the acoustic oscillation about the mean is measured by a piezo-electric sensor. Up to a third of the total fuel flow-rate through the fuel injectors in the nozzle is utilized for the purpose of control. The static pressure in the setup is in the vicinity of 200 psi, and the acoustic oscillations have an amplitude of up to 20 psi. In the combustion system analyzed here, a phase-shifting controller motivated by Rayleigh's criterion reported in [2] has already been incorporated to attenuate the amplitude A of the dominant harmonic. This controller can be approximated by a simple delay when the oscillation is completely dominated by the fundamental harmonic, as shown in Fig. 3. Otherwise, an online frequency-tracker (extended Kalman filter) is used to track the dominant harmonic, which is then phase-shifted and fed back with appropriate gain through the fuel-injectors,

$$\dot{m}_f = kA\sin(\Omega t - \theta), \tag{2}$$

where $\dot{m}_f$ is the actuated/controlled fuel mass flow-rate, k is the gain in the feedback loop, and θ is the phase shift used. Experiments show that a stable oscillation is maintained for all $\theta \in [0, 2\pi]$, with amplitude that depends on θ. The use of an online frequency tracker eliminates the limitation faced by feedback using simple band-pass filtering and a time delay when there are significant changes in the fundamental frequency of the system [21]. In experiments carried out on the UTRC rig, the closed loop frequency varied from $150 - 250$ Hz, and sampling of signals was done at 2 kHz.

In the controller (2) one needs to tune the phase shift θ to ensure the minimum possible oscillation amplitude A. This online optimization can be performed by extremum seeking (presented, for example, in [9]).

3 Identification

Since the system shows a clear separation of time scales between the frequency of the oscillations Ω and the variation in A caused by variation in control parameter θ, we try to identify only the dynamics of the amplitude, i.e., the averaged dynamics. A local model structure for the averaged dynamics is

$$\dot{A} = -\alpha(\theta)\,(A - g(\theta)) \tag{3}$$

$$y = A + w, \tag{4}$$

where $\alpha(\theta) \in \mathbf{IR}^+$ are the time constants of the exponential relaxation processes at the equilibria $A = g(\theta)$, y is the measured output, and w is noise. Henceforth, we identify respectively the equilibria $g(\theta)$, and the decay rates $\alpha(\theta)$ at those equilibria.

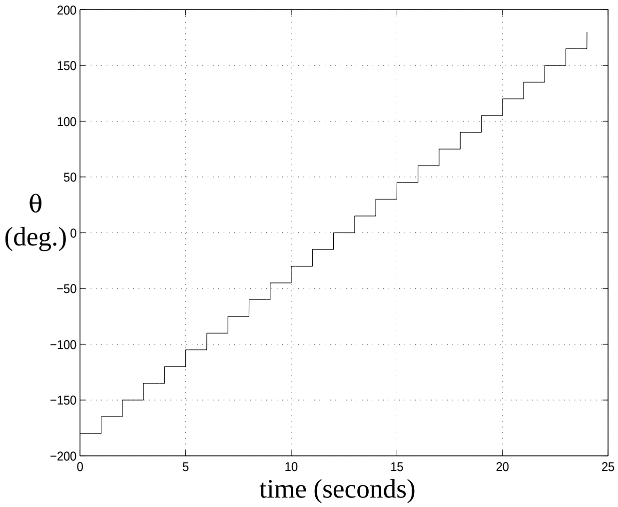

Figure 4: Phase ramp input for identifying the equilibrium map

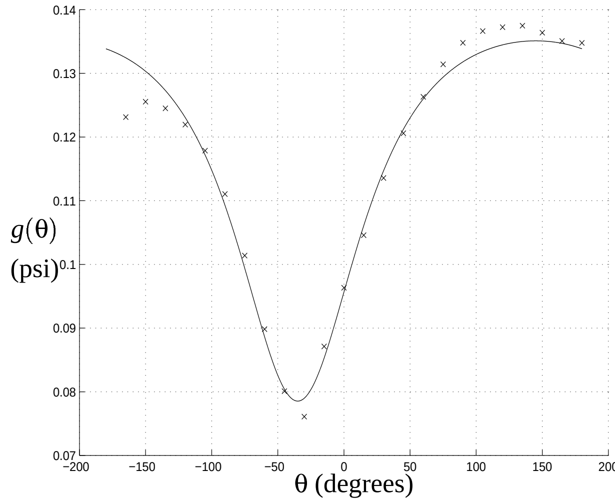

Figure 5: Static map from control phase θ to oscillation amplitude A

3.1 Equilibrium Map ID

An experimental equilibrium map is obtained by varying θ in a phase ramp/staircase of discrete steps from 0° to 360° while running the combustor at 80% full power. In Fig. 4, steps of 15° are used,

$$\theta(t) = \theta_{st}[t],$$

where $[t]$ denotes the greatest integer less than time t, and $\theta_{st} = 15°$ is the discrete increment in θ in each step. The duration of steps is sufficiently long (1 sec) to allow for the transients in oscillation amplitude to settle down. The resulting amplitude values are shown in Fig. 5 in crosses. The amplitude data are obtained from the experimental pressure data by band-pass filtering and amplitude detection. The steady-state values are estimated either by low-pass filtering or by averaging the time-series amplitude data after the system settles. From experimental data in Fig. 5, there does seem to be a smooth variation of the steady-state amplitudes of thermoacoustic instability with θ along a single curve, and there is a definite minimum oscillation amplitude at a certain phase. It can be noted that the ratio of maximum to minimum amplitudes in the static map is about 1.5. Larger ratios were obtained in experiments at other power levels.

A parametrization for the static map, motivated in part by previous work on averaging of a model developed at UTRC [17] by Banaszuk [1], and in part by the experimental static map itself, is:

$$g(\theta) = \Gamma \left\{ \frac{1 + L\sin(\theta + \phi)}{1 + M\sin(\theta + \phi)} \right\}, \qquad (5)$$

where θ is the phase of the phase shifting controller, and $g(\theta)$ is the steady-state oscillation amplitude. The form of the numerator is motivated by model analysis, and the denominator by data, to capture the map in a minimum number of parameters. The phase ϕ is set to be the same in the numerator and denominator, on observation of symmetry in the data. Thus, we get a parametrization with only four parameters.

The parameters are obtained by fitting the parametrization to the experimental data by a least squares fit. Since the parametrization is nonlinear, nonlinear least squares optimization by the Gauss-Newton or Levenberg-Marquandt method is done in MATLAB. Since these techniques only ensure local convergence, a good initial condition needs to be chosen for the optimization routine. It is easy in the present situation to observe the experimental static map, and guess a good initial condition. The parameters obtained as a result, for example $\Gamma = 0.1246$, $L = .7659$, $M = 0.6286$, $\phi = -0.9614$ in the fit (continuous curve) in Fig. 5, can be used for simulation studies.

3.2 Decay Rates ID

Experimentally, the amplitude transients are obtained by performing a large square wave variation in the control phase of the phase shifting controller as in Fig. 6. The duration of the pulses is long enough (2 sec) to allow settling of the transients of each step, and the upper and lower values of θ in the pulses are chosen so as to ensure an observable difference in equilibrium amplitude $g(\theta)$.

To eliminate noise in the transient measurements, a cumulative averaging of the various step responses in a given square wave response is performed. In this instance, taking an average is the best that we can do since we have no *a priori* knowledge of the noise. Time traces of transient responses are averaged cumulatively to obtain the N^{th} estimate of the

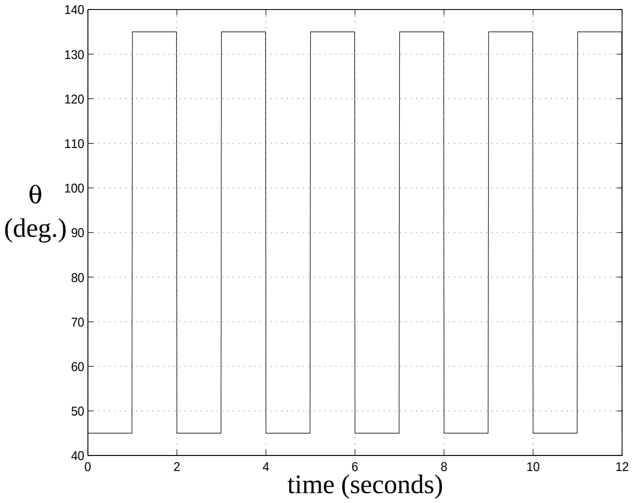

Figure 6: Large square wave variation in control phase θ

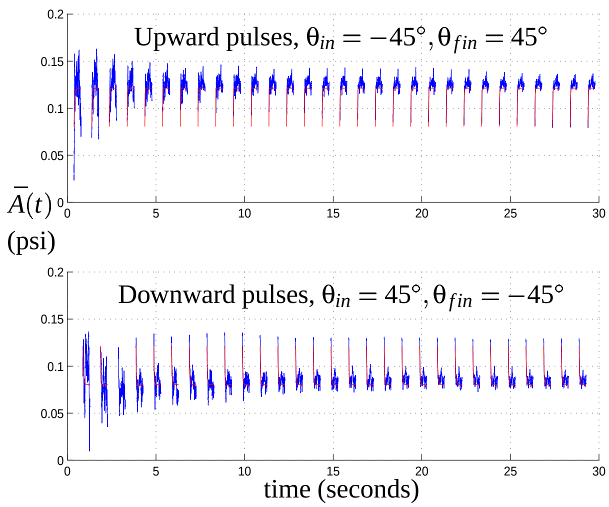

Figure 7: Cumulative average of the pressure amplitude

amplitude time-trace:

$$\bar{A}(t) = \frac{1}{[t/T]+1} \sum_{i=0}^{[t/T]} y(t - iT), \tag{6}$$

where $y(t)$ is the measurement of amplitude as in the previous section, $T = 2$ sec is the time period of the pulses, and $[t/T]$ denotes the greatest integer less than t/T.

The transients due to the upward and downward steps are averaged separately, since they represent transients at different equilibria. The process is shown graphically in Figures 7 and 8, in the top half of the figures for the upward steps, and the lower half of the figures for the downward steps. In the successive cumulative averages, as an additional step response is added into the cumulative total, and a new average taken, there seems to be a clear convergence to a linear exponentially stable process.

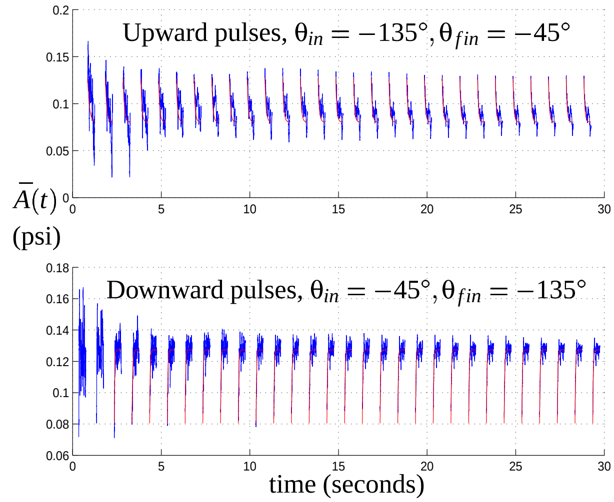

Figure 8: Cumulative average of the pressure amplitude

It can be mentioned here that the purpose of obtaining the transients is two-fold. Firstly, to verify that the equilibria are indeed locally exponentially stable (LES), satisfying the condition implicit in the model structure in Eqn. (3), and secondly, to identify the corresponding time constants. Hence, from the successive cumulative averages in the graph, we can find out if the equilibria are LES. The corresponding final averages are shown in Figures 9 and 10 respectively. In the figures, the smooth curves of the exponential fits are superimposed over the rough curves from data. The assumptions implicit are

1. The noise is uncorrelated in time.
2. The noise, or unmodeled dynamics in the data has zero mean.
3. The transients settle within τ seconds after the step instant. This time interval is obtained from observation of experimental data.

In the case where we can measure the amplitude perfectly without noise, direct integration of Eqn. (3) yields $\alpha(\theta_{fin})$ as

$$\alpha(\theta_{fin}) = \frac{A(t) - A(t+s)}{\int_t^{t+s} A(\sigma)d\sigma - sg(\theta_{fin})} \tag{7}$$

$$\forall t \quad \text{s.t.} \left[\frac{2t}{T}\right] = \left[\frac{2(t+s)}{T}\right], \quad \text{and} \quad \forall s < T/2, \tag{8}$$

where θ_{fin} is the final phase of the phase step. This integration to identify the exponent is identical to the method of modulating functions [4], with a modulating function of unity. However, since we do not have noise-free data, we estimate the exponent from the N^{th} cumulative average of the measured transients (Eqn. (6)) as follows:

$$\hat{\alpha}_N(\theta_{fin}) = \frac{\bar{A}(NT) - \bar{A}(NT+\tau)}{\int_{NT}^{NT+\tau} \bar{A}(\sigma)d\sigma - \tau g(\theta_{fin})}, \tag{9}$$

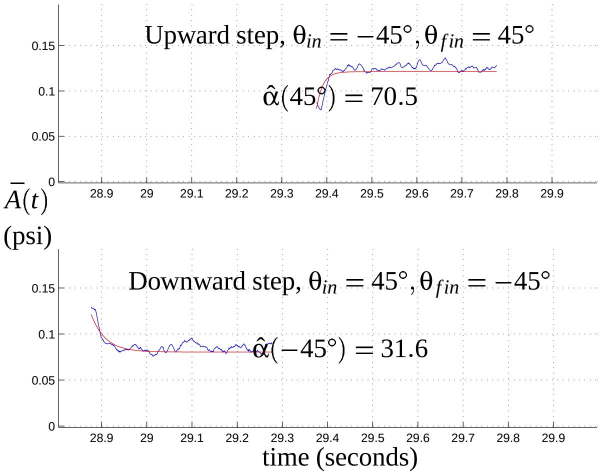

Figure 9: Estimate of the decay rate

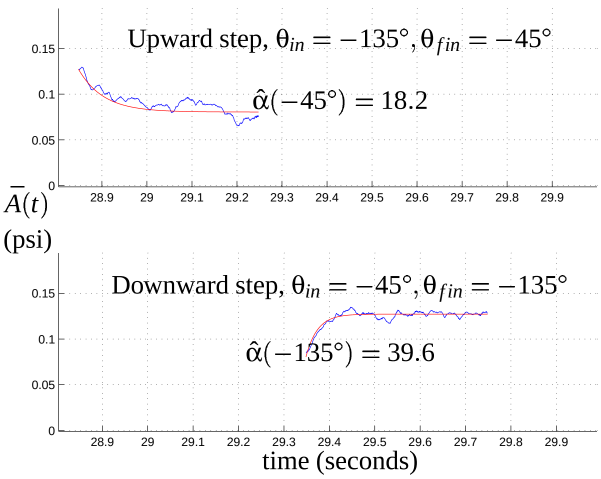

Figure 10: Estimate of the decay rate

Here, we approximate $\bar{A}(NT) \approx g(\theta_{in})$, and $\bar{A}(NT+\tau) \approx g(\theta_{fin})$, where θ_{in} is the starting phase of the phase step, NT is a step time instant, and τ mentioned in the third assumption above is such that the transients settle within it. The exponents $\hat{\alpha}(\theta)$ (θ in degrees) thus calculated are indicated on Figures 9 and 10. Clearly, there is a discrepancy between the values of $\alpha(-45°)$ in the two figures. This can be explained by the presence of noise, and the problem of the fit in Eqn. (5) not being perfect at all points.

Steady state transfer functions $\frac{\alpha(\theta_0)g'(\theta_0)}{(s+\alpha(\theta_0))}$ from control phase to oscillation amplitude were also obtained by small sinusoidal variation in control phase about various control phases θ_0, and they gave values of $\alpha(\theta)$ consistent with those from transient data. But, this method being more sensitive to noise was used only as a consistency check.

The assumption that the noise has zero mean is not restrictive, since the mean only shifts the equilibrium map by a constant value, and does not affect the differences between the estimates of the equilibria.

4 Conclusions

In the process of identifying the equilibrium amplitudes $g(\theta)$ and the time constants $\alpha(\theta)$ at various values of the controller parameter θ, we find that the controlled combustion process has a unique smooth equilibrium map, with locally exponentially stable equilibria, and with a minimum. The identification of averaged dynamics does not impose any assumptions upon the noise other than that it is uncorrelated in time. The identification of local dynamics of the closed loop combustion system provides us the means to optimize the feedback to reduce oscillation amplitude. Further, the identified model can be used in simulation studies.

Acknowledgements

We would like to thank Clas Jacobson and Gonzalo Rey for fruitful discussions at UTRC.

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